1- Calculating Slope

November 28, 2018 1:43 PM

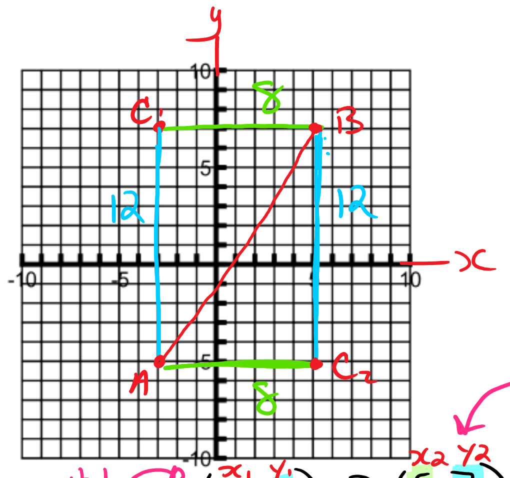
1) calculating slope

One of the most important properties of straight lines is their angle from the horizontal (ie. steepness).
The mathematical term for steepness is "slope".

(x, y)

Investigation #1:

1) On the coordinate grid below, graph the points $A(-3,-5)$ and $B(5,7)$ and join them to form a line segment.



2) Create a right angle triangle where AB is the hypotenuse and C is the third vertex.

3) Count the squares from A to C and B to C and record your numbers.

delta "change in" =

Vertical change (sometimes called rise): $\Delta y = 12$

Horizontal change (sometimes called run): $\Delta x =$

#1 $A(\overset{x_1}{-3}, \overset{y_1}{-5})$ $B(\overset{x_2}{5}, \overset{y_2}{7})$ #2

A vertical change represents the change in y-values of your coordinate points. This is represented by $\boldsymbol{\Delta y}$ and is calculated by subtracting the two y-values of the coordinates, $\boldsymbol{y_2 - y_1}$. $= \Delta y = 7-(-5) = 12$

A horizontal change represents the change in x-values of your coordinate points. This is represented by $\boldsymbol{\Delta x}$ and is calculated by subtracting the two x-values of the coordinates, $\boldsymbol{x_2 - x_1}$. $= \Delta x = 5-(-3) = 8$

4) Calculate Δy and Δx for the line segment AB.

* NOTE * (x_1, y_1) and (x_2, y_2) coordinates

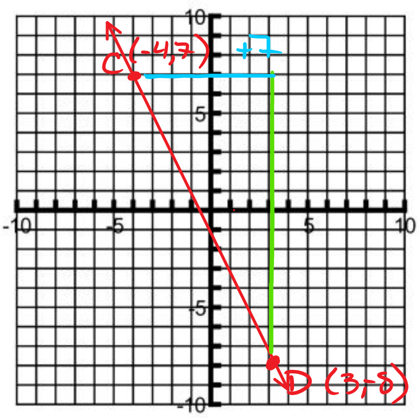
$$\left.\begin{aligned}\Delta y &= \boxed{y_2} - \boxed{y_1} \\ \Delta x &= \boxed{x_2} - \boxed{x_1}\end{aligned}\right\} \text{The order IS important!}$$

2nd coordinate

1st coordinate

Investigation #2:

1) On the coordinate grid below, graph the points $C(-4,7)$ and $D(3,-8)$ and join them to form a line segment.



2) Create a right angle triangle where CD is the hypotenuse and E is the third vertex.

3) Count the squares from C to E and D to E and record your numbers.

Vertical change (sometimes called rise):

from C → D   −15 (down)

Horizontal change (sometimes called run):

from C → D   +7 (right)

4) Calculate Δy and Δx for the line segment CD.

$C(\overset{x_1}{-4},\overset{y_1}{7})$

$D(\overset{x_2}{3},\overset{y_2}{-8})$

$\Delta y = -8 - 7 = -15$

$\Delta x = 3 - (-4) = 7$

$\text{slope} = \frac{\text{rise}}{\text{run}} = \underbrace{\frac{-15}{7} = \frac{15}{-7}}_{= -\frac{15}{7}}$

The slope, m, of a line segment joining $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$$m = \frac{Rise}{Run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{vertical change}}{\text{horizontal change}}$$

When is it most appropriate to use $m = \frac{Rise}{Run}$ to calculate the slope of a line segment?

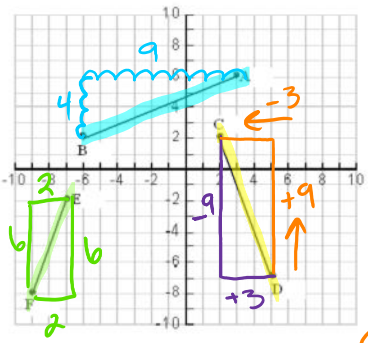
→ only when you are given a graph (count on grid)

When is it most appropriate to use $m = \frac{y_2 - y_1}{x_2 - x_1}$ to calculate the slope of a line segment?

→ when we are given coordinates, or a table of values.

*pick any 2 points ⇒ as long as it is LINEAR
(x_1, y_1) and (x_2, y_2)

Example #1: Determine the slope of the each line segment.



$m_{AB} = \frac{rise}{run} = \frac{4}{9}$ (leave as a fraction in simplest form)

$m_{CD} = \frac{-9}{+3}$ or $\underbrace{\frac{+9}{-3}}$ $= -\frac{9}{3} = -3$ *always simplify!

$m_{EF} = \frac{6}{2} = 3$

(y) RISE ↑+ ↓−

(x) RUN $\xrightarrow{+}$ $\xleftarrow{-}$

Which line segments in have a positive slope? (+) AB, EF

Which line segments in have a negative slope? CD

Example #2: Determine the slope of the each line segment. → $m = \frac{y_2 - y_1}{x_2 - x_1}$

a) $G(0, -7)$ to $H(4, 0)$
$x_1\ y_1$ $\quad x_2\ y_2$

$m = \frac{0-(-7)}{4-0} = \boxed{\frac{7}{4}}$

b) $M(5, -2)$ to $N(-1, 4)$
x_1, y_1 $\quad x_2\ y_2$ $\quad x_2 y_2$ $\quad x_1 y_1$

$m = \frac{4-(-2)}{(-1)-5} = \frac{6}{-6} \boxed{= -1}$

$m = \frac{(-2)-4}{5-(-1)} = \frac{-6}{6} \boxed{= -1}$

* it doesn't matter which coordinate you choose to be #1 or #2

SUMMARY:

Slope	Diagram
positive	↙↗
negative	↖↘
zero	⟷
undefined	↕

horizontal line $\frac{y_2-y_1}{x_2-x_1} = \frac{0}{*} = 0 = m$

vertical line $\frac{y_2-y_1}{x_2-x_1} = \frac{\#}{0} =$ ERROR! cannot ÷ by Ø

Homework

ASSIGNMENT # 1
Pages 1-10 questions #1-34

Characteristics of Linear Relations

Key Terms

Term	Definition	Example
Line		
Line segment		
Linear relation		
Slope		
Positive slope		
Negative slope		
Zero slope		
Undefined slope		
Intercepts		
Parallel lines		
Parallel slopes		
Perpendicular lines		
Perpendicular slopes		
Midpoint formula		
Distance formula		
Parallelogram		

Linear Relations:

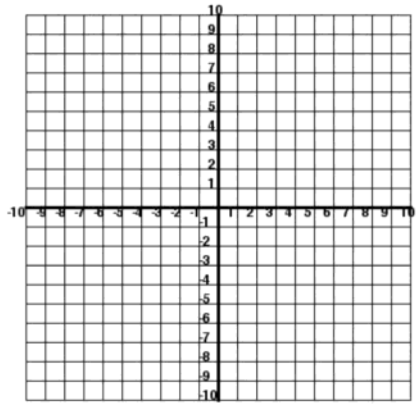
- A relationship between two quantities that when graphed will produce a **straight line**.
- One quantity **increases or decreases at a constant rate** with respect to another.

Eg. $y = -3x$ $C = 50n + 1000$ $p = 40q$

LINE SEGMENT: A part of a line that has two endpoints and includes all the points between the endpoints.

1. Using a dashed or coloured line, graph the relation represented by the equation $y = -3x$.

2. Using a solid or different coloured line graph the same relation if the domain is $0 \le x \le 2$.



The solid section you just plotted is a line segment, a section of the dashed line.

3. What are the endpoints of the line segment? ____________

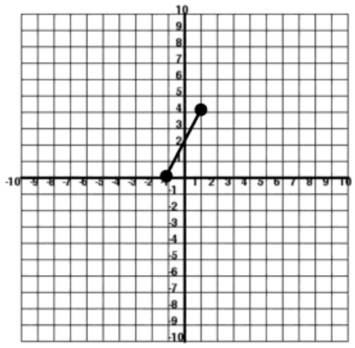
4. What are the endpoints of the dashed line? ______________

5. What are 5 properties you could use to describe the line segment above?

6. Which of these properties are also true for the dashed line above?

Slope of a Line (or Line Segment): (Rate of Change)

Consider the line segment below.



7. What is the vertical change (rise) between the endpoints?

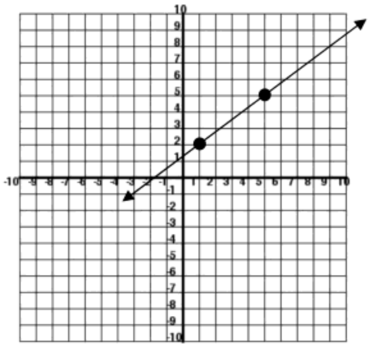
8. What is the horizontal change between the two endpoints?

9. What is the ratio of rise to run as a fraction?

10. How fast does the relationship change in the vertical direction when compared to the horizontal direction?

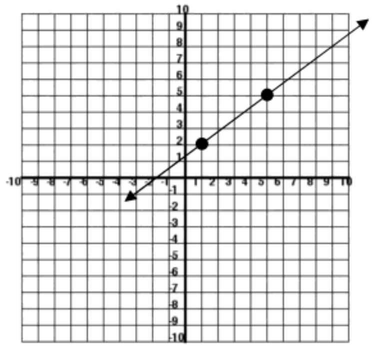
Your notes here…

11. Challenge Question:
Find the slope (rate of change) of the line below.



12. Challenge Question:
Find the slope (rate of change) of the line segment with end points at A(-4,0) and B(0,3).

Find the slope (rate of change) of the line below.



Recall:

Slope is the ratio of $\frac{Rise}{Run}$. We can count gridlines from point-to-point to get $\frac{rise:\ +3}{run:\ +4} = \frac{3}{4}$.

NOTE:

If you started at the right point…

$\frac{rise: -3}{run:\ -4} = \frac{3}{4}$ we would be moving in the "negative" direction but the slope calculated would be the same.

Find the slope (rate of change) of the line segment with end points at A(-4,0) and B(0,3).

Strategy 1: Plot the points on a grid and follow the same solution strategy to the left.

Strategy 2:

We can see the rise is actually a change in the y-direction…a difference in the y-values.
For the points: A(-4,0) and B(0,3)

$$rise: y - y = 3 - 0 = 3$$
$$run: x - x = 0 - (-4) = 4$$

Therefore $slope = \frac{Rise}{Run} = \frac{3}{4}$.

IMPORTANT
TO USE THIS STRATEGY…you must be consistent with your "starting" x and y values in calculating rise and run.

Note the formula on the next page to help you do this.

Slope of a Line (or Line Segment)

Slope is the measure of the "steepness" of a line. It is represented with the symbol (m).
Slope also describes the direction of the line.

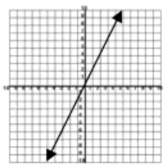
The slope is found by dividing the vertical change (the rise or fall) by the horizontal change (the run).

$$m = \frac{rise}{run} \quad or \quad m = \frac{y_2 - y_1}{x_2 - x_1}$$

Be Careful with Negatives!

Positive Slope

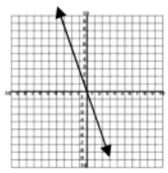
Eg. $m = 2$



Rises from left to right.

Negative Slope

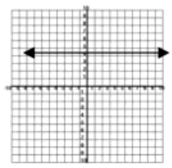
Eg. $m = -3$



Falls from left to right.

Zero Slope

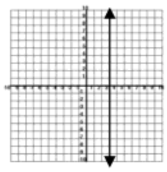
Eg. $m = 0$



Rise is 0.
0 divided by any "run" will still = 0.

Undefined Slope

Eg. $m = \infty$

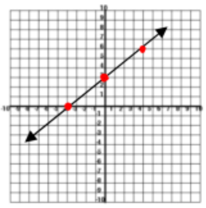


Think... the run is 0.
Division by 0 is undefined.

13. Describe, in your own words, how you find the slope of a line segment.

14. How does a line segment differ from a line?

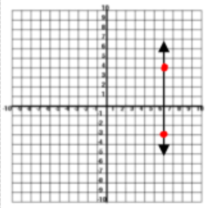
Find the rise, the run and the slope for the following lines by counting units.
In most cases, you will need to pick two points on the line to use.



15. rise=

run=

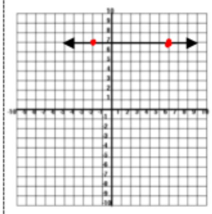
slope=



16. rise=

run=

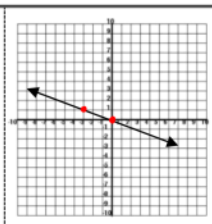
slope=



17. rise=

run=

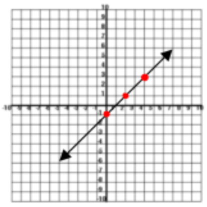
slope=



18. rise=

run=

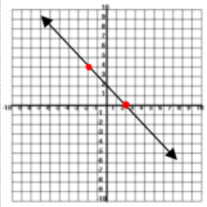
slope=



19. rise=

run=

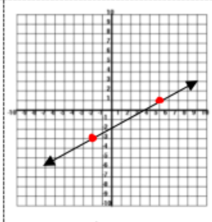
slope=



20. rise=

run=

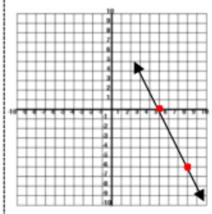
slope=



21. rise=

run=

slope=



22. rise=

run=

slope=

Use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find the slopes of line segments with the following endpoints.

23. $(0,0)$ and $(2,3)$

24. $(1,3)$ and $(2,7)$

25. $(-5,7)$ and $(-4,-2)$

Use the formula $m = \frac{y_2-y_1}{x_2-x_1}$ to find the slopes of line segments with the following endpoints.

26. $(5,7)$ and $(5,3)$

27. Find the coordinates of any another point on this line.

28. $(-4,5)$ and $(6,5)$

29. Find the coordinates of any another point on this line.

30. $\left(\frac{1}{2},4\right)$ and $(2,-6)$

31. Find the coordinates of any another point on this line.

32. The slope of a line is -2. If the line passes through (t, -1) and (-4,9), find the value of t.

33. The slope of a line is $-\frac{3}{2}$. If the line passes through (5,2) and (b,-4), find the value of b.

34. Challenge

Given a point on the line and the slope, sketch the graph of the line.

$(2,3), m = -2$