12) parallel & perpendicular equations

We Always Need Two Pieces of Information to Write Equations:

1. Slope of the line
2. Point or intercept

REMEMBER:

Parallel: Same slope

Perpendicular: negative reciprocal slope (flip!)

ex $\Rightarrow m = 5/3 \Rightarrow -3/5$ & change the sign +/–

Example 1: Write the equation of a line *parallel* to $2x - 4y + 3 = 0$ with the point $(5,1)$ in general form.

① find the slope of the line

ie: put in $y = mx + b$ form   ← slope

$2x - 4y + 3 = 0$
$-3 \quad -3$

isolate y
– no fractions
– coefficient of x cannot be negative

$2x - 4y = -3$
$-2x \qquad -2x$

$\dfrac{-4y}{-4} = \dfrac{+2x - 3}{-4}$

$y = \dfrac{1}{2}x + \dfrac{3}{4}$ ← slope

Parallel: same slope
$\Rightarrow m = \dfrac{1}{2}$

always use this form when you have slope + 1 point at least 1 point

Put in Slope-Point Form:

② use $m = \dfrac{1}{2}$ & given point $(5,1)$
   you just found

$y - y_1 = m(x - x_1)$

$2x \quad \left(y - 1 = \dfrac{1}{2}(x - 5) \right)$

$2y - 2 = 1(x - 5)$
$2y - 2 = x - 5$
$-2y + 2 \qquad -2y + 2$
$0 = x - 2y - 3$

Example 2: Write the equation of a line *perpendicular* to $5y - 15x = 1$ with the point $(7, -3)$ in general form.

① find slope: $5y - 15x = 1$
$+15x \quad +15x$

isolate y

$\dfrac{5y}{5} = \dfrac{15x}{5} + \dfrac{1}{5}$

$y = 3x + \dfrac{1}{5}$

$\Rightarrow m = \dfrac{3}{1}$
perp. $m = -\dfrac{1}{3}$

perpendicular slopes are the negative reciprocal.

② use $m = -\dfrac{1}{3}$ and $(7, -3)$

$y - y_1 = m(x - x_1)$

$y (-(-3)) = -\dfrac{1}{3}(x - 7)$

$\times 3 \quad \left(y + 3 = -\dfrac{1}{3}(x - 7) \right)$

$3y + 9 = -1(x - 7)$
$+3y + 9 = -1x + 7$
$+1x - 7 \qquad +1x - 7$

$\boxed{x + 3y + 2 = 0}$

Example 3: Write the equation of a line *perpendicular* to the line through $(7,5)$ & $(10,9)$ passing through the point $(-1,8)$ in general form.

$x_1\, y_1 \quad x_2\, y_2$

① If given 2 points use : to find the slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-5}{10-7} = \frac{4}{3}$$

perp. slope $= \frac{-3}{4}$

② use $m = \frac{-3}{4}$ & $(-1,8)$

$y - y_1 = m(x - x_1)$

solve $\Rightarrow$ $\left(y - 8 = \frac{-3}{4}(x - -1)\right)$

$4y - 32 = -3(x + 1)$
$4y - 32 = -3x - 3$
$+3x + 3 \quad +3x + 3$
$3x + 4y - 29$
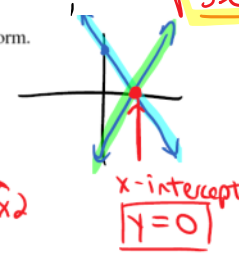
Example 4: Two *perpendicular* lines intersect on *x axis*. One line is $y = \frac{-1}{2}x + 5$ in general form.

① $m = \frac{2}{1}$ (perp. to $m = \frac{-1}{2}$)

② find x-intercept $\Rightarrow y = 0$

$\left(0 = \frac{-1}{2}x + 5\right) \times 2$

$0 = -1x + 10$
$-10 \qquad -10$
$-10 = -1x \Rightarrow x = 10$

x-intercept
$y = 0$

$(10, 0)$

③ use $m = \frac{2}{1}$ & $(10,0)$ $\Rightarrow$ SOLVE

Example 5: Write the equation of a line *parallel* to $-2y + x = 8$ with the same *y-intercept* as $10y + 32x = 100$ in general form.

① Find slope of : $-2y + x = 8$

$+x \qquad -x$

isolate y

$-2y = +x + 8$
$\overline{-2} \quad \overline{+2} \quad \overline{-2}$

$y = \frac{1x}{2} - 4 \Rightarrow m = \frac{1}{2}$

if its parallel, it will have the same slope

② Find point : find y-int

is where $x = 0$ (always)

$32(0)$
$10y + \cancel{32x} = 100$
$10y = 100$
$\overline{10} \quad \overline{10}$
$y = 10$

$(0, 10)$

The y-int of the line is therefore $(0,10)$

③ Use $m = \frac{1}{2}$ & $(0,10)$

$\Rightarrow$ Solve $y - y_1 = m(x - x_1)$

$\left(y - 10 = \frac{1}{2}(x - 0)\right)$

expand.
$2y - 20 = 1(x - 0)$
$2y - 20 = x - 0$
$+2y +20 \qquad +2y +20$
$x + 2y + 20 = 0$
general form.

Homework

assignment # 12
pages #43-48 questions #189-200

Parallel and Perpendicular Lines

Recall:
- Parallel lines have equal slopes.
- Perpendicular lines have slopes that are negative reciprocals.

For each line below, state the slope of a line that would be (a) parallel (b) perpendicular.

189. $y = 3x - 5$	190. $y - 5 = -\frac{2}{3}x$	191. $5x - 3y = 14$
a)	a)	a)
b)	b)	b)

192. CHALLENGE.

Write the equation of the line parallel to $5x - 8y + 12 = 0$ and through the point (-2,3).

SOLUTION to Q. 192.
Write the equation of the line parallel to $5x - 8y + 12 = 0$ and
through the point (-2,3).

**Parallel means same slope. So we need to find slope of $5x - 8y + 12 = 0$.

$5x - 8y + 12 = 0$	Convert to slope intercept form.
$-8y = -5x - 12$	
$y = \frac{5}{8}x + \frac{12}{8}$	This gives us the slope. $m = \frac{5}{8}$
	Use the slope, $m = \frac{5}{8}$ and the point (-2,3) to write the equation.
$m = \frac{y_2 - y_1}{x_2 - x_1}$	Fill in what you know. $m = \frac{5}{8}$. Substitute point (-2,3)
$\frac{5}{8} = \frac{y-3}{x--2}$	Cross-Multiply.
$5(x + 2) = 8(y - 3)$	Simplify.
$5x + 10 = 8y - 24$	
$5x - 8y + 34 = 0$	General Form
$y = \frac{5}{8}x + \frac{17}{4}$	Slope-Intercept Form

193. Write the equation of the line parallel to $4x - 6y + 12 = 0$ and through the point (5,7).

Explain your reasoning

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Eg.2. Write the equation of the line perpendicular to $3x + 2y - 4 = 0$ and through the point (2,3).

Perpendicular means slopes are negative reciprocals.
Step 1: Find the slope of $3x + 2y - 4 = 0$.

$3x + 2y - 4 = 0$ Convert to slope-intercept form.

$2y = -3x + 4$

$y = \frac{-3}{2}x - \frac{4}{2}$ This line has a slope, $m = \frac{-3}{2}$.   Negative reciprocal!

The perpendicular line will have a slope of $m = \frac{2}{3}$

Use: $m = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{2}{3} = \frac{y-3}{x-2}$ Fill in what you know. $m = \frac{2}{3}$. Substitute point (2,3)

$2(x - 2) = 3(y - 3)$ Cross-Multiply.

$2x - 4 = 3y - 9$ Simplify.

$2x - 3y + 5 = 0$ General Form

$y = \frac{2}{3}x + \frac{5}{3}$ Slope-Intercept Form

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194. Write the equation of the line perpendicular to $4x + 3y - 24 = 0$ and through the point $(1,4)$.

Eg.3. Write an equation for the line through C(2,4) that is perpendicular to the line through A(1,2) and B(4,8).

First find slope AB. $\quad m = \frac{8-2}{4-1} = \frac{6}{3} = 2 \quad\quad$ Therefore, the perpendicular line has slope, $m = \frac{-1}{2}$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad\quad$ Fill in what you know: $m = \frac{-1}{2}$. & substitute point (2,4)

$\frac{-1}{2} = \frac{y-4}{x-2}$ $\quad\quad\quad\quad$ Cross-Multiply.

$-1(x-2) = 2(y-4)$ $\quad\quad$ Simplify.

$-x + 2 = 2y - 8$

$x + 2y - 10 = 0$ $\quad\quad\quad\quad$ General Form

$y = -\frac{1}{2}x + 5$ $\quad\quad\quad\quad$ Slope-Intercept Form

Know which of these forms you are being asked to answer in. If it is not specified, you can choose.

Both describe the same line.

$\quad$

195. Write an equation for the line through $C(1,2)$ that is perpendicular to the line through $A(2,4)$ and $B(5,5)$.

Explain your reasoning

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196. Write an equation for the line through $Q(1,2)$ that is perpendicular to the line through $R(-2,0)$ and $S(3,5)$.

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Determine the equation of the following lines. Answer in general form.

197. The line parallel to $2x - 3y + 1 = 0$ and passing through the point $(1, 2)$.	198. The line perpendicular to $x - 5y + 2 = 0$ and passing through the point $(-2, 5)$.
199. The line perpendicular to $3x - 12y + 16 = 0$ and having the same y-intercept as $14x - 13y - 52 = 0$.	200. Two perpendicular lines intersect on the x-axis. An equation of one line is $y = 3x + 9$ Find the equation of the other line.