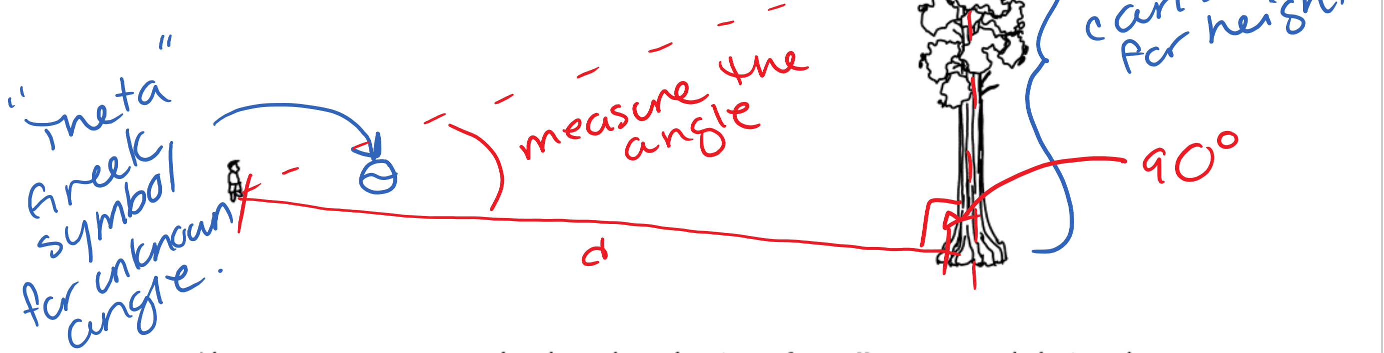
Intro to Trigonometry

September 20, 2018 10:21 PM

Why Trigonometry?

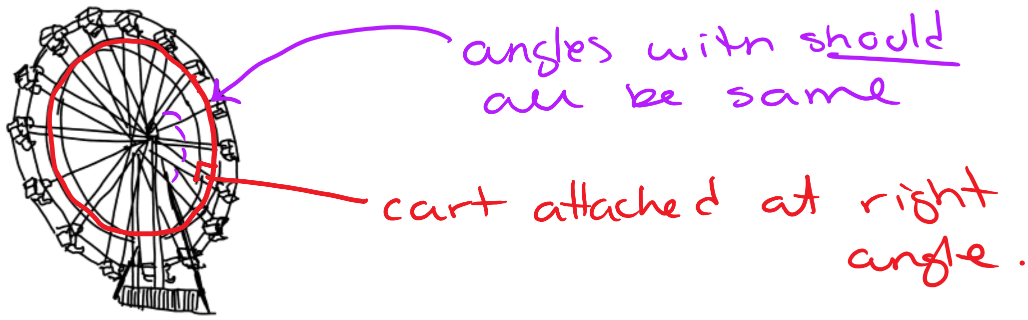
There is an application of trigonometry that could solve each problem below.

A student approaches a large Sequoia tree outside the entrance to the school and wonders how tall the tree is.



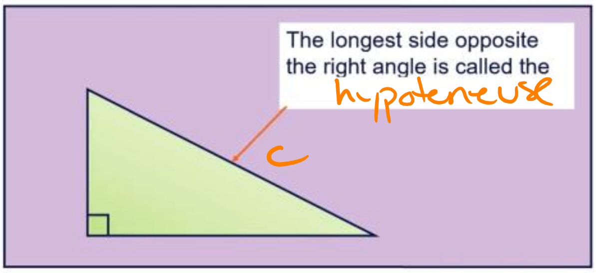
A homeowner wants to cut a new board to replace a decaying roof truss. He can measure the horizontal distance and the angle of inclination but needs to know how long to cut the board.

An engineer is constructing a Ferris wheel for a downtown park. There are 16 passenger carts.

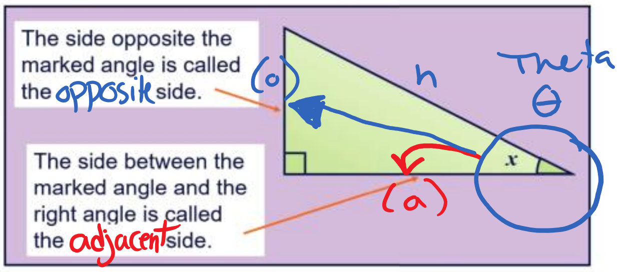


THE RIGHT TRIANGLE: OPPOSITE AND ADJACENT SIDES

A **right-angled triangle** contains a right angle.



The two shorter sides of a right-angled triangle are named with respect to one of the acute angles.

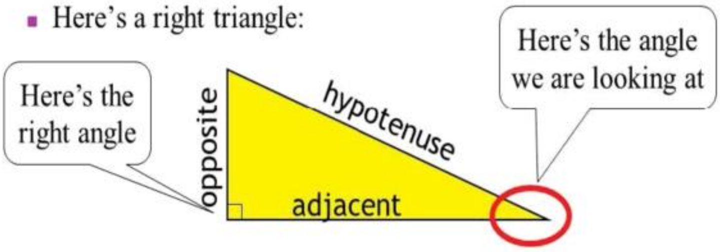


Trigonometry

The word trigonometry comes from the Greek meaning 'triangle measurement'.

Trigonometry uses the fact that the side lengths of similar triangles are the same ratio (but different lengths) to find unknown sides and angles.

- We only care about right triangles
  - A right triangle is one in which one of the angles is 90°
  - Here's a right triangle:



- We call the longest side the hypotenuse
- We pick one of the other angles--*not* the right angle
- We name the other two sides relative to that angle

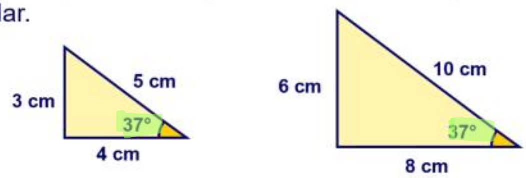
For example, when one of the angles in a right-angled triangle is 30° the side opposite this angle is always half the length of the hypotenuse.

Similar Triangles:

If two right-angled triangles have an acute angle of the same size they must be similar triangles

- internal angles are same/equal
- ratio of side lengths are equal.

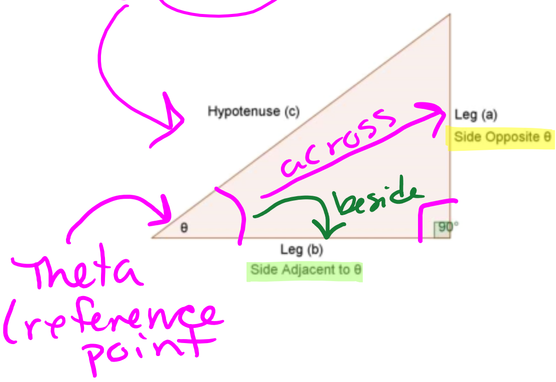
For example, two triangles with an acute angle of 37° are similar.



The ratio of the side lengths in each triangle is the same.

$\frac{\text{opp}}{\text{adj}} = \frac{3}{4} = \frac{6}{8}$ $\quad$ $\frac{\text{opp}}{\text{hyp}} = \frac{3}{5} = \frac{6}{10}$ $\quad$ $\frac{\text{adj}}{\text{hyp}} = \frac{4}{5} = \frac{8}{10}$

The Right Triangle



A right triangle is a triangle with one right angle (90°).

The side opposite the right angle is called the hypotenuse.

The other two sides are called "legs".

The sides of the right triangle form a Pythagorean Triple. That is, they satisfy the Pythagorean Theorem: $a^2 + b^2 = c^2$.

Some Pythagorean Triples: (whole numbers that obey Pythagorean Theorum)

(3,4,5), (5,12,13), (7,24,25), (8,15,17), (9,40,41), (11,60,61,), …

(6, 8, 10)

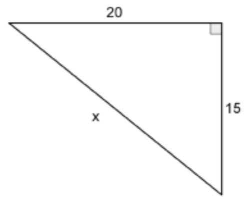
$a^2 + b^2 = c^2$

$(6^2) + (8^2) = (10^2)$

$36 + 64 = 100$

Find the indicated side length (nearest tenth) in the following right triangles. (1 d.p.)

1.



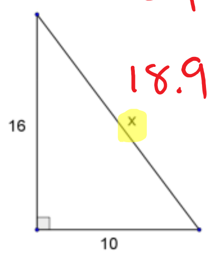
$a^2 + b^2 = c^2$

$20^2 + 15^2 = c^2$

$625 = c^2$

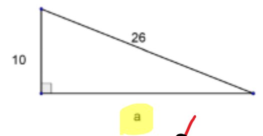
$25 = c$

2.



18.9

3.



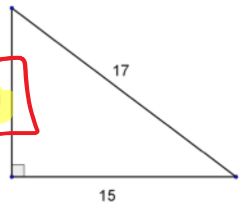
a = 24

$a^2 + b^2 = c^2 - b^2$

$\sqrt{a^2} = \sqrt{c^2 - b^2}$

$a = \sqrt{c^2 - b^2}$

4.

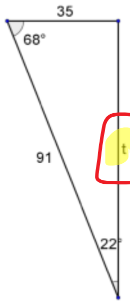


8 = j

$a = \sqrt{17^2 - 15^2}$

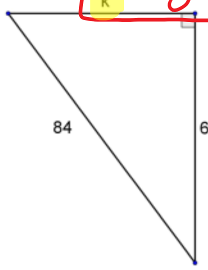
$\sqrt{289 - 225} = \sqrt{64} = 8$

5.



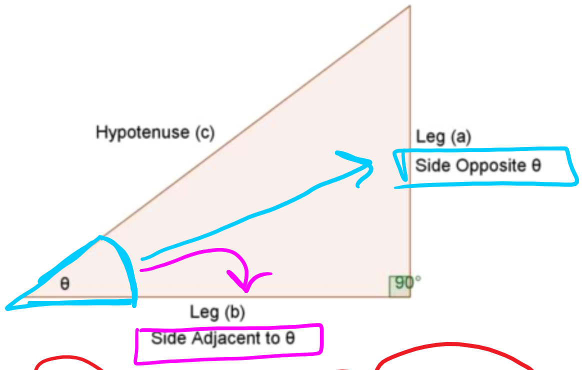
t = 84

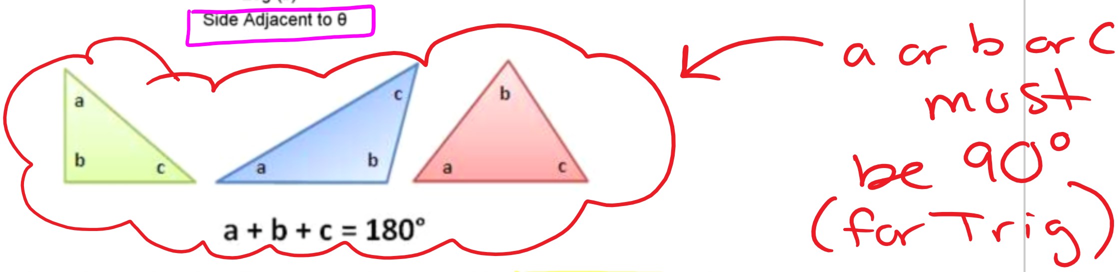
6.



k = 54.4

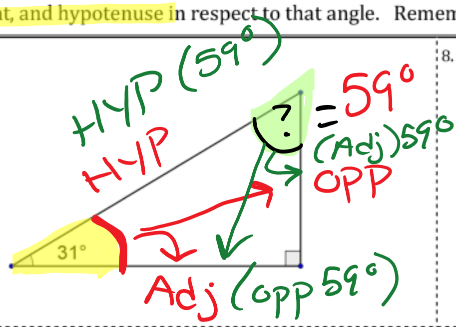
Labeling the Right Triangle for use with Trigonometry.



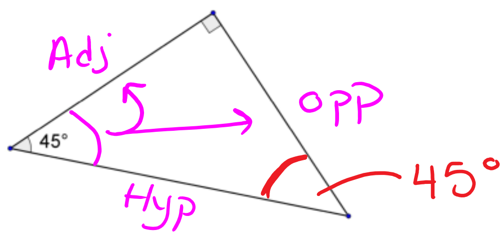


One acute angle is indicated on each of the following triangles. If possible, label each triangle with: opposite, adjacent, and hypotenuse in respect to that angle. Remember, only **right** triangles can be labeled this way.

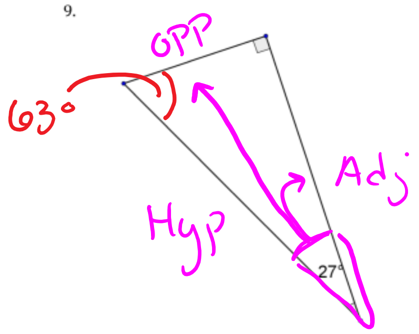
7.



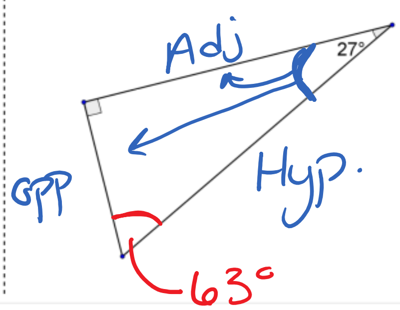
8.



9.



10.



Trigonometry of Right Triangles – **THE RATIOS**

Since similar right triangles have equivalent ratios for corresponding angles, we can use those ratios to find unknown angles and/or side lengths.

These ratios have been calculated and stored in our calculator for many angles to help us solve problems.

We will use the three **primary trigonometric ratios:**

Tangent ratio Sine ratio Cosine ratio

The Primary Trig. Ratios

For an acute angle in a right triangle: θ = measured angle

the ratio of $\frac{\textit{opposite} \angle\theta}{\textit{hypotenuse}}$ is called the SINE RATIO. $\sin\theta = \frac{O}{H}$ REMEMBER AS SOH or SO/H "SOH" = $\sin\theta = \frac{O}{H}$

the ratio of $\frac{\textit{adjacent}\angle\theta}{\textit{hypotenuse}}$ is called the COSINE RATIO. $\cos\theta = \frac{A}{H}$ REMEMBER AS CAH or CA/H "CAH" = $\cos\theta = \frac{A}{H}$

the ratio of $\frac{\textit{opposite} \angle\theta}{\textit{adjacent} \angle\theta}$ is called the TANGENT RATIO. $\tan\theta = \frac{O}{A}$ REMEMBER AS TOA or TO/A "TOA" = $\tan\theta = \frac{O}{A}$

"SOH" "CAH" "TOA"

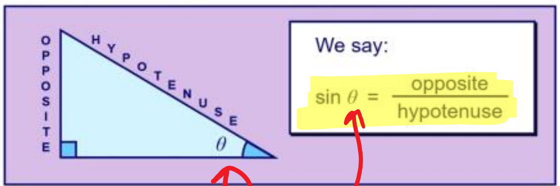
Since the ratios of the legs of a right triangle remain constant despite reducing or enlarging the triangle, we can use these ratios to solve proportional problems.

The ratios have been calculated and stored in our calculator for many angles to help us solve problems.

THE SINE RATIO

The ratio of $\frac{\text{the length of the opposite side}}{\text{the length of the hypotenuse}}$ is the **sine ratio**.

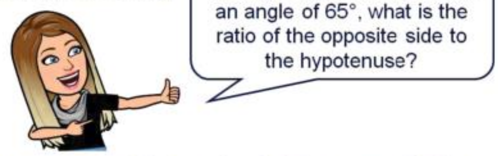
The **value of the sine ratio** depends on the **size of the angles in the triangle**.



sin(65°) = 0.

What is the value of sin 65°?

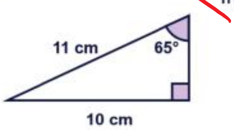
This is the same as asking:



It doesn't matter how big the triangle is because all right-angled triangles with an angle of 65° are similar.

The length of the opposite side divided by the length of the hypotenuse will always be the same value as long as the angle is the same.

In this triangle,



$$\sin 65° = \frac{\text{opposite}}{\text{hypotenuse}}$$

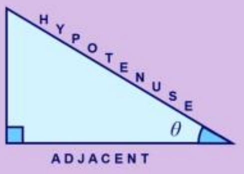
$$= \frac{10}{11}$$

= (to 2 d.p.)

THE COSINE RATIO

The ratio of $\dfrac{\text{the length of the adjacent side}}{\text{the length of the hypotenuse}}$ is the **cosine ratio**.

The value of the cosine ratio depends on the size of the angles in the triangle.



We say,

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

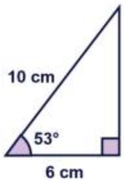
What is the value of cos 53°?

This is the same as asking:

In a right-angled triangle with an angle of 53°, what is the ratio of the adjacent side to the hypotenuse?

It doesn't matter how big the triangle is because all right-angled triangles with an angle of 53° are similar.

The length of the opposite side divided by the length of the hypotenuse will always be the same value as long as the angle is the same.



In this triangle,

$$\cos 53^\circ = \frac{\text{adjacent}}{\text{hypotenuse}}$$

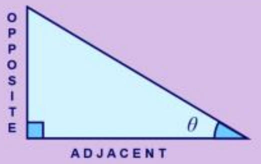
$$= \frac{6}{10}$$

$$=$$

THE TANGENT RATIO

The ratio of $\dfrac{\text{the length of the opposite side}}{\text{the length of the adjacent side}}$ is the **tangent ratio**.

The value of the tangent ratio depends on the size of the angles in the triangle.



We say,

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

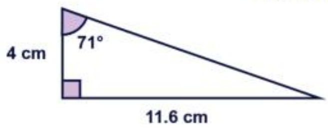
What is the value of tan 71°?

This is the same as asking:

In a right-angled triangle with an angle of 71°, what is the ratio of the opposite side to the adjacent side?

It doesn't matter how big the triangle is because all right-angled triangles with an angle of 71° are similar.

The length of the opposite side divided by the length of the adjacent side will always be the same value as long as the angle is the same.



In this triangle,

$$\tan 71^\circ = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{11.6}{4}$$

$$=$$

Writing the Trigonometric Ratios

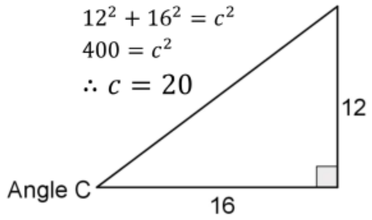
Remember, the ratios can be remembered using SO/H CA/H TO/A.

$a^2 + b^2 = c^2$

$12^2 + 16^2 = c^2$

$400 = c^2$

$\therefore c = 20$



From the diagram we see that...

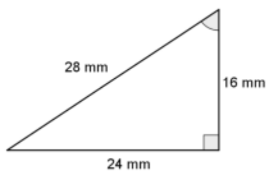
$tanC = \frac{12}{16}\ or\ \frac{3}{4}$

$sinC = \frac{12}{20}\ or\ \frac{3}{5}$

$cosC = \frac{16}{20}\ or\ \frac{4}{5}$

Find the three trig. ratios for the indicated angles below. Answer in fraction form.

12.

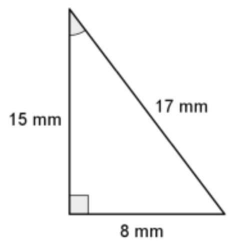


$sin\theta =$ ——

$cos\theta =$ ——

$tan\theta =$ ——

13.

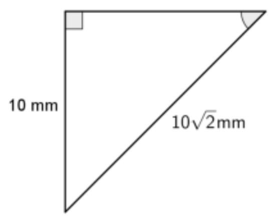


$sin\theta =$ ——

$cos\theta =$ ——

$tan\theta =$ ——

14.

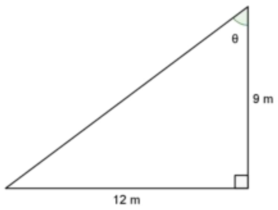


$sin\theta =$ ——

$cos\theta =$ ——

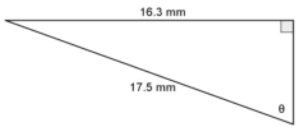
$tan\theta =$ ——

15. Find $sin\theta$.



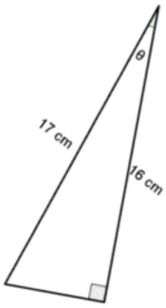
$sin\theta$=

16. Find $cos\theta$.



$cos\theta$=

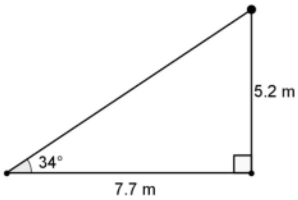
17. Find $tan\theta$.



$tan\theta$=

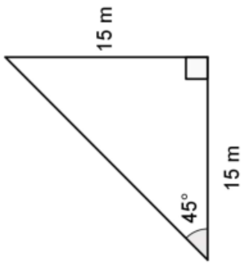
In each of the following diagrams, identify which ratio is represented (Sine, Cosine or Tangent).

18.



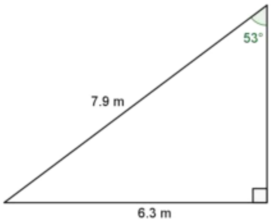
Ratio________________

19.



Ratio________________

20.



Ratio________________

Use a scientific calculator to determine a decimal approximation for each of the following. Round to 4 decimals if necessary.

21. $\sin 30° =$	22. $tan70° =$	23. $cos35° =$
24. $\sin 42° =$	25. $\tan 45° =$	26. $\cos 60° =$

27. Notice that $tan45° = 1$ in the question above. Refer to another question above to help you describe what that means.

28. Explain what it means for a right triangle to have a sine ratio equal to $\frac{1}{2}$.

Skill Reminder:

Solve the following equations. Answer to the nearest hundredth if necessary.

29. $\frac{12}{x} = 4$

$\frac{12}{x} = \frac{4}{1}$ set up equivalent fractions

$12 = 4x$ cross-multiply

$3 = x$ isolate the variable

30. $\frac{x}{4} = 6$

31. $\frac{x}{4} = 0.55$

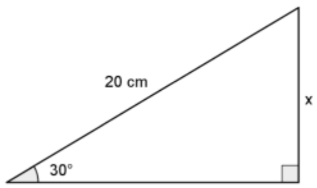
32. $\frac{x}{5} = \frac{10}{2}$

33. $\frac{2}{x} = 1.3$

34. $\frac{x}{2.5} = 6$

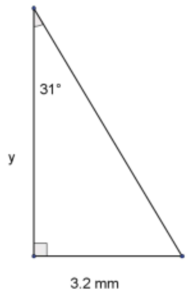
35. Challenge.

If we know $\sin 30° = 0.5000$, find the length of the missing side in the following diagram.



36. Challenge.

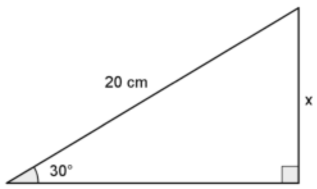
Find the length of the indicated side.



Finding Side Lengths Using Trigonometry

Find the length of the indicated side using an appropriate trigonometric ratio. Answer to tenths.

37.



We know:

$\sin 30° = 0.5000$

We also know: $\sin 30° = \frac{x}{20}$

We can say:

$0.5000 = \frac{x}{20}$

Solve the proportion:

$20(0.5000) = x$

$10.0 = x$

NOTE

To solve this problem…we can write…

$\sin 30° = \frac{x}{20}$

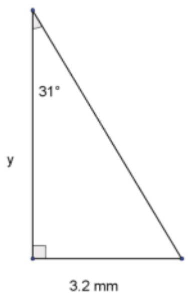
Multiply both sides by 20 to give:

$20sin30° = x$

Type 20 × $sin30$into calculator…

$10.0 = x$

38.



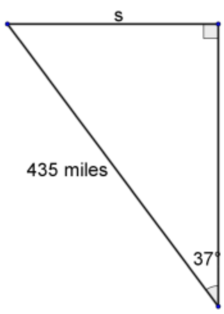
$tan\theta = \frac{opp}{adj}$

$tan31 = \frac{3.2}{y}$

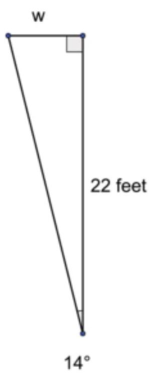
$ytan31 = 3.2$

$y = \frac{3.2}{tan31} = 5.3$ mm

39.



40.



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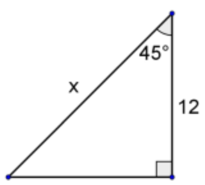

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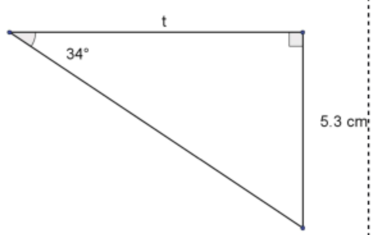

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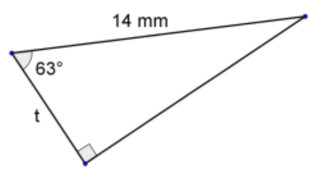
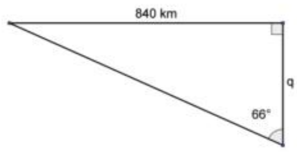
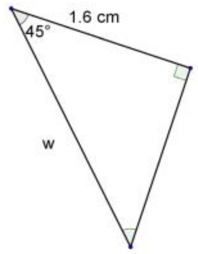


Find the length of the indicated side using an appropriate trigonometric ratio. Answer to tenths.

44.



45.



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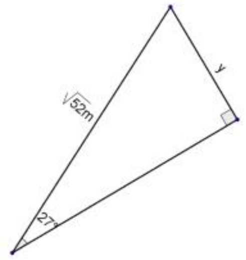
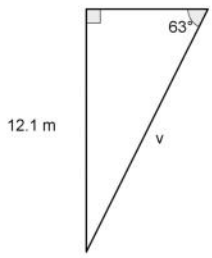


47.



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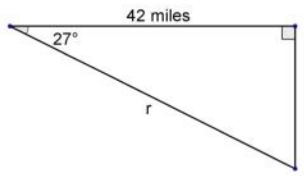

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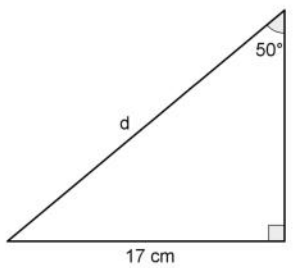

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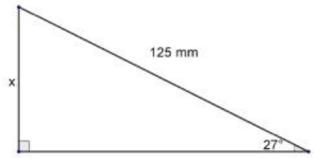

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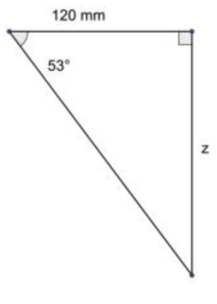

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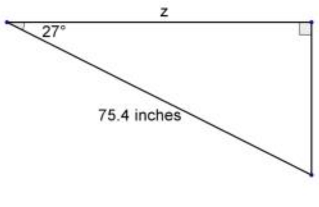
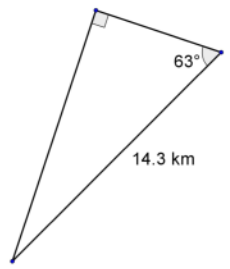


Find the length of the indicated side using an appropriate trigonometric ratio.

53. Find the length of each leg.



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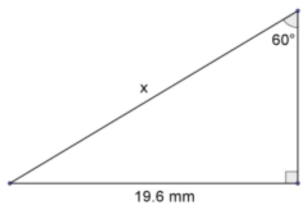

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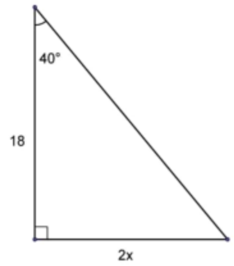


56. The tangent ratio is a ratio of what two sides in a right triangle?

57. Can you use the tangent ratio to find the hypotenuse of a right triangle?

58. Can you use the sine ratio to find the hypotenuse of a right triangle?

59. The sine ratio of a right triangle is $\frac{4}{5}$. If the hypotenuse is 20 cm long, what are the lengths of the other two sides?

60. The cosine ratio of a right triangle is $\frac{9}{20}$. If the hypotenuse is 8 m long, what are the lengths of the other two sides?

61. The cosine ratio for a right triangle is 2.1042. Find the opposite side if the hypotenuse is 4 mm.

62. The sine ratio for a right triangle is 7.1004. Find the hypotenuse if the opposite side is 17 cm.

63. Draw a diagram illustrating the cosine ratio for $\angle J$ in $\triangle JKL$. if

$$\angle K = 90°,$$
$$\overline{JK} = 11\ cm,$$
$$\overline{KL} = 11\ cm$$

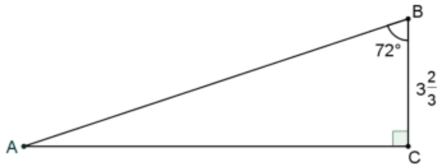
64. Draw a diagram illustrating the tangent ratio for $\angle P$ in $\triangle PQR$ if

$$\angle R = 90°,$$
$$\overline{PQ} = 10cm,$$
$$\overline{PR} = 8\ cm$$

Solving Triangles:

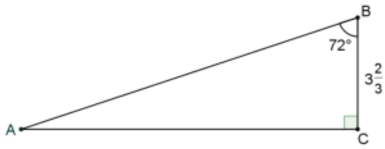
To "solve a triangle" means to find the length of all unknown sides and measure of unknown angles.

65. Explain the steps you would take to solve the following triangle.



Solve the following triangles. Answer to tenths.

66.



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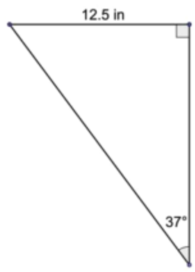

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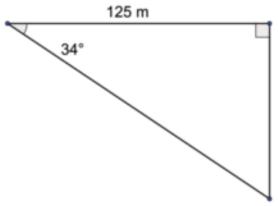
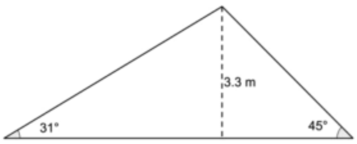


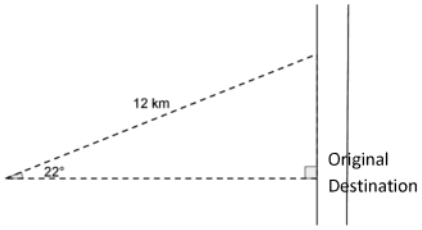
69. The dotted line is an altitude (perpendicular to base).



Solve each of the following word problems. Include a diagram in your solution.

70. From a point 220 m from the Empire State Building, a tourist measures the angle of inclination to the top to be 60°. Calculate the height of the building to the nearest metre...

71. A hiker loses track of her direction and wanders 22 degrees off course. If she continues to walk for 12 km to the river destination, how far away from her original destination will she be? (nearest tenth of a kilometer)



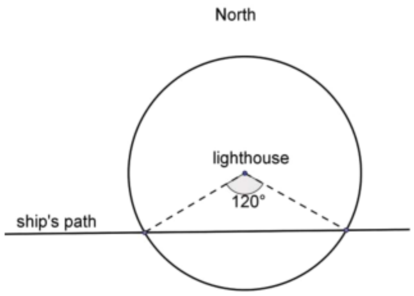
72. An airplane approaches a control tower. The angle of depression from the pilot to the tower is 12°. If the plane is flying at an altitude of 1500 m, how far is the plane from being directly above the tower (to the nearest kilometer)?

73. Find the area of a rectangle with a diagonal of 20 m if the angle between the diagonal and longer side is 25 degrees. (nearest unit)

74. A student crossing to the west building casts a shadow on the path. She is 165 cm tall and the angle to the sun is 25°. How long is the shadow on the path to the nearest centimetre?

75. A radio tower is 396 feet tall. How far from the base of the tower is a technician if the angle of inclination to the top of the tower is 27°? Answer to the nearest foot.

76. A lighthouse attendant has a range of visibility of 24 km. A ship on the horizon passes by the lighthouse. The attendant sees the ship for a total of 120 degrees. For how many kilometers was the ship within the attendant's range of sight? (nearest tenth)

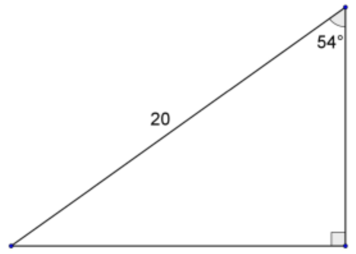


Draw a scale diagram that would **represent** each of the following.

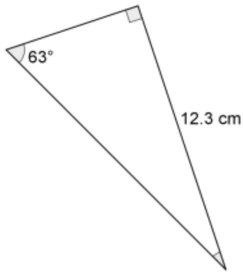
77. Draw a triangle that has a the following:
$sin\alpha = \frac{2}{5}$, hypotenuse is 10 cm long.

78. Draw a triangle that has a the following:
$tan\beta = \frac{12}{5}$, hypotenuse is 26 cm long.

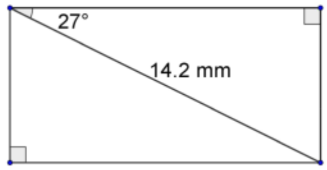
79. Solve the triangle.



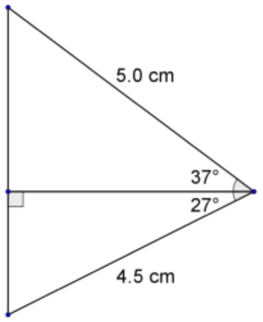
80. Solve the triangle.



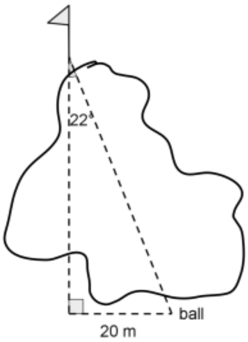
81. Find the perimeter of the following rectangle.



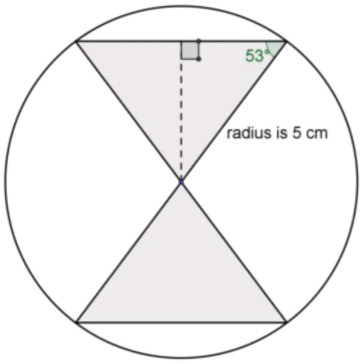
82. Find the total area.



83. While golfing with his father-in-law, Mr. J hits a shot short of a pond. He walks 20 m to his left to a point directly across the pond from the hole. The angle between the two lines of sight is 22°. Find the distance from his ball to the hole to the nearest tenth of a metre.



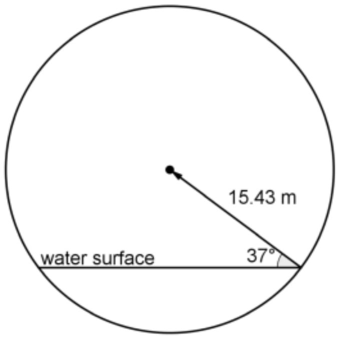
84. Find the area of the circle that is not covered by the shaded triangles. Answer to the nearest tenth.



85. Anya lets out 125 feet of kite string at Clover Point. The wind pulls the kite string tight at an angle of 55° to the ground. Approximate the height of the kite to the nearest foot .

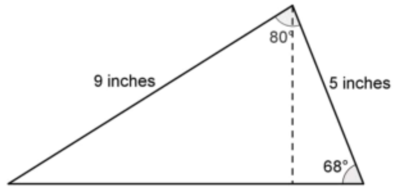
What assumptions did you make?

86. The radius of a circular tunnel in Shanghai is 15.43 m. During a flood, a worker in the water at the side of the tunnel measured an angle to the centre to be 37°. Find the depth of the water at its deepest point. (The water surface forms a chord across the tunnel.)



Solve the following problems involving triangles.

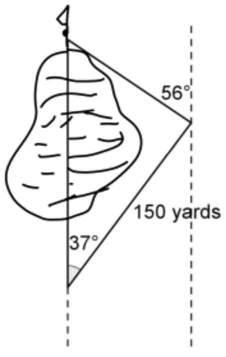
87. Find the area of the triangle below to the nearest square inch.



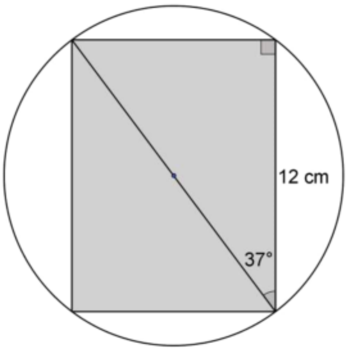
88. At 11:00 in the morning, the angle of elevation to the sun 58°. A tree in the school yard casts a shadow of 56 m. How tall is the tree to the nearest metre?

89. Tucker has two choices to get his ball to the hole at the 17th at Cordova Bay, go around the lake or go over it. He decides to go around the lake as shown on the diagram. How much farther does he have to hit the ball going around the lake instead of going straight over it? Answer to the nearest yard.

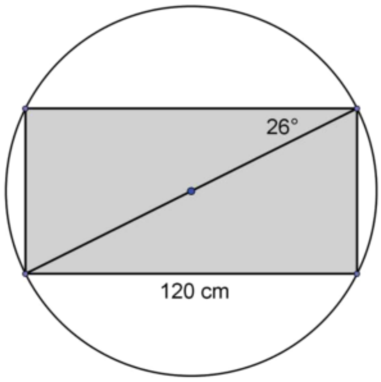
(Careful, not a right triangle shown.)



90. Find the area of the circle that is not covered by the shaded rectangle to the nearest square unit.

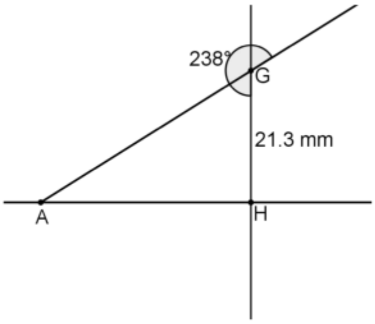


91. Find the area of the circle not covered by the shaded **rectangle** to the nearest 100 cm.

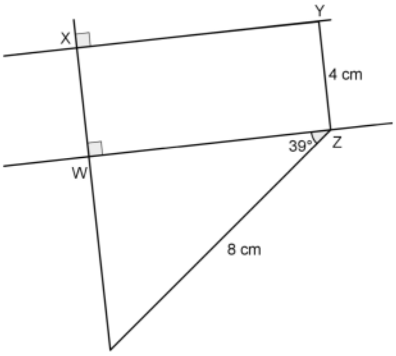


92. Sandra stands at the midpoint between two buildings and measures the angles of elevation to their tops to be 14° and 18°. If the two buildings are 80 metres apart, what is the difference in their heights? Answer to the nearest metre.

93. Find the length of AG to the nearest tenth of a millimetre.



94. Find the area of rectangle WXYZ to the nearest square unit.



Finding Angles Using the Three Ratios

Unless otherwise stated, calculate the measure of angles to the nearest tenth of a degree.

Eg. 42.8°

Recall:

The three primary trig. ratios:

Tangent Ratio: $\tan\theta = \frac{length\ of\ side\ opposite\ \theta}{length\ of\ side\ adjacent\ \theta}$

Sine Ratio: $sin\theta = \frac{length\ of\ side\ opposite\ \theta}{length\ of\ hypotenuse}$

Cosine Ratio: $cos\theta = \frac{length\ of\ side\ adjacent\ \theta}{length\ of\ hypotenuse}$

The stored values in your calculator allow you to find angles using the ratios.

The magic of sin^{-1}, cos^{-1}, and tan^{-1}.

The "inverse trigonometric functions". These functions convert the stored ratios in your calculator to the angle.

Challenge

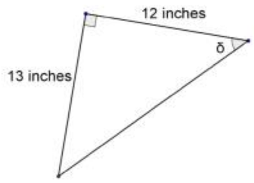
95. Find the measure of angle A in a right triangle if $tanA = 1.000$.

Challenge

96. Find the measure angle B in a right triangle if $sinB = \frac{1}{2}$

Challenge

97. What ratio would you use to find the measure of the indicated angle?



Find the measure of the indicated angle.

Use the Inverse functions to find the indicated angle to the nearest tenth.

98. Find the measure angle A in a right triangle if tanA=1.000.

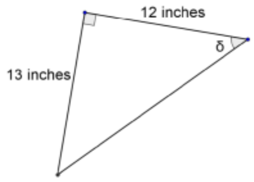
Use the $\tan^{-1}$ button.
Type:
$\tan^{-1}(1.000)=A$
$A = 45°$

99. Find the measure angle B in a right triangle if sinB=0.5000.

Use the $\sin^{-1}$ button.
Type:
$sin^{-1}(1 \div 2)=B$
$B = 30°$

100. What ratio would you use to find the measure of the indicated angle?

Use the tangent ratio: $tan\delta = \frac{13}{12}$



Find the measure of the indicated angle.

Type:
$\tan^{-1}(13 \div 12)= \delta$
$\delta = 47.3°$

101. Find angle A, if $sinA = 0.2654$.

102. Find angle B, if $cosB = \frac{5}{7}$.

103. Find angle Q, if $tanQ = \frac{15}{8}$.

104. Find angle T, if $sinT = \frac{15}{22}$.

105. Find angle D, if $cosD = \frac{11}{10}$.

106. Find angle U, if $tanU = 2.6784$.

107. In a right triangle, one acute angle has sine ratio of 0.5. Find the sine ratio of the other acute angle.

108. In a right triangle, one acute angle has cosine ratio of $\frac{1}{\sqrt{2}}$. Find the sine ratio of the other acute angle.

109. In a right triangle, one acute angle has cosine ratio of $\frac{1}{2}$. Find the tangent ratio of the other acute angle.

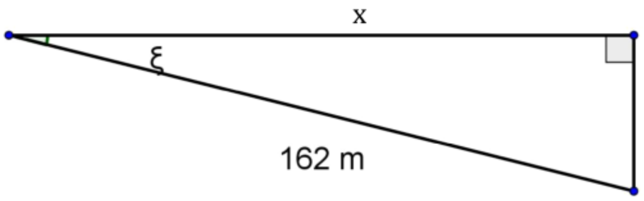
110. Which of the three trigonometric ratios (sine, cosine, tangent) can have a value greater than 1 ?

111. Draw a right triangle and use it to explain your answer to the previous question.

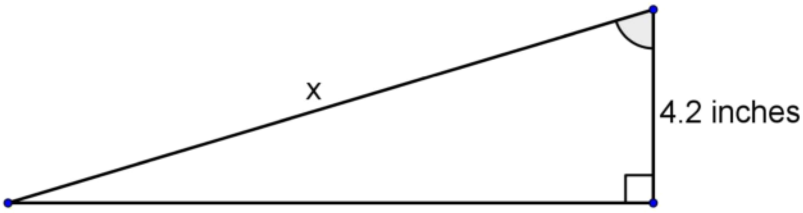
112. Draw a right triangle with an acute angle that has an adjacent side equal in length to the opposite side. Find the cosine ratio for that angle. (Round your answer to 3 decimals.)

113. Draw a right triangle with an acute angle that has a hypotenuse 50% longer than the adjacent side. Find the cosine ratio for that angle.

114. Use a protractor to measure the indicated angle. Then determine the length of side x using the cosine ratio.



115. Use a protractor to measure the indicated angle. Then determine the length of side x using the cosine ratio.

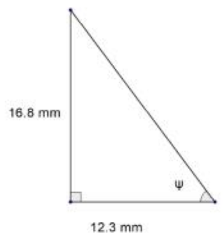


Working with the ratios to find angles.

Have a plan…

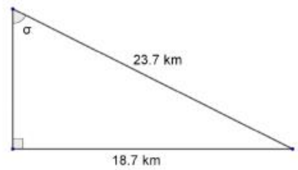
1. Choose the correct ratio {sine, cosine, or tangent}.
2. Fill in the known side lengths into your chosen ratio.
3. Use the "inverse trig. function" to convert ratio → angle.

116. What ratio do the given sides form for the indicated angle?



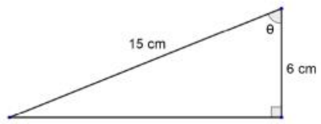
Sine Cosine Tangent

117. What ratio do the given sides form for the indicated angle?



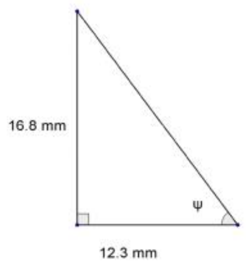
Sine Cosine Tangent

118. What ratio do the given sides form for the indicated angle?

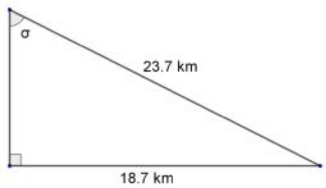


Sine Cosine Tangent

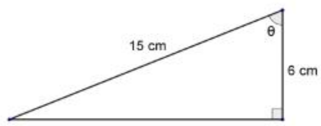
119. Calculate the measure of angle ψ to the nearest tenth of a degree.



120. Calculate the measure of angle σ to the nearest tenth of a degree.

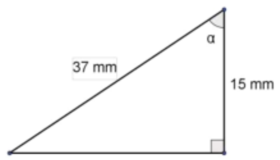


121. Calculate the measure of angle θ to the nearest tenth of a degree.



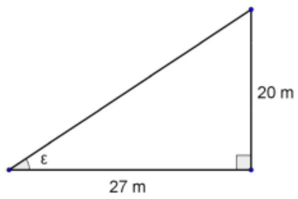
Working with the ratios to find angles.

122. What ratio do the given sides form for the indicated angle?



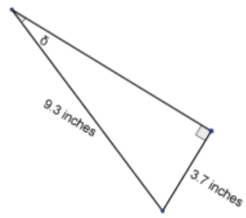
Sine Cosine Tangent

123. What ratio do the given sides form for the indicated angle?



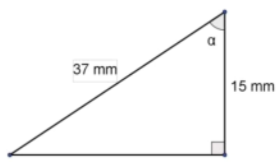
Sine Cosine Tangent

124. What ratio do the given sides form for the indicated angle?

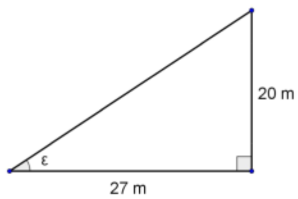


Sine Cosine Tangent

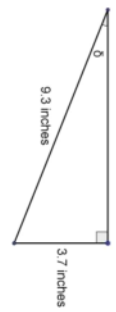
125. Calculate the measure of angle ∝ to the nearest tenth of a degree.



126. Calculate the measure of angle ε to the nearest tenth of a degree.

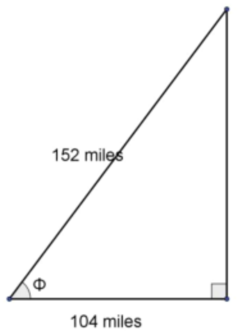


127. Calculate the measure of angle θ to the nearest tenth of a degree.



Find the measure of the indicated angle. Round answers to the nearest tenth of a degree.

128.



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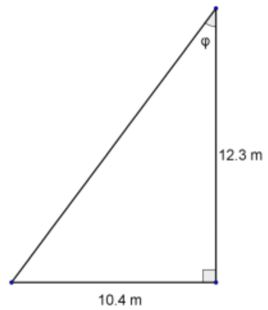

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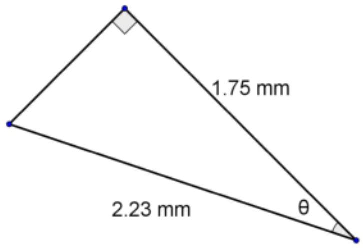

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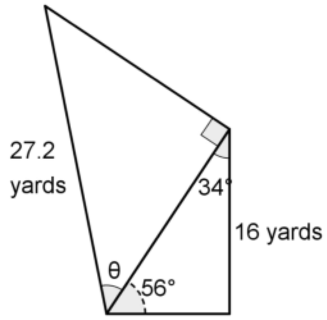
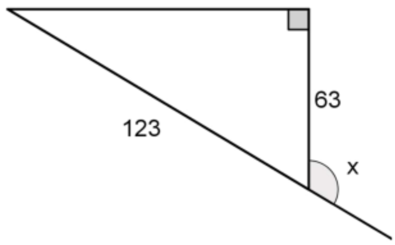
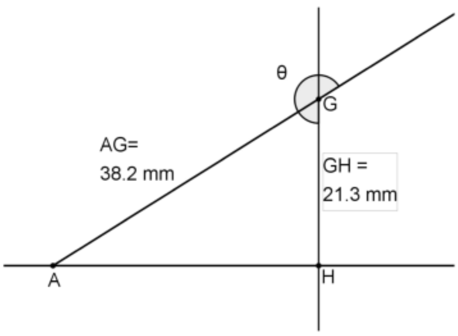


132. Find the measure of angle x to the nearest tenth of a degree..

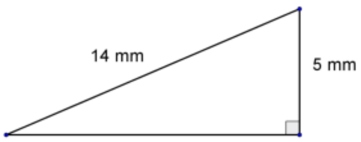


133. Find the measure of angle θ to the nearest degree.

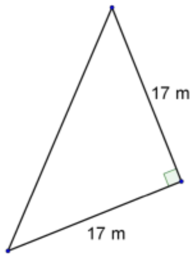


Solve the following triangles. Calculate answers to the nearest tenth.

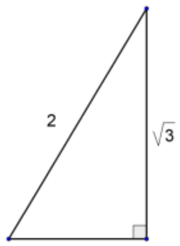
134.



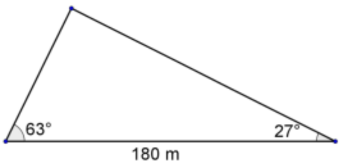
135.



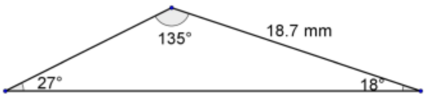
136.



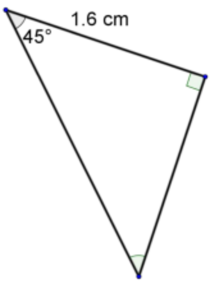
137.



138.

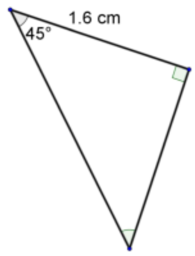


139. Challenge.
Find the Area of the following triangle to the nearest tenth of a square unit.



Find the area of the following triangles. Units for each question are indicated.

140. Nearest tenth.



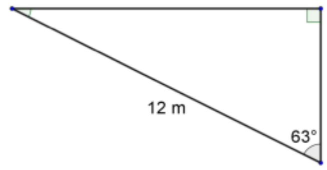
$$\text{Area} = \frac{base \times height}{2}$$

Find base:

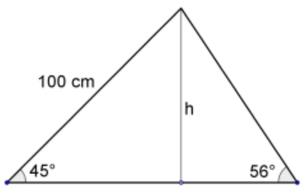
$$tan45 = \frac{opposite}{1.6}$$
$$\therefore base = 1.6\,cm$$

$$\text{Area} = \frac{1.6 \times 1.6}{2}$$
$$\text{Area} = 1.2cm^2$$

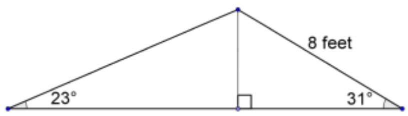
141. Nearest square metre.



142. Nearest hundred square centimetres.



143. Nearest square foot.



144. A triangle has side lengths of 8 cm, 7cm and 12 cm. Find the area of the triangle if the angle between the 8 cm and 12 cm side is 34°. Answer to the nearest square cm.

145. A triangle has side lengths of 10 km, 23 km and 32 km. The angle opposite the 10 km side is 9.2° Find the area of the triangle. Answer to the nearest square km.

Applications of trigonometry.

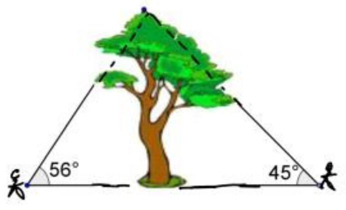
146. A kite stuck in a nearby tree. A child standing 25 m from the base of a tree pulls the string tight. If the tree is 30 m tall, approximately how far is the kite from the child to the nearest metre?

147. A surveyor measures the angle of elevation to the top of a building to be 23°. If the surveyor is 1345 feet from the base of the building, how tall is the building to the nearest foot?

148. From the top of a 20 m cliff above a road, the angle of depression to two approaching cars is 25° and 40° respectively. How far apart are the cars to the nearest metre?

149. Two hot air balloons float above the ocean at a height of 1000 feet. From a sailboat an observer measures the angle of elevation to one balloon is 60° and to the other balloon is 50°. [both balloons are on the same bearing from the observer] How far apart are the balloons to the nearest foot?

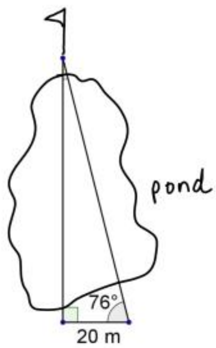
150. Two boys on opposite sides of the tree below measure the angle of elevation to the top of the tree. If the tree is 175 feet tall, how many feet apart are the boys?



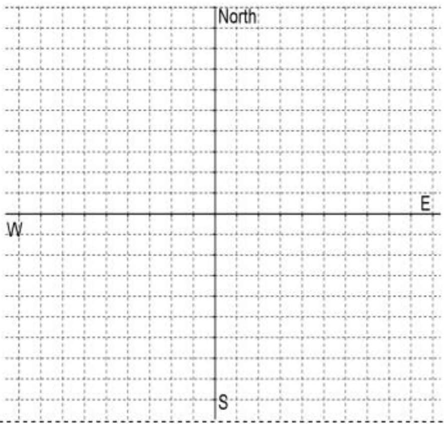
151. Highway sign shows that the road descends at a rate of 8%. Draw a diagram that shows what this means.

If a 3 km section of straight road descends at this grade, what is the drop in elevation?

152. While golfing with his father-in-law, Mr. J hits a shot short of a pond. The flag (hole) is directly across the pond from his ball. He paces 20 m to the right of his ball and measures the angle back to the hole to be 76°. How far is the ball from the hole to the nearest metre?

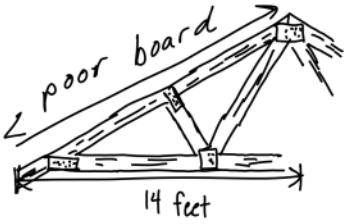


153. A hiker leaves base camp travelling due north at 5 km/h. After two hours, she turns and travels east. Three hours later, she sprains her ankle. At what bearing would a rescue team need to travel to reach the injured hiker? How far away is she from base camp? (nearest tenth)

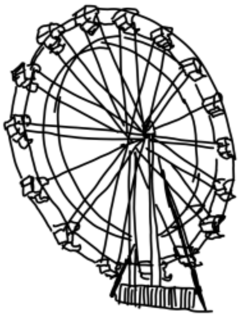


154. A student approaches a large Sequoia tree outside the entrance to the school and wonders how tall the tree is. He paces 150 metres from the base of the tree and measures the angle of elevation to the top of the tree to be 35°. Find the height of the tree to the nearest metre.

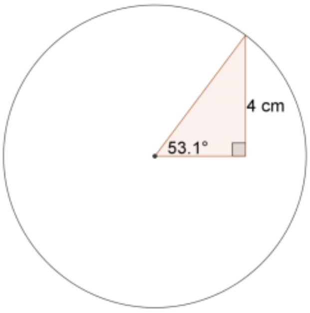
155. A homeowner wants to cut a new board to replace a decaying roof truss. He can measure the horizontal distance and the angle of inclination but needs to know how long to cut the board. The horizontal distance is 14 feet and the angle of inclination is 24°. Find the distance to the nearest tenth of a foot.



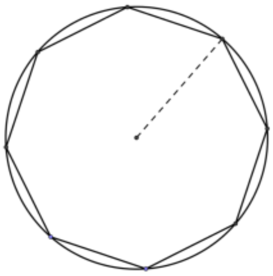
156. An engineer is constructing a Ferris wheel for a downtown park. There are 16 passenger carts and the radius of the wheel is 10 metres. How far apart are the passenger carts to the nearest hundredth of a metre?



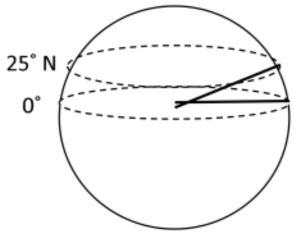
157. Find the area of the circle to the nearest square centimetre. $[A = \pi r^2]$



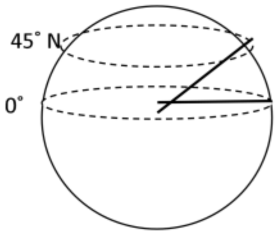
158. Find the perimeter of the octagon inscribed in a circle of radius 8 cm. (Nearest cm)



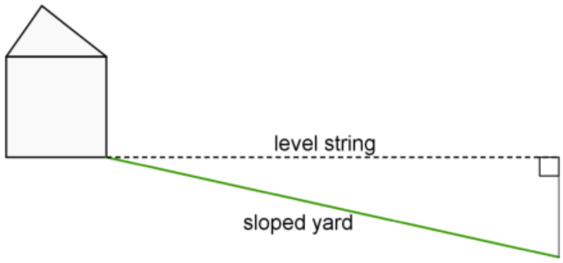
159. Find the length of the 25° line of latitude. The Earth's radius is 6380 km. Answer to the nearest km.



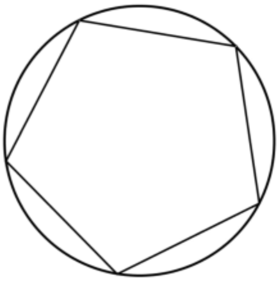
160. Find the length of the 45° line of latitude. The Earth's radius is 6380 km. Answer to the nearest km.



161. Mr. Teespré 's backyard slopes away from his house towards the beach. The instructions for his new lawnmower state that the mower should not be used if the slope is greater than 15°. Being a trigonometry specialist, he extends a level string 125 feet from the base of his house. From that point, he measures that the distance along the ground back to his house is 130 m. Is his yard too steep for this mower?

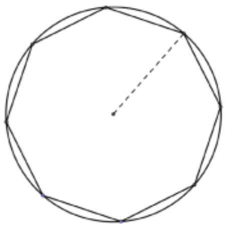


162. A regular pentagon is inscribed in a circle of radius 10 cm. Calculate the perimeter of the pentagon. Answer to the nearest cm.



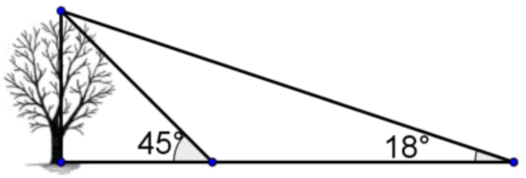
163. A regular decagon (10 sides) is inscribed inside a circle of radius 8 cm. Find the perimeter of the decagon. Answer to the nearest cm.

164. Find the area of the octagon inscribed in a circle of radius 8 cm. Answer to the nearest square cm.



165. A regular hexagon is inscribed in a circle with a radius 18 cm. What would be the side length of the hexagon? Answer to the nearest cm.

166. From a point 15 m from the base of a tree, a woman found the angle of inclination to the top of the tree to be 45°. Her sister found the angle to be 18° from a point farther away from the base of the tree. How far away are the two women away from each other?
(nearest tenth of a metre)

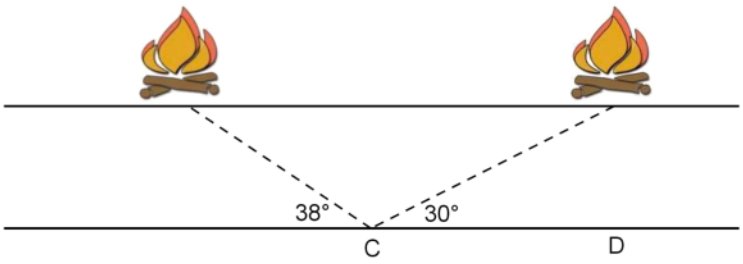


More word problems using right triangles:

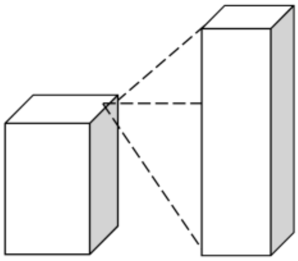
- Draw a diagram.
- Fill in known values.
- Let a variable represent unknown(s)
- Choose an appropriate strategy to solve for the unknown(s).
- Interpret the problem.

167. Solve the triangle given the following.
ΔXYZ
$x = 9\ cm$
$\angle Y = 90°$
$\angle Z = 36°$

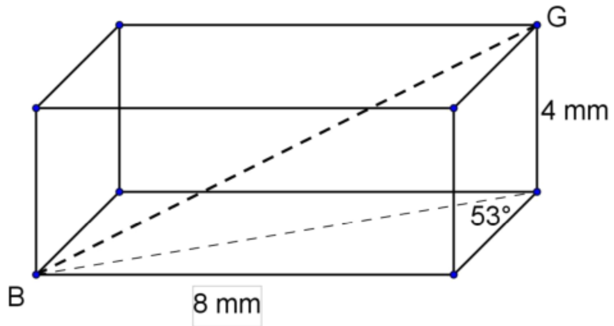
168. A firefighter is walking along the river at point C when she spots two fires on the opposite river bank. She measures the angles below and paces a distance of 300 m from point C to point D. Point D is directly across the river from one of the fires. How far apart are the fires to the nearest metre?



169. Anya stands on top of a building in downtown Victoria. From her position, the angle of elevation to the top of an adjacent building is 47°. The angle of depression to the base of the building is 62°. She is told that the buildings are 45 m apart. Based on this information, what is the height of the taller building to the nearest metre?



170. Find the length of diagonal BG in the rectangular prism. Answer to the nearest tenth of a millimeter.



171. The line of sight from an inflatable boat to the top of an oil derrick is 24 degrees. If the derrick is 45 m tall, how far is the boat from its base? (nearest tenth)

172. A pilot on a level path knows she should descend at an angle of 3 degrees to maintain comfort and safety. If she is flying at an altitude of 12 000 feet, how many miles from the runway should she begin her descent?

173. An aircraft ascends after takeoff at an angle of 22 degrees. What will be the altitude of the aircraft after it flies at that angle for 1200 m? (nearest metre)

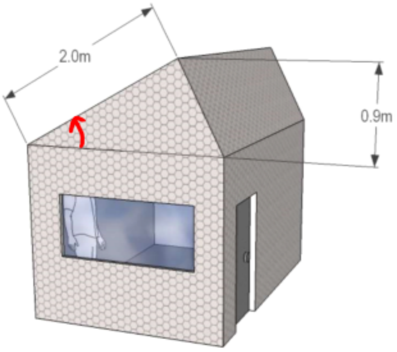
174. A hamster scurries up a ramp at a speed of 1.5 m/s. The ramp is inclined at an angle of 18 degrees. How many metres above the ground will the hamster be after 30 seconds?

175. Anya travels down a zip line at 25 km/h. The angle of descent of the zip line is 11 degrees. How many vertical metres has she fallen after 3 minutes?

What assumptions did you make?

176. The Earth's radius is 6380 km. Find the length of the 35° latitude to the nearest 10 km.

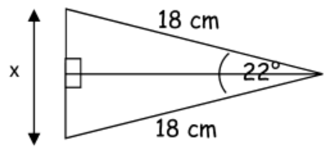
177. Find the angle of inclination at the back of the roof. The "rise" of the roof is 0.9 m. (nearest tenth)



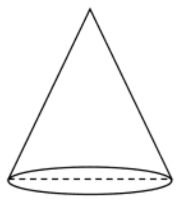
178. A ladder should make an angle of 72° with the ground for maximum safety. If the ladder is 4 m long, how far should it reach up the wall? (nearest tenth)

179. The angle of elevation to the top of a tree, measured on a 1.5 m transit from a distance of 30 m, is 15°. Find the height of the tree. (nearest tenth)

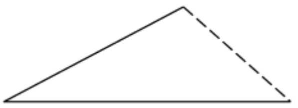
180. Find the value of 'x'.



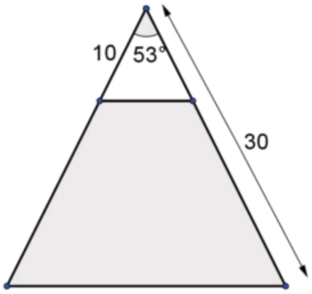
181. Mr. J has developed the ideal ice cream cone. The cone has a slant height of 13 cm and a diameter of 7.8 cm. Find the angle that the curved surface makes with the diameter.



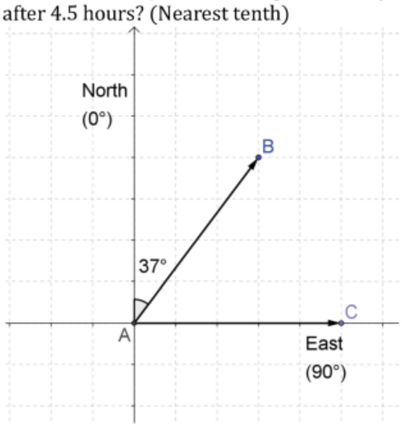
182. Mr. J continues to work on his isolated surf hut. Below is two-thirds of a roof truss he wants to complete. Find the length of wood he must cut (nearest tenth) to complete the truss. The long side is 8.2 m and the short side is 6.8 m. The angle between them is 35°.



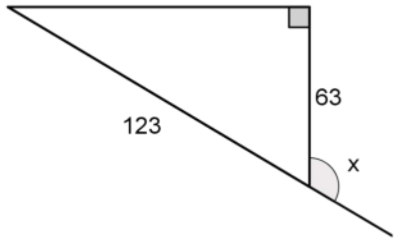
183. Both triangles (large and smaller inset) are isosceles. Find the area of the shaded trapezoid to the nearest tenth of a square unit.



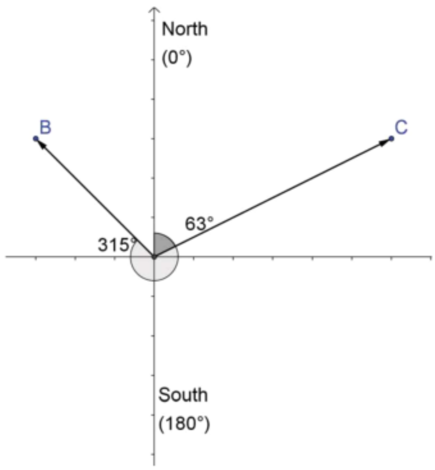
184. From a fire station in central BC, Georgia travels on a bearing of 37° at 6 km/h. Shelby leaves the station at the same time travelling due east at 5 km/h. How far apart are they after 4.5 hours? (Nearest tenth)



185. Find the measure of angle x to the nearest tenth of a degree.



186. At 9:00 am, a ship leaves port traveling at 30 km/h on a bearing of 63°. At the same time, another ship leaves port on a bearing of 315° at a speed of 19 km/h. When the boats stop after two hours, how far east is the boat at point C?



Draw an accurate diagram to answer each of the following questions.

187. In ΔQRS, $\angle QSR = 90°$, $QR = 12\ cm$ and $QS = 10\ cm$. Find the measure of $\angle QRS$

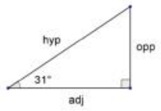
188. In ΔTUV, $\angle TVU = 90°$, $TU = 115\ m$ and $TV = 99\ m$. Find the measure of $\angle UTV$

189. In ΔDEF, $\angle DFE = 90°$, $DE = 12\ cm$ and $\angle DEF = 30°$. Find the length of FE.

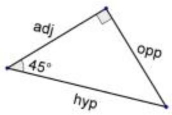
190. In ΔABC, $\angle ACB = 90°$, $BC = 5\ cm$ and $\angle ABC = 12°$. Find the length of AC.

Answers:

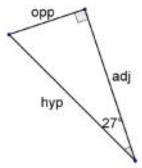
1. 25.0
2. 18.9
3. 24.0
4. 8.0
5. 84.0
6. 54.4
7.



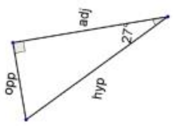
8.



9.



10.



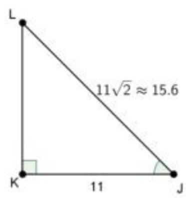
11. $tan\theta = \frac{3}{4}$
    $sin\theta = \frac{3}{5}$
    $cos\theta = \frac{4}{5}$

12. $sin\theta = \frac{6}{7}$
    $cos\theta = \frac{4}{7}$
    $tan\theta = \frac{3}{2}$

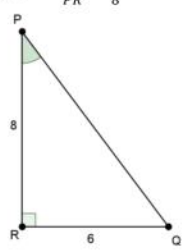
13. $sin\theta = \frac{8}{17}$
    $cos\theta = \frac{15}{17}$
    $tan\theta = \frac{8}{15}$

14. $sin\theta = \frac{1}{\sqrt{2}}$
    $cos\theta = \frac{1}{\sqrt{2}}$
    $tan\theta = 1$

15. $sin\theta = \frac{4}{5}$
16. $cos\theta = \frac{6.4}{17.5}$
17. $tan\theta = \frac{5.7}{16}$
18. Tangent
19. Tangent
20. Sine
21. 0.5000
22. 2.7475
23. 0.8192
24. 0.6691
25. 1.0000
26. 0.5000
27. A right triangle with an acute angle of 45° is an isosceles triangle with equal legs therefore $\frac{opp}{adj}$ will always equal 1, tangent 45 will always equal 1.
28. Sine is a ratio of opposite to hypotenuse. If the sine ratio is $\frac{1}{2}$ it means the hypotenuse is twice as long as the opposite side.
29. $x = 3$
30. $x = 24$
31. $x = 2.2$
32. $x = 25$
33. $x = 1.54$
34. $x = 15$
35. $x = 10\ cm$
36. $y = 5.3$
37. Answered on page.
38. Answered on page.
39. $261.8\ miles$
40. $w = 5.5\ feet$
41. $x = 17.0$
42. $t = 7.9\ cm$
43. $t = 6.4\ mm$
44. $q = 374.0\ km$
45. $w = 2.3\ cm$
46. $y = 3.3\ m$
47. $v = 13.6\ m$
48. $r = 47.1\ miles$
49. $d = 22.2\ cm$
50. $x = 56.7\ mm$
51. $z = 159.2\ mm$
52. $z = 67.2\ inches$
53. $12.7\ km$, $6.5\ km$
54. $x = 22.6\ mm$
55. $x = 7.6$
56. Opposite and adjacent
57. Not directly. The tangent ratio does not involve the hypotenuse.
58. Yes, the sine ratio involves the hypotenuse.
59. 16 cm and 12 cm
60. 3.6 m and 7.1 m
61. Not possible, the hypotenuse would need to be shorter than the adjacent side to have a cosine ratio greater than 1.
62. Not possible, like the answer above, a sine ratio cannot be greater than 1.
63. $\cos J = \frac{JK}{JL} = \frac{11}{11\sqrt{2}} = \frac{1}{\sqrt{2}}$

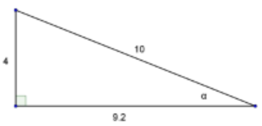


64. $tanP = \frac{QR}{PR} = \frac{6}{8}$

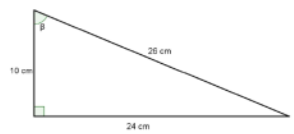


65. Answer will vary. But you will need to use the given side and angle to find another side length. Then choose to find another side or remaining angles.
66. AB = 11.9, AC = 11.3
67. 16.6 in, 20.8 in

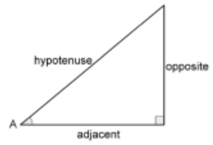
68. 84.3 m, 150.8 m
69. 4.7 m, 6.4 m, 8.8 m
70. 381 m
71. 4.5 km
72. 7 km (7057 m)
73. 153.2 m^2
74. 354 cm
75. 777 ft
76. 41.6 km
77.



78.

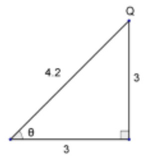


79. 11.7, 16.2, 36°
80. 6.3 cm, 13.8 cm, 27°
81. 38.2 mm
82. 10 cm^2
83. 53.4 m
84. 54.5 cm^2
85. 102 feet, assuming the ground is level and the string is straight.
86. 6.14 m
87. 22 in^2
88. 90 m
89. 78 yd
90. 69 cm^2
91. 7000 cm^2
92. 3 m
93. 40.2 mm
94. 24.9 cm^2
95. 45°
96. 30°
97. Tangent ratio, 47.3°
98. Answered on page.
99. Answered on page.
100. Answered on page.
101. 15.4°
102. 44.4°
103. 61.9°
104. 42.3°
105. No Solution
106. 69.5°
107. 0.8660 or $\frac{\sqrt{3}}{2}$
108. 0.7071 or $\frac{1}{\sqrt{2}}$
109. 0.5774 or $\frac{1}{\sqrt{3}}$
110. Tangent
111.

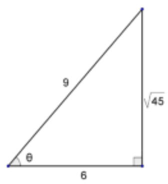


The side opposite to angle A can be greater than the side adjacent to angle A. As a ratio, $\frac{\text{opposite}}{\text{adjacent}}$ would be greater than 1. The sine and cosine ratios can not produce values greater than 1 because the denominator in the ratio will always be larger than the numerator.

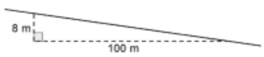
112. $cos\theta = 0.707$(side lengths of 3 were arbitrarily chosen)



113. $cos\theta = 0.6667$



114. 14°, 157 m
115. 73.5°, 14.8 in
116. Tangent
117. Sine
118. Cosine
119. 53.8°
120. 52.1°
121. 66.4°
122. Cosine
123. Tangent
124. Sine
125. 66.1°
126. 36.5°
127. 23.4°
128. 46.8°
129. 40.2°
130. 38.3°
131. 44.8°
132. 120.8°
133. 236.1°
134. 20.9°, 69.1°, 13.1 mm
135. 45°, 45°, 24.0 m
136. 30°, 60°, 1
137. 81.7 m, 160.4 m, 90°
138. 12.7 mm, 29.1 mm
139. 1.2 m^2
140. Answered on page.
141. 29 m^2
142. 4190 cm^2
143. 34 square feet
144. 27 cm^2
145. 59 km^2
146. 39 m
147. 571 ft
148. 19 m
149. 262 ft
150. 293 ft
151.



The units are simply an example. A descent of 8% means that the road "falls" 8 units for every 100 units of horizontal travel.
A 3 km section of road falls 0.24km.

152. 80 m
153. A rescue team would need to travel 18.0 km at 56.3°.
154. 105 m
155. 15.3 ft
156. 3.90 m
157. 78.5 cm^2
158. 49 cm
159. 36 331 km
160. 28 346 km
161. Yes. His yard slopes at an angle of 16°. Too steep for the mower.
162. 59 cm
163. 49 cm

164. 181 cm^2
165. 18 cm
166. 31.2 m
167. 6.5 cm, 11.1 cm, 54°
168. 522 m
169. 133 m
170. 10.8 mm
171. 101.1 m
172. 43 miles
173. 450 m
174. 14 m
175. 239 m
176. 32 840 km
177. 26.7°
178. 3.8 m
179. 9.5 m
180. 6.9 cm
181. 72.5°
182. 4.7 m
183. 319.6 square units
184. 22.4 km
185. 120.8°
186. 80 km
187. $\angle QRS = 56.4°$
188. $\angle UTV = 30.6°$
189. 10.4 cm
190. 1.1 cm

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I. Model Problems
II. Practice
III. Challenge Problems
IV. Answer Key

Web Resources

SOHCAHTAO
www.mathwarehouse.com/trigonometry/sine-cosine-tangent-home.php
Real World Applications
www.mathwarehouse.com/trigonometry/sohcahtoa-real-world-applications.html

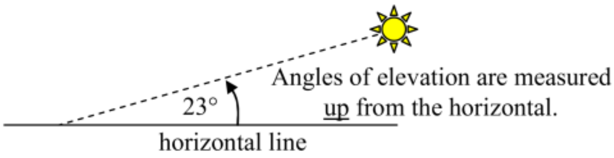
Graph Paper Maker (free): www.mathworksheetsgo.com/paper/

Online Graphing Calculator(free): www.mathworksheetsgo.com/calculator/

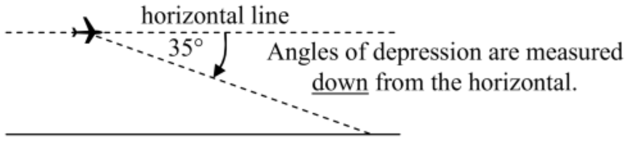
Applications of Right Triangle Trigonometry: Angles of Elevation and Depression

Preliminary Information: On most maps, it is customary to orient oneself relative to the direction north: for this reason, north is almost always indicated on every map. Likewise, when working with real-life trigonometry problems, it is very common to orient angles relative to a horizontal line.

An angle of elevation refers to the acute angle a line (or ray, segment, etc.) makes with a horizontal line, when measured above the horizontal (hence an angle of *elevation*). For example, the sun's rays could form a 23° angle of elevation (above the horizon).

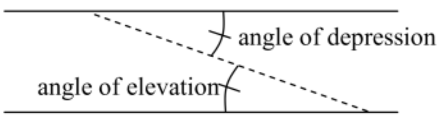


An angle of depression refers to the acute angle a line makes with a horizontal line, when measured below the horizontal (hence an angle of *depression*). For example, an airplane pilot could look down and see a feature on the ground below at a 35° angle of depression (below the horizon).



Angles of elevation and depression typically have their vertex at the point where an observer is positioned. In the previous example, notice that the vertex of the 35° angle is located at the pilot's location.

Because horizontal lines are everywhere parallel, angles of depression and elevation are numerically equivalent because they form alternate interior angles of parallel lines:



Students should always be encouraged to consider the following two ideas when they see either phrase mentioned in a problem:

- You may always draw an additional horizontal line on any diagram extending from any point in the diagram. Just as you did in Geometry, drawing such an *auxiliary line* can help to make a complex problem simpler.

- The most common error students make when they encounter these terms is they mark an angle relative to a vertical line (such as an angle with a wall, building, or tree) instead of with a horizontal line. As stated earlier, always feel free to draw in an auxiliary horizontal line.

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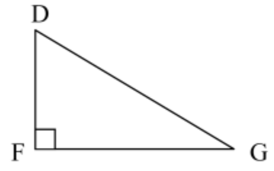
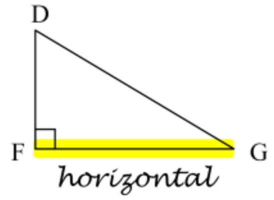
Part I) Model Problems



Example 1: Consider right ΔDFG pictured at right. Classify each angle as an angle of elevation, an angle of depression, or neither.

Step 1: Highlight the horizontal line(s) in the figure.



Step 2: Determine which acute angles are formed with the horizontal. In this example, $\angle F$ and $\angle G$ are formed with the horizontal, but only $\angle G$ is acute.

Step 3: Classify each angle:

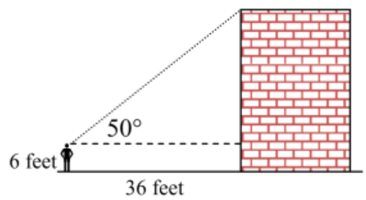
- $\angle D$ is neither an angle of depression nor an angle of elevation, as it is formed with vertical line segment $\overline{DF}$.

- $\angle F$ is formed with a horizontal line segment, but it is not an acute angle. So it is neither. (In reality, there would be no harm in specifying it as a 90° angle of elevation, but it is simpler just to say that $\angle F$ is a right angle.

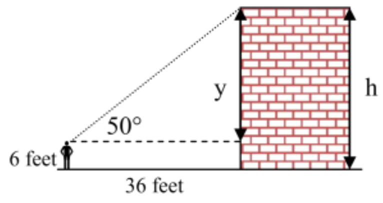
- $\angle G$ is the only acute angle measured from the horizontal; because line segment $\overline{DG}$ is above the horizontal, it is an angle of elevation.

Example 2: Michael, whose eyes are six feet off the ground, is standing 36 feet away from the base of a building, and he looks up at a 50° angle of elevation to a point on the edge of building's roof. To the nearest foot, how tall is the building?

Step 1: Make a detailed sketch of the situation. Make sure to include auxiliary horizontal lines as needed.



Step 2: Assign variables to represent the relevant unknowns. In this example, we shall use the variable y to represent the vertical leg of the right triangle, and h to represent the height of the building.



Step 3: Use SOHCAHTOA to solve for the unknown side of the right triangle:

$$\tan 50° = \frac{y}{36}$$

$y = 36 \tan 50°$ (Note that units of feet were dropped for simplicity.)

$$y = 36(1.19175)$$

$$y = 42.90\,ft$$

Step 4: Determine the height of the building: Since Michael's eyes are six feet from the ground, we must add six feet to variable y to get h:

$$h = 6 + y$$

$$h = 6 + 42.90$$

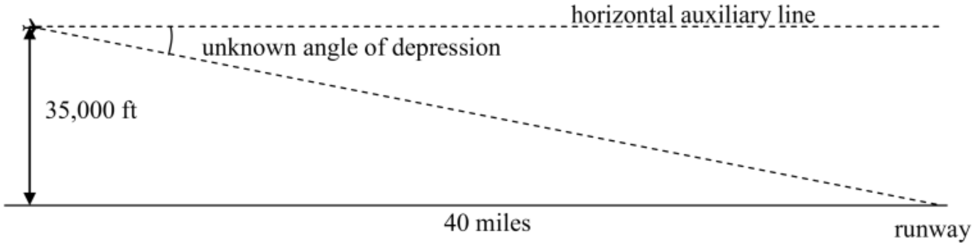
$$h = 48.90\,ft$$

$$h = 49\,ft \quad (rounded)$$

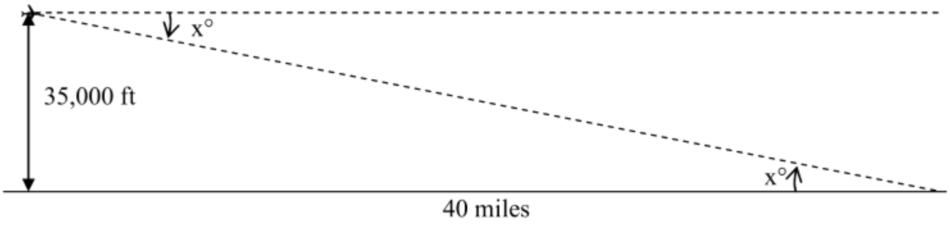
Step 5: Check for reasonableness: If Michael were looking up at a 45° angle of elevation, y would be 36 feet due to the isosceles triangle created. Because he is looking up at a greater angle, it is reasonable that y is greater than 36 feet. Adding 6 feet accounts for the fact that his eyes are 6 feet from the ground.

Example 3: A pilot is traveling at a height of 35,000 feet above level ground. According to her GPS, she is 40 miles away from the airport runway, as measured along the ground. At what angle of depression will she need to look down to spot the runway ahead?

Step 1: Make a detailed sketch of the situation. Make sure to include auxiliary horizontal lines as needed.



Step 2: Assign variables to represent the relevant unknowns. In this example, we shall use the variable x to represent the unknown angle of depression:



Because the ground is horizontal, and the auxiliary line is horizontal, we can properly assume that both angles marked x in the figure are congruent, as they are both alternate interior angles of parallel lines.

Step 3: Use SOHCAHTOA to solve for the unknown angle:

$$\tan x° = \frac{35{,}000\,ft}{40 miles}$$

Because we have mixed units, we recall that there are 5280 feet in a mile to convert 40 miles to feet:

$$\tan x° = \frac{35{,}000\,ft}{40 miles \cdot \frac{5280 ft}{mile}} = \frac{35{,}000\,ft}{211{,}200\,ft} = \frac{35{,}000}{211{,}200}$$

To solve for the unknown angle, we use the inverse tangent function:

$$\tan x° = \frac{35{,}000}{211{,}200}$$
$$x° = \tan^{-1}\left(\frac{35{,}000}{211{,}200}\right)$$
$$x° = 9.41°$$

Step 4: Check for reasonableness: 40 miles is much larger than 35,000 feet, so it seems reasonable that the pilot would look down at an angle of only a few degrees.

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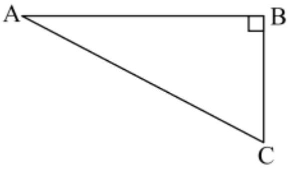
Part II) Practice Problems

1. Classify each of the three angles in the figure at right as an angle of elevation, an angle of depression, or neither.



2. Multiple-Choice: A 15 foot ladder rests against a tree on level ground and forms a 75° angle of elevation. Where is the correct location of the 75° angle?

A) Between the ladder and the ground
B) Between the ladder and the tree
C) Between the tree and the ground
D) It is not possible to place a 75° angle on such a figure.

3. Tammi Jo, whose eyes are five feet off the ground, is standing 50 feet away from the base of a building, and she looks up at a 73° angle of elevation to a point on the edge of building's roof. To the nearest foot, how tall is the building?

4. A pilot is traveling at a height of 30,000 feet above level ground. She looks down at an angle of depression of 6° and spots the runway. As measured along the ground, how many miles away is she from the runway? Round to the nearest tenth of a mile.

5. A dog, who is 8 meters from the base of a tree, spots a squirrel in the tree at an angle of elevation of 40°. What is the direct-line distance between the dog and the squirrel?

6. A ship is on the surface of the water, and its radar detects a submarine at a distance of 238 feet, at an angle of depression of 23°. How deep underwater is the submarine?

7. The sun is at an angle of elevation of 58°. A tree casts a shadow 20 meters long on the ground. How tall is the tree?

8. Two observers on the ground are looking up at the top of the same tree from two different points on the horizontal ground. The first observer, who is 83 feet away from the base of the tree, looks up at an angle of elevation of 58°. The second observer is standing only 46 feet from the base of the tree. (Note: you may ignore the heights of the observers and assume their measurements are made directly from the ground.)

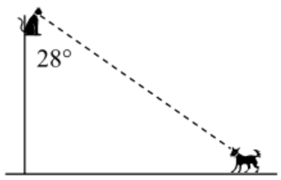
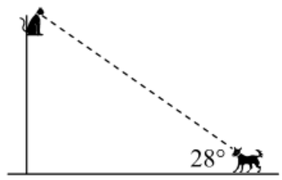
a) How tall is the tree, to the nearest foot?

b) At what angle of elevation must the second observer look up to see the top of the tree?

9. Error Analysis: Consider the following problem, which Stephanie and Adam are both trying to solve:

> "A cat, who has climbed a tree, looks down at a dog at a 28° angle of depression. If the dog is 34 meters from the base of the tree, how high up is the cat?"

The first steps of their work are shown below. Analyze their work and determine who, if anyone, has set it up correctly.

Stephanie's work	Adam's work
 34 meters $\tan 28^\circ = \frac{34}{x}$	 34 meters $\tan 28^\circ = \frac{x}{34}$

10. Complete problem 9: How high up in the tree is the cat?

Part III) Challenge Problems

11. A person starts out 17 miles from the base of a tall mountain, and looks up at a 4° angle of elevation to the top of the mountain. When they move 12 miles closer to the base of the mountain, what will be their angle of elevation when they look to the top? Answer to the nearest degree.

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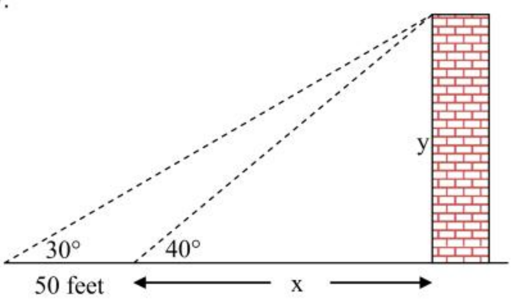
12. A pilot maintains an altitude of 25,000 feet over level ground. The pilot observes a crater on the ground at an angle of depression of 5°. If the plane continues for 16 more miles, what will be the angle of depression to the crater? Answer to the nearest degree.

13. An observer on the ground looks up to the top of a building at an angle of elevation of 30°. After moving 50 feet closer, the angle of elevation is now 40°. Consider the diagram below:



a) Set up an equation representing the situation from the first vantage point. Your equation will incorporate the 30° angle, x, y, and the 50 feet.

b) Set up an equation representing the situation from the second vantage point. Your equation will incorporate the 40°, x, and y.

c) You now have two equations in two variables. Solve them simultaneously to determine the value of x, the distance from the second vantage point to the base of the building.

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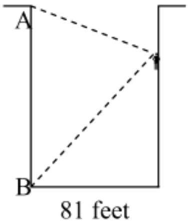
d) Solve for y, the height of the building.

14. Two observers (located at points A and B in the diagram) are watching a climber on the opposite face of a chasm. The chasm is 81 feet wide. When observer A looks down to the bottom of the opposite wall of the chasm, he must look down at an angle of depression of 51°. However, observer A sees the climber at an angle of depression of 20°. Observer B will see the climber at what angle of elevation?



Foundations of Mathematics & Pre-Calculus 10

Solving for Angles

Name: ______________________________ **Date:**________________ **Block:** _____

Teacher: Miss Zukowski

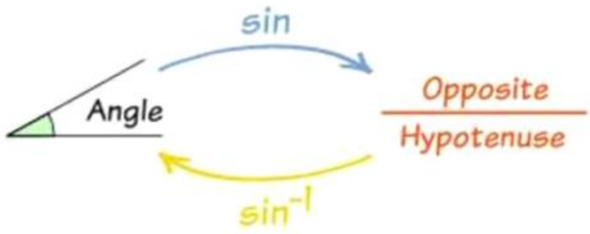
Inverse Functions

$$\theta = \cos^{-1}(x) \quad \Leftrightarrow \quad x = \cos(\theta)$$

$$\theta = \sin^{-1}(x) \quad \Leftrightarrow \quad x = \sin(\theta)$$

$$\theta = \tan^{-1}(x) \quad \Leftrightarrow \quad x = \tan(\theta)$$

For example, if we are doing the "inverse of sinθ"…we are trying to FIND the angle, when we are GIVEN both side lengths.



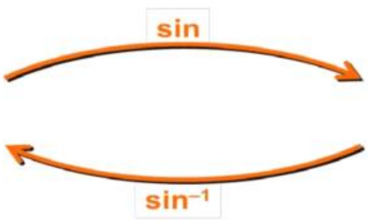
The inverse of sin

sin θ = 0.5, what is the value of θ?

To work this out use the **$\sin^{-1}$** key on the calculator.

$\sin^{-1} 0.5 =$ ☐

$\sin^{-1}$ is the inverse of sin. It is sometimes called arcsin.



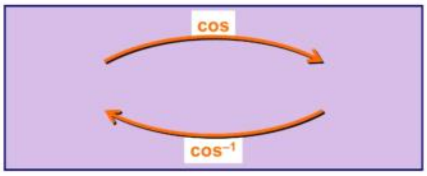
The inverse of cos

Cos θ = 0.5, what is the value of θ?

To work this out use the **cos^{-1}** key on the calculator.

$\cos^{-1} 0.5 =$ ☐

$\cos^{-1}$ is the inverse of cos. It is sometimes called arccos.



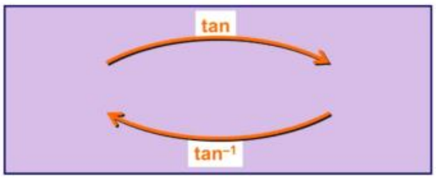
The inverse of tan

tan θ = 1, what is the value of θ?

To work this out use the **tan^{-1}** key on the calculator.

$\tan^{-1} 1 =$ ☐

$\tan^{-1}$ is the inverse of tan. It is sometimes called arctan.



4 steps we need to follow:

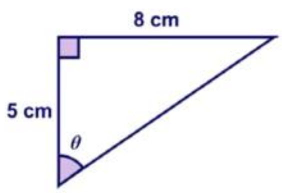
Step 1 Find ________________________________→out of Opposite, Adjacent and Hypotenuse.

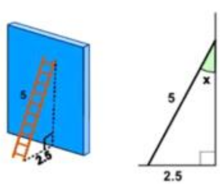
Step 2 Use ________________________ to decide which one of Sine, Cosine or Tangent ratio to use in this question.

Step 3 For ________calculate Opposite/Hypotenuse, for ______ calculate Adjacent/Hypotenuse or for _______calculate Opposite/Adjacent.

Step 4 ____________________________ from your **calculator**, using one of $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$ *(these are inverse, or 2nd function settings)*

Examples: Finding angles





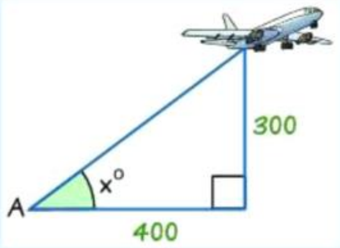
Example

The ladder leans against a wall as shown.

What is the **angle** between the ladder and the wall?

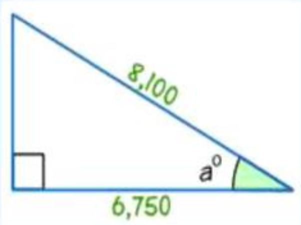
Example

Find the angle of elevation of the plane from point A on the ground.



Example

Find the size of angle a°



Example: Find the angle **"a"**

We know

- The distance down is 18.88 m.
- The cable's length is 30 m.

And we want to know the angle "a"