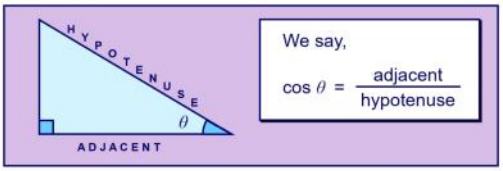
2-3 Trig Ratios & Finding Side Length

September 24, 2018 9:20 PM

THE COSINE RATIO

The ratio of $\frac{\text{the length of the adjacent side}}{\text{the length of the hypotenuse}}$ is the **cosine ratio**.

The value of the cosine ratio depends on the size of the angles in the triangle.



We say,

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

SOH CAH TOA

$\cos\theta = \frac{A}{H}$

What is the value of cos 53°?

This is the same as asking:

In a right-angled triangle with an angle of 53°, what is the ratio of the adjacent side to the hypotenuse?

It doesn't matter how big the triangle is because all right-angled triangles with an angle of 53° are similar.

The length of the opposite side divided by the length of the hypotenuse will always be the same value as long as the angle is the same.

(Hyp) (Adj)

In this triangle,

$\cos 53° = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\cos 53° = \frac{6}{10}$

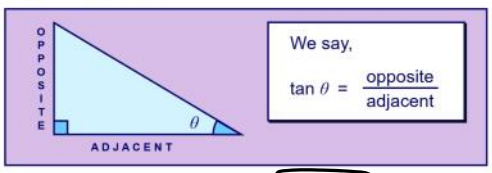
$\cos(53) = 0.6$

ratio to represent side lengths

THE TANGENT RATIO

The ratio of $\frac{\text{the length of the opposite side}}{\text{the length of the adjacent side}}$ is the **tangent ratio**.

The value of the tangent ratio depends on the size of the angles in the triangle.



We say,

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

SOH CAH TOA

$\tan\theta = \frac{\text{opp}}{\text{adj}}$

What is the value of tan 71°?

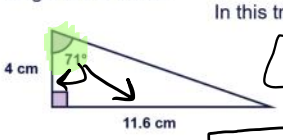
This is the same as asking:

In a right-angled triangle with an angle of 71°, what is the ratio of the opposite side to the adjacent side?

It doesn't matter how big the triangle is because all right-angled triangles with an angle of 71° are similar.

The length of the opposite side divided by the length of the adjacent side will always be the same value as long as the angle is the same.

In this triangle,



$\tan 71° = \frac{\text{opposite}}{\text{adjacent}}$

$= \frac{11.6}{4}$

$\tan(71°) = 2.90$

Writing the Trigonometric Ratios

Remember, the ratios can be remembered using SO/H CA/H TO/A.

From the diagram we see that...

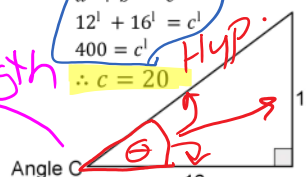
$\tan C = \frac{12}{16}$ or $\frac{3}{4}$

$\sin C = \frac{12}{20}$ or $\frac{3}{5}$

$\cos C = \frac{16}{20}$ or $\frac{4}{5}$

From side length

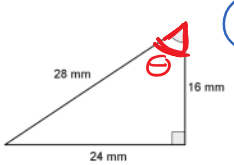
$a^2 + b^2 = c^2$
$12^2 + 16^2 = c^2$
$400 = c^2$
$\therefore c = 20$



SOH CAH TOA

Find the three trig. ratios for the indicated angles below. Answer in fraction form.

12.

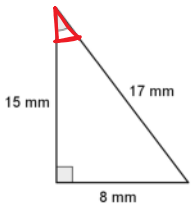


$\sin\theta = \frac{O}{H} = \frac{24}{28} = \frac{6}{7}$

$\cos\theta = \frac{A}{H} = \frac{4}{7}$

$\tan\theta = \frac{O}{A} = \frac{3}{2}$

13.

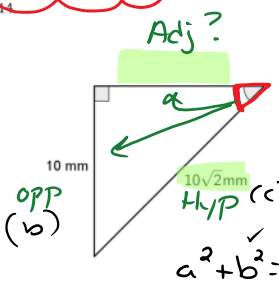


$\sin\theta = \frac{O}{H} = \frac{8}{17}$

$\cos\theta = \frac{A}{H} = \frac{15}{17}$

$\tan\theta = \frac{O}{A} = \frac{8}{15}$

14.



$a^2 + b^2 = c^2$

$\frac{O}{H}\ \sin\theta = \frac{10}{10\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\frac{A}{H}\ \cos\theta = \frac{10}{10\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\frac{O}{A}\ \tan\theta = \frac{10}{10} = 1$

$a = \sqrt{c^2 - b^2}$

$a = \sqrt{(10\sqrt{2})^2 - (10)^2}$

$a = \sqrt{200 - 100}$

$a = \sqrt{100}$

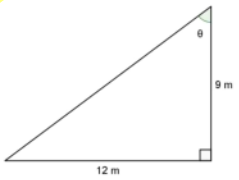
$a = 10$

$(10\sqrt{2})^2$

$10^2 \cdot (\sqrt{2})^2$

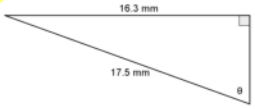
$100 \cdot 2 = 200$

15. Find $sin\theta$.



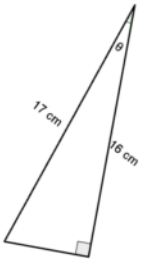
$sin\theta =$

16. Find $cos\theta$.



$cos\theta =$

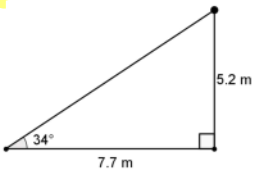
17. Find $tan\theta$.



$tan\theta =$

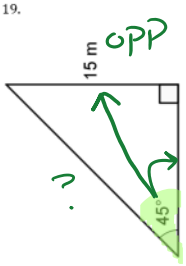
In each of the following diagrams, identify which ratio is represented (Sine, Cosine or Tangent).

18.



Ratio_____

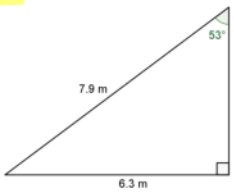
19.



opp
Adj

Ratio Tangent = opp/Adj

20.



Ratio_____

Use a scientific calculator to determine a decimal approximation for each of the following. Round to 4 decimals if necessary.

21. sin 30 ° =

22. $tan70° =$

23. $cos35° =$

24. sin 42 ° =

25. tan 45 ° = 1.0000

26. cos 60 ° =

27. Notice that $tan45° = 1$ in the question above. Refer to another question above to help you describe what that means.

Q#19

$Tan = \frac{opp}{Adj} = \frac{15}{15} = 1$ 2 side lengths same

∴ always 45° angle

28. Explain what it means for a right triangle to have a sine ratio equal to $\frac{1}{1}$.

→ all Trig ratios have 3 variables eg. $\tan\theta = \frac{O}{A}$

Skill Reminder:

Solve the following equations. Answer to the nearest hundredth if necessary.

29. $\frac{12}{x} = 4$

$\frac{12}{x} = \frac{4}{1}$ set up equivalent fractions

$12 = 4x$ cross-multiply

$3 = x$ isolate the variable

30. $\frac{x}{4} = 6$ ×4

$\therefore x = 6 \times 4$

$= 24$

31. $\frac{x}{4} = 0.55$

32. $\frac{x}{5} = \frac{10}{2}$

variable is on bottom → FLIP

33. $\frac{1}{x} = 1.3$ ×x

$1 = 1.3 \cdot x$

$\frac{1}{1.3} = \frac{1.3x}{1.3}$

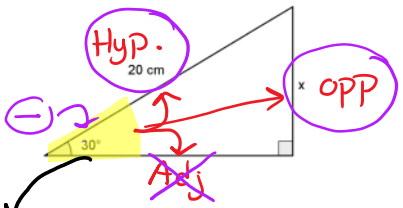
$\frac{1}{1.3} = x$

$0.769 = x$

34. $\frac{x}{2.5} = 6$

35. Challenge.

If we know $\sin 30° = 0.5000$, find the length of the missing side in the following diagram.



$\sin\theta = \frac{O}{H}$

$\sin(30) = \frac{x}{20}$

$0.5 = \frac{x}{20}$ ×20

$\frac{x}{20} = 0.5$ ×20

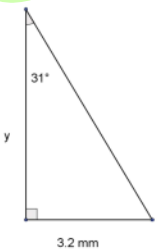
$\therefore x = (0.5)(20)$

$x = 10$

1-step calc = $(\sin(30))(20)$

36. Challenge.

Find the length of the indicated side.



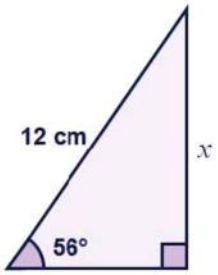
Homework: up to Q #36

Finding Side Length Using Trigonometry

If we are given _____and _____in a right-angled triangle we can use one of the three trigonometric ratios to find the lengths of other sides.

For example,

Find x to 2 decimal places.



We are given the_____and we want to find the length of the side_____the angle, so we use:

$$\_\_\_\_\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\_\_\_\_\_ = \frac{x}{12}$$

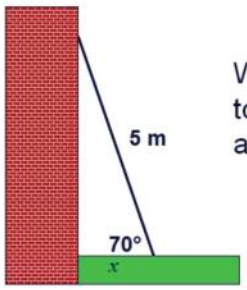
$$x = \_\_\_\_\_$$

$$= \_\_\_\_\_$$

Example 2:

A 5 m ladder is resting against a wall. It makes an angle of 70° with the ground.

What is the distance between the base of the ladder and the wall?



We are given the_____and we want to find the length of the side_____to the angle, so we use:

$$\_\_\_\_\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\_\_\_\_70° = \boxed{}$$

$$x = \_\_\_\_\_$$

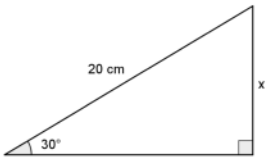
$$= \_\_\_\_\_$$

1 decimal Place

Finding Side Lengths Using Trigonometry

Find the length of the indicated side using an appropriate trigonometric ratio. Answer to tenths.

37.



We know:

$\sin 30° = 0.5000$

We also know: $\sin 30° = \frac{x}{20}$

We can say:

$0.5000 = \frac{x}{20}$

Solve the proportion:

$20(0.5000) = x$

$10.0 = x$

NOTE

To solve this problem...we can write...

$\sin 30° = \frac{x}{20}$

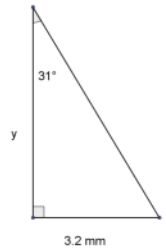
Multiply both sides by 20 to give:

$20sin30° = x$

Type $20 \times sin30$ into calculator...

$10.0 = x$

38.



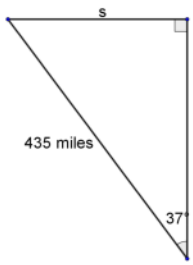
$tan\theta = \frac{opp}{adj}$

$tan31 = \frac{3.1}{y}$

$ytan31 = 3.2$

$y = \frac{3.1}{tan\ 31} = 5.3$ mm

39.

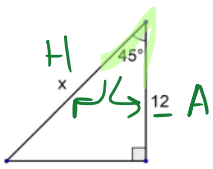


40.

SOH CAH TOA

O/H A/H O/A

41.



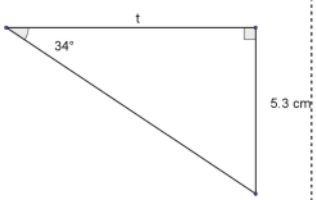
H A

$\cos\theta = \frac{A}{H}$

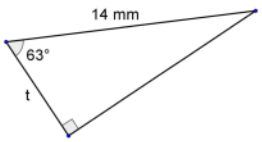
$\cos(45) = \frac{12}{x}$

$x = \frac{12}{\cos(45)} = 16.97 \cong 17.0$

42.



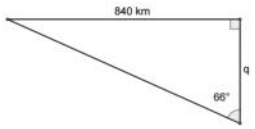
43.



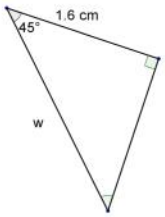
P 12. HW

Find the length of the indicated side using an appropriate trigonometric ratio. Answer to tenths.

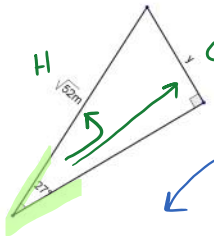
44.



45.



46.



sinθ = O/H

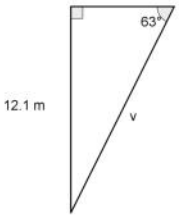
sin(27) = y/√52

y = (sin(27))(√52)

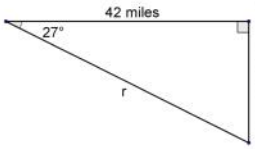
y = 3.3

sin(27) x √52

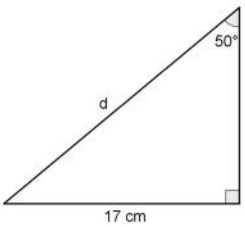
47.



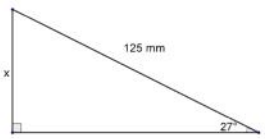
48.



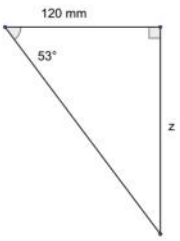
49.



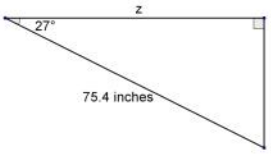
50.



51.

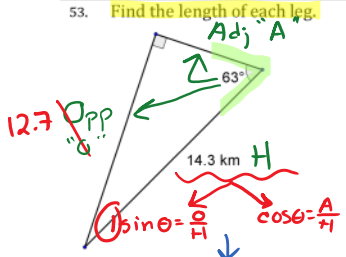


52.

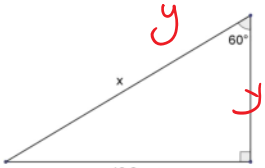


Find the length of the indicated side using an appropriate trigonometric ratio.

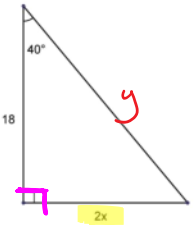
53. Find the length of each leg.



54. solve x and y



55. Solve for x and y



Hint: $\tan(40°) = \frac{2x}{18}$ means, $2x = (\tan 40°)(18)$ so... $x = \frac{(\tan 40°)(18)}{2}$

① $\sin(63) = \frac{Opp}{14.3}$

$Opp = \sin(63) \cdot 14.3$

$Opp = 12.7$ km

② $\cos\theta = \frac{A}{H}$ OR $\tan\theta = \frac{O}{A}$

$\cos(63) = \frac{A}{14.3}$ $\tan\theta = \frac{12.7}{A}$

$A = (\cos 63)(14.3)$ $A = \frac{12.7}{\tan(63)}$

$A = 6.5$ km $A = 6.5$ km

56. The tangent ratio is a ratio of what two sides in a right triangle?

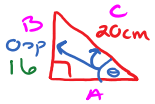
57. Can you use the tangent ratio to find the hypotenuse of a right triangle?

58. Can you use the sine ratio to find the hypotenuse of a right triangle?

HW up to #58

Class Examples

59. The sine ratio of a right triangle is $\frac{4}{5}$. If the hypotenuse is 20 cm long, what are the lengths of the other two sides?



$a^2 + b^2 = c^2$

$a = \sqrt{c^2 - b^2}$

$a = \sqrt{(20^2) - (16^2)}$

$\sin\theta = \frac{4}{5} = \frac{O}{H}$

$\frac{4}{5} = \frac{Opp}{20}$ $Opp = 16$

60. The cosine ratio of a right triangle is $\frac{9}{20}$. If the hypotenuse is 8 m long, what are the lengths of the other two sides?

61. The cosine ratio for a right triangle is 2.1042. Find the opposite side if the hypotenuse is 4 mm.

$\cos\theta = \frac{A}{H}$

$(2.1042) = \frac{A}{4}$

$A = (2.1042)(4)$

62. The sine ratio for a right triangle is 7.1004. Find the hypotenuse if the opposite side is 17 cm.

$\sin\theta < 1$

$\sin\theta = \frac{O}{H}$

$7.1004 = \frac{17}{H}$

$a = \sqrt{c^2 - b^2}$

$a = \sqrt{(20^2) - (16^2)}$

$a = 12$

$\frac{4}{5} = \frac{16}{20}$ (×4)

$A = (2.1042)(4)$

$A = 8.4168$

• not possible to use cosθ to find opp.

$\cos\theta < 1$

$7.1064 = \frac{17}{H}$

$\therefore H = 2.39$

*NOT possible.

63. Draw a diagram illustrating the cosine ratio for $\angle J$ in $\triangle JKL$. if

"angle J"

"Triangle JKL"

$\angle K = 90°$,
$\overline{JK} = 11\ cm$,
$\overline{KL} = 11\ cm$

"side from J → K"

$\cos\angle J$

$\cos\theta = \frac{A}{H}$

$\cos\theta = \frac{11}{H}$

$c = \sqrt{a^2 + b^2}$

$c = \sqrt{11^2 + 11^2}$

$c = \sqrt{242}$

$\cos\theta = \frac{11}{\sqrt{242}}$

$\cos\theta = \frac{11}{11\sqrt{2}}$

$\sqrt{242} = 11\cdot\sqrt{2}$

$\sqrt{121} \times 2$

$11\sqrt{2}$

64. Draw a diagram illustrating the tangent ratio for $\angle P$ in $\triangle PQR$ if

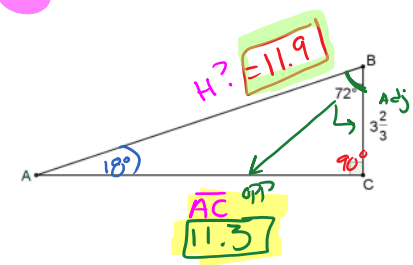
$\angle R = 90°$,
$\overline{PQ} = 10cm$,
$\overline{PR} = 8\ cm$

Solving Triangles:

To "solve a triangle" means to find the length of all unknown sides and measure of unknown angles.

180 - 72 - 90 = 18

65. Explain the steps you would take to solve the following triangle.



72°

① use Tan θ = opp/adj to solve for side AC

Tan(72) = opp/(3 2/3) opp = tan(72)(3 2/3)

opp = 11.3

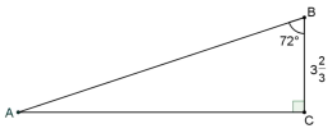
② use sin θ = o/H OR cos θ = A/H

sin 72 = 11.3/H H = 11.3/sin(72) = 11.9

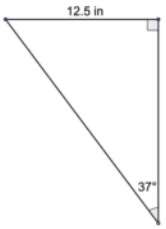
cos(72) = 3 2/3 / H H = 3 2/3 / cos(72) = 11.9

Solve the following triangles. Answer to tenths.

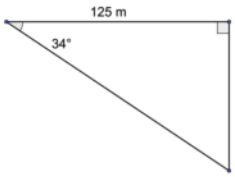
66.



67.



68.



69. The dotted line is an altitude (perpendicular to base).