2) 

**Consistent & Inconsistent Solutions**

**Warm-Up:** Solve each system of equations graphically and verify algebraically.

\[ \begin{align*}
\text{a) } & \begin{cases} 
y = 3x + 2 \\
2x - y = -4 
\end{cases} \\
\text{Solution: } & (2, 8) \\
\text{Verification: } & y = 3x + 2 \\
& (8) = 3(2) + 2 \\
& 8 = 6 + 2 \\
& \text{Yes, } (2, 8) \text{ is a solution for both lines.}
\end{align*} \]

\[ \begin{align*}
\text{b) } & \begin{cases} 
3x - y = 0 \\
6x + 2y = -8 
\end{cases} \\
\text{Solution: } & (3, -4) \\
\text{Verification: } & 3x - y = 0 \\
& (3) - (3) = 0 \\
& 0 - 4 = 0 \\
& \text{Yes.}
\end{align*} \]
### IMPORTANT IDEAS:

A system of linear equations can have **one** solution, **no** solution, or an **infinite** number of solutions. Before solving, you can predict the number of solutions for a linear system by comparing the **slope** and **y-intercept** of the equations.

<table>
<thead>
<tr>
<th>Intersecting Lines</th>
<th>Parallel Lines</th>
<th>Coincident Lines</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1 solution</strong> (intersect once)</td>
<td><strong>0 solution(s)</strong></td>
<td><strong>∞ solution(s)</strong></td>
</tr>
<tr>
<td>Different slopes</td>
<td>Same slopes</td>
<td>Same slopes</td>
</tr>
<tr>
<td>Point of intersection</td>
<td>y-intercepts</td>
<td></td>
</tr>
</tbody>
</table>

**Example #1:** Predict the number of solutions for each linear system. Justify your answer.

1. \(6x + y = 3\)
   - \(y = -x + 3\)
   - \(m_1 = -1\)
   - \(b_1 = 3\)
   - \(-2x - y + 2 = 0\)
   - \(y = 2x + 2\)
   - \(m_2 = 2\)
   - \(b_2 = 2\)
   - **m1 \neq m2** \(\Rightarrow\) **different slopes**
   - \(b_1 \neq b_2\) \(\Rightarrow\) **different y-intercepts**
   - **ONE SOLUTION**

**Example #2:** Given the equation \(2x - y + 4 = 0\) write another linear equation that will form a linear system with the following number of solutions:

1. \(\text{ax} - y + 1 = 0\)
   - **Exactly one solution**
   - \(m = 2\)
   - \(b = -4\)

2. \(y = 2x + 4\)
   - **No solution**

3. \(y = \frac{3}{2}x - \frac{2}{3}\)
   - **Infinite solutions**
   - \(m_1 = \frac{3}{2}\)
   - \(b_1 = -\frac{2}{3}\)
   - \(m_2 = \frac{3}{2}\)
   - \(b_2 = -\frac{2}{3}\)
a) Exactly one solution

\[ y = \square x + \square \]

diff. slope same or different

eg. \( y = 5x + 4 \) (other answers)

b) No solution

\[ y = \square x + \square \]

same slope must be different (parallel lines) b ≠ 4

eg. \( y = 2x + 7 \) (other answers)

c) Infinite solutions. = same line

\[ y = \square x + \square \]

same slope same y-int

must be \( y = 2x + 4 \)

-----

Example #3: For the linear system \( x - 2y + 4 = 0 \) and \( 7x - 14y + C = 0 \), what value(s) of \( C \) would give:

\[ y = \frac{-1}{3}x + 2 \]

\[ m_1 = \frac{-1}{3} \]

\[ y \text{-int} = 2 \]

-----

a) No solution

parallel no intersection

\( m_1 = m_2 \)

\( b_1 \neq b_2 \)

\( C \) must = anything other than 28

\( C \neq 28 \)

b) An infinite number of solutions

same line

\( m_1 = m_2 \)

\( b_1 \neq b_2 \)

\( \frac{28}{141} = 2 \)

\( C = 28 \) (must)

-----

A finite number of solutions

\( m, \neq m_2 \)

\( y \text{-int} (b) = \text{same or dif.} \)

\( m_1 = \frac{1}{2} \)

\( m_2 = \frac{1}{2} \)

NOT possible. The value of \( C \), will not change the slope.

-----

ASSIGNMENT #2

Pages 9-11 Questions #25-50

1-2
28. Challenge

On the three graphs below, draw a system of linear equations with ...

![Graphs showing one solution, no solution, and infinite solutions.]

a) One solution  
b) No solutions  
c) Infinite Solutions

30. Challenge

How many solutions are there to the system

\[ y = 3x + 3 \]
\[ y = x + 1 \]

Explain your reasoning.

<table>
<thead>
<tr>
<th>Types of Solution Sets:</th>
<th>One solution</th>
<th>No Solutions</th>
<th>Infinite Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lines intersect once.</td>
<td>Parallel Lines</td>
<td>Same Slopes</td>
<td>Same Lines</td>
</tr>
<tr>
<td>Different Slopes.</td>
<td>Same Slopes</td>
<td>Different ( y )-intercepts</td>
<td>Same Slopes</td>
</tr>
</tbody>
</table>

We say the system is CONSISTENT

We say the system is INCONSISTENT (no solution)

We say the system is CONSISTENT
Determine if the following systems have one solution, no solutions, or infinite solutions.

<table>
<thead>
<tr>
<th>31. [ y = 3x + 3 ]</th>
<th>32. [ y = 2x + 5 ]</th>
<th>33. [ 3y = 9x + 12 ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ y = x + 1 ]</td>
<td>[ y = 3x - 5 ]</td>
<td>[ 3x - 9y = 12 ]</td>
</tr>
</tbody>
</table>

One solution because the slopes are different.

Lines will intersect once.

<table>
<thead>
<tr>
<th>34. [ 6x + 4y = 1 ]</th>
<th>35. [ 2x + y = 5 ]</th>
<th>36. [ \frac{7}{3}x + 5 ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ 3x - 2y = 4 ]</td>
<td>[ y = -2x - 5 ]</td>
<td>[ 3y = 2x - 5 ]</td>
</tr>
</tbody>
</table>

Find the value of k that makes each system inconsistent.

<table>
<thead>
<tr>
<th>37. [ y = kx - 3 ]</th>
<th>38. [ 2y = kx + 1 ]</th>
<th>39. [ 4kx = y - 2 ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ 2y = 2x + 6 ]</td>
<td>[ 2x - y = 7 ]</td>
<td>[ 5x + 3y - 12 = 0 ]</td>
</tr>
</tbody>
</table>

Find the value of b that will produce a system with infinite solutions.

<table>
<thead>
<tr>
<th>40. [ y = x - b ]</th>
<th>41. [ 3x - y = 7 ]</th>
<th>42. [ 2x + 3y - 2b = 0 ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ 2y = 2x - 4 ]</td>
<td>[ 4y = 12x + b ]</td>
<td>[ y = -\frac{2}{3}x + 1 ]</td>
</tr>
</tbody>
</table>
43. Solve:
   \[ 2x + 3y - 6 = 0 \]
   \[ 3x - y + 2 = 0 \]

44. The system above is
   a) Consistent
   b) Inconsistent

45. Solve:
   \[ x - y = 1 \]
   \[ 5x + 2y = 5 \]

46. Add the two equations above and graph the new equation.

47. What do you notice?

48. Graph the system of equations:
   \[ y = x + 2 \]
   \[ 3y = 2x - 5 \]

49. What is the problem when solving this system by graphing?

50. Challenge

Solve the system of linear equations: \[ y = x + 2 \] and \[ 3y = 2x - 5 \].