2-Consistent & Inconsistent Solutions

January 6, 2019 6:08 PM

2) consistent & Inconsistent solutions

Warm-Up: Solve each system of equations graphically and verify algebraically..

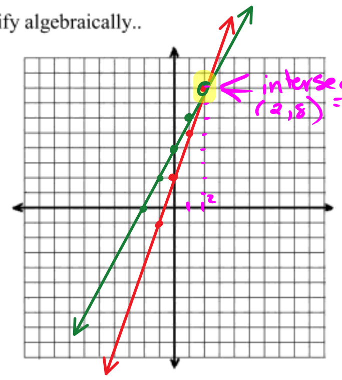
a) $\begin{cases} y = 3x + 2 \\ 2x - y = -4 \end{cases}$

$y = 3x + 2$

$m = \frac{3}{1}$

$y\text{-int} = 2$

$2x - y = -4$
$\quad +4 \quad\quad +4$
$2x + 4 - y = 0$
$\quad\quad +y \quad +y$
$2x + 4 = y$
or $y = 2x + 4$
$m = \frac{2}{1}$
$y\text{-int} = 4$



Solution: (2,8)
(x,y) substitute

Verification:

$y = 3x + 2$
$(8) = 3(2) + 2$
$8 = 6 + 2$
$8 = 8$ ✓

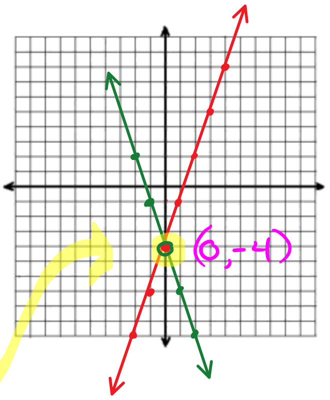
$2x - y = -4$
$2(2) - (8) = -4$
$4 - 8 = -4$
$-4 = -4$ ✓

yes (2,8) is a solution for BOTH lines.

b) $\begin{cases} 3x - y - 4 = 0 \\ 6x + 2y = -8 \end{cases}$

$3x - y - 4 = 0$
$\quad +y \quad\quad +y$
$3x - 4 = y$
or $y = 3x - 4$
$m = \frac{3}{1}$
$y\text{-int} = -4$

$6x + 2y = -8$
$-6x \quad\quad -6x$
$\frac{2y}{2} = \frac{-6x - 8}{2}$
$y = -\frac{6}{2}x - 4$
$y = -3x - 4$
$m = -\frac{3}{1}$
$y\text{-int} = -4$



Solution: (0,-4)

Verification:

$3x - y - 4 = 0$
$3(0) - (-4) - 4 = 0$
$0 + 4 - 4 = 0$
$0 = 0$ ✓

$6x + 2y = -8$
$6(0) + 2(-4) = -8$
$0 + (-8) = -8$
$-8 = -8$ ✓

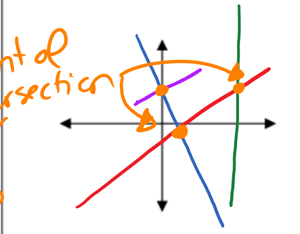
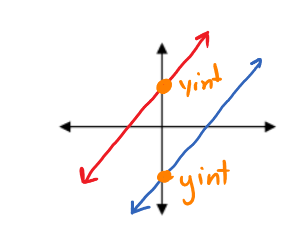
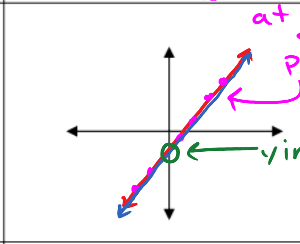
1

10

IMPORTANT IDEAS:

A system of linear equations can have one solution, no solution, or an infinite number of solutions. Before solving, you can predict the number of solutions for a linear system by comparing the slope and y-intercept of the equations. => Formula is Slope-Intercept Form

$y = mx + b$ (slope, y-int)

Intersecting Lines	Parallel Lines	Coincident (SAME) Lines
1 solution (intersect once)	∅ solution(s) do not intersect	∞ solution(s) (same line = intersect at every single point on the line)
		
Different slopes $m_1 \neq m_2$	SAME slopes $m_1 = m_2$	SAME slopes $m_1 = m_2$
same or diff y-intercepts (doesn't matter)	Different y-intercepts $b_1 \neq b_2$	SAME y-intercepts $b_1 = b_2$
"Consistent"	"Inconsistent"	"Consistent"

... systems of equations ....

point of intersection

if lines cross, MUST have .... Different slopes

Example #1: Predict the number of solutions for each linear system. Justify your answer.

#1 Step convert to Slope-Int. Form: $y = mx + b$

2+ equations ... this is a "system"

a) ① $x + y = 3$ ($-x$)
② $-2x - y + 2 = 0$

① $y = -x + 3$
$m_1 = -1$
(y-int) $b_1 = +3$

② $-2x - y + 2 = 0$
$+y \quad +y$
$-2x + 2 = y$
$y = -2x + 2$
$m_2 = -2$
(y-int) $b_2 = +2$

∴ $m_1 \neq m_2$ diff slopes
$b_1 \neq b_2$ diff y-int.
ONE SOLUTION

b) ① $4x + 6y + 10 = 0$
② $-2x - 3y = 5$

① $4x + 6y + 10 = 0$
$-4x \quad -10 \quad -4x - 10$
$\frac{6y}{6} = \frac{-4x - 10}{6}$
$y = -\frac{4}{6}x - \frac{10}{6}$ simplify
$\boxed{y = -\frac{2}{3}x - \frac{5}{3}}$
$m_1 = -\frac{2}{3}$ $b_1 = -\frac{5}{3}$

② $-2x - 3y = 5$
$+2x \quad +2x$
$\frac{-3y}{-3} = \frac{2x + 5}{-3}$
$\boxed{y = -\frac{2}{3}x - \frac{5}{3}}$
$m_2 = -\frac{2}{3}$ $b_2 = -\frac{5}{3}$

∴ $m_1 = m_2$ parallel
$b_1 = b_2$ ⇐ means same line
∞ solutions

c) ① $2x - 4y + 1 = 0$
② $3x - 6y - 2 = 0$

① $2x - 4y + 1 = 0$
$+4y \quad +4y$
$\frac{2x + 1}{4} = \frac{4y}{4}$
$\frac{2}{4}x + \frac{1}{4} = y$ => $y = \frac{1}{2}x + \frac{1}{4}$
$m_1 = \frac{1}{2}$ $b_1 = \frac{1}{4}$

② $3x - 6y - 2 = 0$
$+6y \quad +6y$
$\frac{3x - 2}{6} = \frac{6y}{6}$
$y = \frac{3}{6}x - \frac{2}{6}$
$y = \frac{1}{2}x - \frac{1}{3}$
$m_2 = \frac{1}{2}$ $b_2 = -\frac{1}{3}$

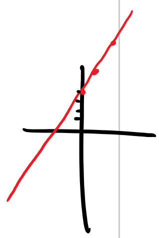
$m_1 = m_2$ => parallel lines
$b_1 \neq b_2$
∅ solutions

Example #2: Given the equation $2x - y + 4 = 0$ write another linear equation that will form a linear system with the following number of solutions.

① Convert into $y = mx + b$

$2x - y + 4 = 0$
$+y \quad +y$

$y = 2x + 4$
$m = 2 = \frac{2}{1}$
b: y-int = +4



a) Exactly one solution

b) No solution

c) Infinite solutions. = same line

a) Exactly one solution

$y = \boxed{}x + \boxed{}$

diff. slope $m \neq 2$

same or diff (any #)

eg. $y = 5x + 4$ (other answers)

b) No solution

$y = \boxed{2}x + \boxed{}$

same (parallel lines)

must be diff. $b \neq 4$

eg $y = 2x + 7$ (other answers)

c) Infinite solutions. = same line

$y = \boxed{}x + \boxed{}$

same slope

same y-int

must be => $y = 2x + 4$

Example #3: For the linear system $x - 2y + 4 = 0$ and $7x - 14y + C = 0$, what value(s) of C would give:

*put in slope-int. form.

① +2y +2y ②

$\frac{\cancel{2}y}{\cancel{2}} = \frac{x + 4}{2}$

$y = \frac{1}{2}x + 2$

$m_1 = \frac{1}{2}$ y-int = 2

$7x - \cancel{14y} + C = 0$

$+\cancel{14y}$ $+14y$

$\frac{7x + C}{14} = \frac{\cancel{14}y}{\cancel{14}}$

$y = \frac{7}{14}x + \frac{C}{14}$

$y = \frac{1}{2}x + \frac{C}{14}$

$m_2 = \frac{1}{2}$

y-int = $\frac{C}{14}$

a) No solution

• parallel

• no intersection

$m_1 = m_2$

$b_1 \neq b_2$

C must = anything other than 28

$C \neq 28$

b) An infinite number of solutions

• same line

$m_1 = m_2$

$b_1 = b_2 \leftarrow 28$

$2 = \frac{C}{14}$

$\frac{28}{14} = 2$

$C = 28$ (must)

c) Exactly one solution

• intersect at 1 point

$m_1 \neq m_2$

y-int (b) = same or diff.

$m_1 = \frac{1}{2}$

$m_2 = \frac{1}{2}$

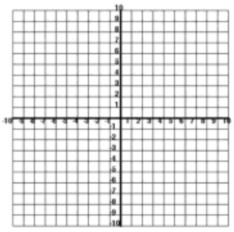
NOT possible. The value of C, will not change the slope.

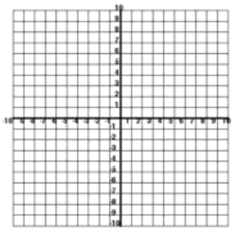
ASSIGNMENT # 2

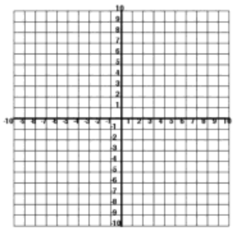
pPages 9-11 Questions #25-50

29. ## Challenge

On the three graphs below, draw a system of linear equations with . . .


a) One solution


b) No solutions


c) Infinite Solutions

30. ## Challenge

How many solutions are there to the system

$y = 3x + 3$

$y = x + 1$

Explain your reasoning.

Types of Solution Sets:		
One solution	No Solutions	Infinite Solutions
• Lines intersect once. • Different Slopes.	• Parallel Lines • Same Slopes • Different y-intercepts	• Same Lines • Same Slopes • Same y-intercepts
We say the system is **CONSISTENT**	We say the system is **INCONSISTENT** (no solution)	We say the system is **CONSISTENT**

Determine if the following systems have one solution, no solutions, or infinite solutions.

31. $y = 3x + 3$ $y = x + 1$ One solution because the slopes are different. Lines will intersect once.	32. $y = 2x + 5$ $y = 3x - 5$	33. $3y = 9x + 12$ $3x - 9y = 12$
34. $6x + 4y = 1$ $3x - 2y = 4$	35. $2x + y = 5$ $y = -2x - 5$	36. $y = \frac{2}{3}x + 5$ $3y = 2x - 5$

Find the value of k that makes each system **inconsistent.**

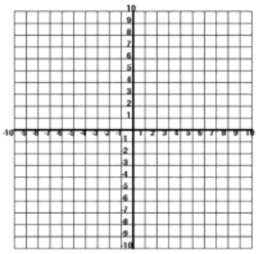
37. $y = kx - 3$ $2y = 2x + 6$	38. $2y = kx + 1$ $2x - y = 7$	39. $4kx = y - 2$ $5x + 3y - 12 = 0$

Find the value of b that will produce a system with **infinite solutions**.

40. $y = x - b$ $2y = 2x - 4$	41. $3x - y = 7$ $4y = 12x + b$	42. $2x + 3y - 2b = 0$ $y = -\frac{2}{3}x + 1$

43. Solve:

$$2x + 3y - 6 = 0$$
$$3x - y + 2 = 0$$



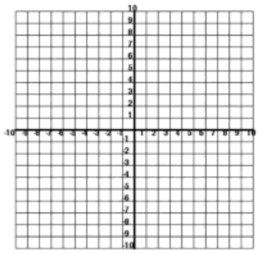
44. The system above is

a) Consistent

b) Inconsistent

45. Solve:

$$x - y = 1$$
$$5x + 2y = 5$$



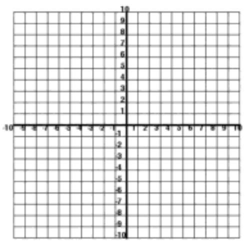
46. Add the two equations above and graph the new equation.

47. What do you notice?

48. Graph the system of equations:

$y = x + 2$

$3y = 2x - 5$



49. What is the problem when solving this system by graphing?

50. ## Challenge

Solve the system of linear equations: $y = x + 2$ and $3y = 2x - 5$.