3- Parallel + Perpendicular Lines

November 28, 2018 1:43 PM

① Journal => Hand In

② Warm-up Questions (on your own!)

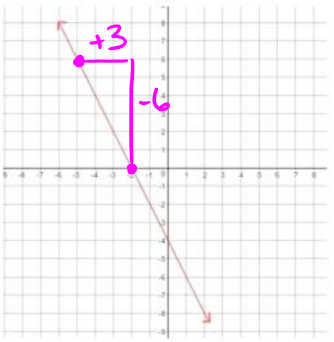
3) parallel and perpendicular lines

slope = $\frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$

Warm-Up:

1. Find the slope of the following lines using the graphs below:

a)

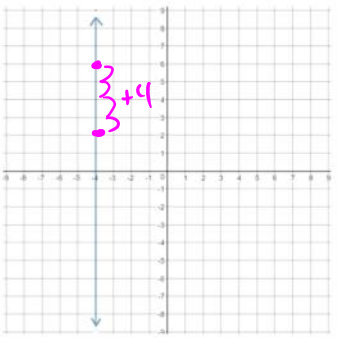


rise: -6 } you may have different values here.

run: +3 }

m = $\frac{-6}{3} = -2$

b)



rise: +4 > any number

run: 0

m = $\frac{4}{0}$ = undefined (can't divide by 0)

2. a) Calculate the slope of the line that passes through A (2, 6) and B (8, 15). Give your answer in lowest terms.

x_1, y_1 x_2, y_2

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{15-6}{8-2} = \frac{9}{6}$ simplify $\div 3 = \boxed{\frac{3}{2}}$

b) Find the co-ordinates of any other point on this line.

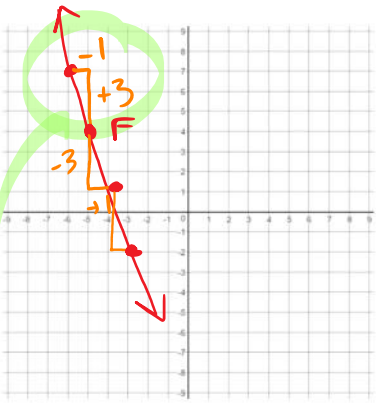
add the run (x) (2 +2 , 6 +3) add the rise (y)

(4, 9)

slope $= \frac{3}{2} = \frac{\Delta y}{\Delta x}$

3. Graph the line that passes through the point F (-5, 4) and has a slope of m = -3. (Plot at least 4 points)

remember m = -3 is really $\frac{-3}{1} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$



why have I reversed the ± sign of the slope here?

plot a point in the reverse direction, need to also reverse slope!

Part 1: Parallel Lines

Parallel lines NEVER intersect (cross)

Parallel lines have EQUAL = the exact same slope

Example #1: State the slope that is parallel:

a) $m_A = \frac{-3}{8}$: $m_B = \frac{-3}{8}$

b) $m_A = 4$: $m_B = \frac{4}{1} = \frac{\text{rise}}{\text{run}}$

Example #2: Determine if AB is parallel to CD.

A (17, 82) x_1, y_1	B (21, 92) x_2, y_2
C (6, 20) x_1, y_1	D (10, 30) x_2, y_2

$m_{AB} = \frac{92-82}{21-17} = \frac{10 \div 2}{4 \div 2} = \frac{5}{2}$

$m_{CD} = \frac{30-20}{10-6} = \frac{10}{4} = \frac{5}{2}$

① Find slope m_{AB}
② Find slope m_{CD}
③ Are they equal?

$m_{AB} = m_{CD}$
$\frac{5}{2} = \frac{5}{2}$ yes, parallel

Example #3: Find "k" if the following slopes are parallel.

a) $m_1 = \frac{4}{3}$ and $m_2 = \frac{k}{2}$ $\quad m_1 = m_2$

$\frac{4}{3} = \frac{k}{2}$
$4 \times 2 = 3k$
$\frac{8}{3} = \frac{3k}{3}$
$\therefore k = \frac{8}{3}$

b) $m_1 = -\frac{4}{5}$ and $m_2 = \frac{10}{k}$

$m_1 = m_2$
$-\frac{4}{5} = \frac{10}{k}$
$(-4 \cdot k) = (5 \cdot 10)$
$-4k = 50$
$k = \frac{50}{-4} = \frac{25}{-2}$
$\therefore k = -\frac{25}{2}$

Example #4: Determine the co-ordinates of Point D, on the y-axis, so that MN is parallel to CD.

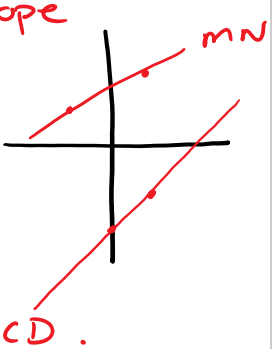
M (-3, 3) x_1, y_1	N (1, 5) x_2, y_2
C (4, -3)	D (0, y_2)

D: $x = 0$

same slope

① Find slope of MN

$m_{MN} = \frac{5-3}{1-(-3)} = \frac{2}{4} = \frac{1}{2}$



② Find "y_2" of CD

same $m_{MN} = \frac{1}{2}$

$m_{CD} = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{1}{2} = \frac{y_2 - (-3)}{0 - 4}$

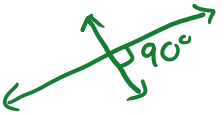
$\frac{1}{2} = \frac{y_2 + 3}{-4}$

$1 \cdot (-4) = 2(y_2 + 3)$

$-4 = 2y_2 + 6$
$-6 \quad -6$
$-10 = 2y_2$
$\frac{-10}{2} = \frac{2y_2}{2}$

$-5 = y_2$ $\quad D(0, -5)$

Part 2: Perpendicular Lines



Perpendicular lines intersect at a 90° angle ⊥ (perpendicular)

Perpendicular lines have a negative reciprocal slopes (switch sign) (flip the fraction)

Perpendicular slopes will always multiply to −1

Example #1: State the slope that is perpendicular to the following:

a) $m = \frac{-12}{5}$ ① Flip ② switch the sign $= \frac{5}{-12} = \frac{5}{12}$

b) $m = \frac{20}{1}$ ① Flip ② switch sign $= \frac{1}{20} = -\frac{1}{20}$

Example #2: Determine if AC is perpendicular to BD.

A (1,10) x_1, y_1	B (-3,7) x_1, y_1
C (-2,-1) x_2, y_2	D (8,10) x_2, y_2

$m_{AC} = -\frac{1}{m_{BD}}$

$m_{AC} = \frac{(-1) - 10}{(-2) - 1} = \frac{-11}{-3} = \frac{11}{3}$

$m_{BD} = \frac{10 - 7}{8 - (-3)} = \frac{3}{11}$

NO! NOT ⊥

☑ reciprocal fractions

☒ opposite signs

Telling us they ARE ⊥

Example #3: Find "k" if the following slopes are perpendicular.

① Take the neg. recip. of ONE ② set as EQUAL ③ Cross multiply ④ solve for k

a) $m_1 = \frac{12}{5}$ and $m_2 = \frac{k}{2}$

$\frac{-5}{12} = \frac{k}{2}$

$(-5 \cdot 2) = 12k$

$-10 = 12k$

$\frac{-10}{12} = \frac{12k}{12}$

$k = \frac{-5}{6}$

b) $m_1 = -\frac{4}{5}$ and $m_2 = \frac{10}{k}$

$+\frac{5}{4} = \frac{10}{k}$

$5k = 40$

$k = 8$

$m_1 \perp m_2$: $-\frac{4}{5}$ ⊥ $\frac{10}{8} = \frac{5}{4}$

Example #4: Is the triangle with vertices A (-2, -3), B (2, 1) and C (-4, -2) a right triangle?

① Plot the points

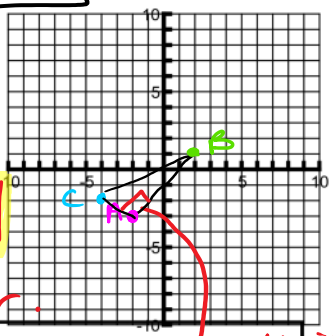
② Do we have a 90° angle?

③ Compare slopes of 2 Lines

$m_{AC} = \frac{-2 - (-3)}{-4 - (-2)} = \frac{1}{-2}$

$m_{AB} = \frac{1 - (-3)}{2 - (-2)} = \frac{4}{4} = 1$

☒ neg. reciprocal? NOT perpendicular.



is this 90°

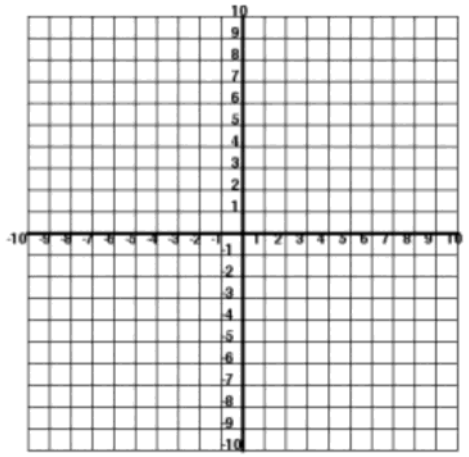
Homework

ASSIGNMENT # 3

Pages 16-20 questions #61-84+#100-103

61. Challenge # 5

Determine if AB is parallel to CD given the following points: A(1,2), B(5,4), C(0,-2), D(6,1).

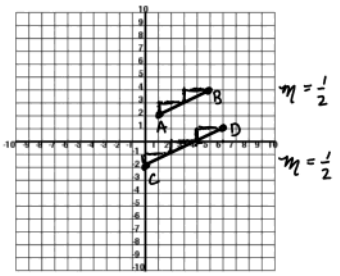


62. What can you say about the slopes of parallel line segments?

Slopes of Parallel Lines (or segments)

Recall two lines are parallel if they do not ever intersect.

Parallel lines have *equal slopes*.



Any two horizontal lines are parallel.
Any two vertical lines are parallel.

To determine if line segments are parallel, calculate their slopes.

Eg.1. Determine if AB is parallel to CD. A(1,2), B(5,4), C(0,-2), D(6,1).

Slope of AB:
$m_{AB} = \frac{4-2}{5-1} = \frac{2}{4} = \frac{1}{2}$

Slope of CD:
$m_{CD} = \frac{1-(-2)}{6-0} = \frac{3}{6} = \frac{1}{2}$ SAME SLOPES ∴ PARALLEL

Eg.2. The following are slopes of two lines. Find the value of k so that the two lines are parallel.

$m_1 = 2$ and $m_2 = -\frac{6}{k}$ Since the lines are parallel, slopes must be equal. $2 = \frac{-6}{k}$

Cross Multiply: $\frac{2}{1} = \frac{-6}{k}$ $2k = -6$ $k = -3$

Determine if the following pairs of line segments are parallel.

63. A(-2,-1), B(1,5) and C(2, -1), D(4,3)

64. E(-3, 0), F(1, 5) and G(0, -6), H(2, -1)

65. I(-4,0), J(8, 2) and K(2, 8), L(-2, 4)

The following are slopes of two lines. Find the value of k so that the two lines are parallel.

66. $m_1 = -\frac{2}{3}$ and $m_2 = -\frac{k}{9}$

67. $m_1 = -3$ and $m_2 = \frac{k}{4}$

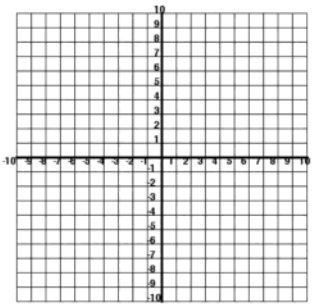
68. $m_1 = \frac{k}{3}$ and $m_2 = \frac{1}{2}$

70. The points A(6,3), B(2,9), and C(2,3) are given. Determine the coordinates of point D so that CD is parallel to AB and D is on the y-axis.

Slopes of Perpendicular Line Segments.

- The slopes of perpendicular lines are negative reciprocals.
- The product of perpendicular slopes is -1.

71. Plot the right triangle with vertices:
A(2,2), B(5,7), and C(10,4).



72. Find the slope of AB. m =

73. Find the slope of BC. m =

These segments form the right angle in the triangle.

74. What do you notice about the slopes of the two segments.

75. Multiply the two slopes. What is the result?

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76. Is the triangle with vertices X(-9,-1), Y(-7,7), Z(3,-4) a right triangle?

Perpendicular Lines will have slopes that are NEGATIVE RECIPROCALS.

Examples of perpendicular slopes are: $m_1 = 5,\ m_2 = -\frac{1}{5}$.

Examples of perpendicular slopes are: $m_1 = -\frac{5}{3},\ m_2 = \frac{3}{5}$.

Perpendicular slopes will have a product of -1.

Look at the example above... $-\frac{5}{3} \times \frac{3}{5} = -\frac{15}{15} = -1$

Determine the slope of a line segment perpendicular to a segment with each given slope.

77. $m = -3$

78. $m = -\frac{2}{3}$

79. $m = \frac{4}{5}$

The following are slopes of two lines. Find the value of k so that the two lines are perpendicular.

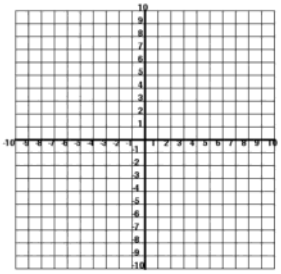
80. $m_1 = -\frac{2}{3}$ and $m_2 = -\frac{k}{9}$

81. $m_1 = -3$ and $m_2 = \frac{k}{4}$

82. $m_1 = \frac{k}{3}$ and $m_2 = \frac{1}{2}$

Graph each pair of line segments. Determine if they are perpendicular or not.

83. A(0,0), B(6,4) and C(7,3), D(-11,1)



84. G(2,10), H(-7,-2) and J(7,0), K(-5,9)