3.4 USING EXPONENTS TO SOLVE PROBLEMS

Name:________________ Block_____

A lot of mathematical and scientific formula's and equations involve variables AND EXPONENTS!

area of a circle — quadratic

For Example: $A = \pi r^2$ $\quad a^2 + b^2 = c^2$ $\quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\quad E_k = \frac{1}{2}v^2$

Pythagoras — kinetic energy

Now that you have practiced working with exponents you ***can apply your knowledge*** to a variety of practical problems.

A) SOLVING WORD PROBLEMS WITH EXPONENTS

Example #1:

Mountain pine beetles can double their population in one year if conditions are right. They live in mature lodgepole and jack pine trees by boring into the bark. Only 5 mm long, these small beetles can kill pine trees if their numbers are great enough. Suppose the beetle population in a particular area is 10,000 and it doubles each year. What will the population be in 1 year? 2 years? 3 years?

→ ×2

Ø is always the number you started with

a) Create a table to show the growth of the population of pine beetles over 3 years.

a) Time (years)	a) Beetle population (#)	b) Product of Power
0	10,000	$10000(2)^0$
1	20,000	$10000(2)^1$
2	40,000	$10000(2)^2$
3	80,000	$10000(2)^3$

b) Express the population as a product of 10,000 and a power of 2. Add this information to your table.

0 years 10000 ?
1 years 10 000 × 2 $= 2^1$
2 years 10 000 × 2 × 2 $= 2^2$
3 years 10 000 × 2 × 2 × 2 $= 2^3$

exponent is equal to year number

what is year Ø? $= 2^0$

c) What patterns do you notice in your table?

- 10 000 is the coefficient
- when a population is doubled, the number of times it is ×2 increases each year.

d) Write an expression in exponential form to determine the number of beetles in n years. Explain what each part of the expression represents.

where n = # of years

$\#\text{ of Beetles} = 10000(2)^n$

B) THE PYTHAGOREAN THEOREM

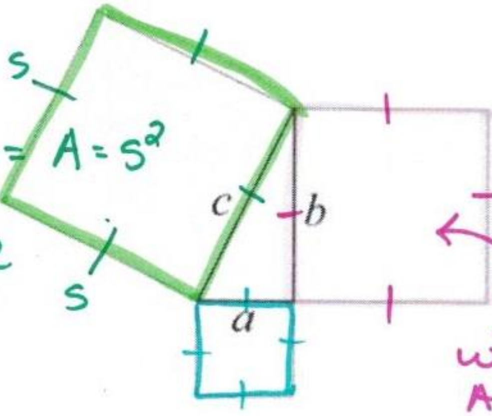
90° angle

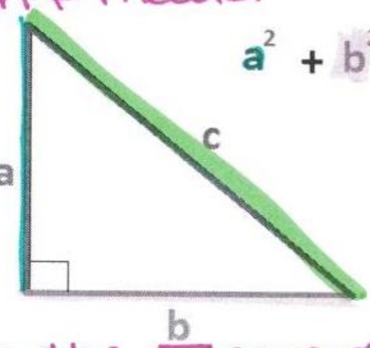
The Pythagorean theorem states that for any RIGHT triangle, the length of the hypotenuse squared is equal to the sum of the squares of the other two sides.

'a' and 'b' can switch, doesn't matter

$\hookrightarrow$ longest side

recall: Area of a square = $A = s^2$ b/c all sides are equal.





when the Area is known, the $\sqrt{\ }$ square root can provide side length

This can be rearranged in different ways so you ***can solve for different sides***:

DO ALL OPERATIONS TO BOTH SIDES!!

$a^2 + b^2 = c^2$		
$a^2 + b^2 = c^2$ $-b^2 \quad -b^2$ $\sqrt{a^2} = \sqrt{(c^2 - b^2)}$ $\therefore a = \sqrt{(c^2 - b^2)}$	$a^2 + b^2 = c^2$ $-a^2 \quad -a^2$ $\sqrt{b^2} = \sqrt{(c^2 - a^2)}$ $\therefore b = \sqrt{(c^2 - a^2)}$	$a^2 + b^2 = c^2$ so, $\sqrt{c^2} = \sqrt{a^2 + b^2}$ $\therefore c = \sqrt{(a^2 + b^2)}$

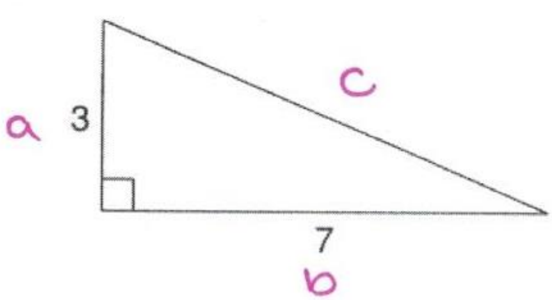
taking the $\sqrt{\ }$ eliminates the exponent

* as long as we know 2 sides (any 2) we can always find the length of the other

Example 2:

Solve for the unknown side length: (label sides)

a)



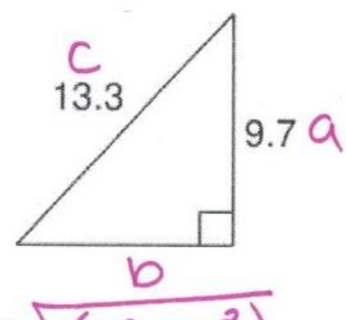
$c = \sqrt{(a^2 + b^2)}$

$c = \sqrt{(3^2 + 7^2)}$

$c = \sqrt{(9 + 49)}$

$c = \sqrt{58}$ $\boxed{= 7.62}$

b)



$b = \sqrt{(c^2 - a^2)}$

$b = \sqrt{(13.3^2 - 9.7^2)}$

$b = \sqrt{(176.89 - 94.09}$

$b = \sqrt{82.8}$ $\boxed{= 9.1}$

PRACTICE

64. The population of certain forms of bacteria double every day. If the population began with 1 million, how large would the population be after 7 days? Write your answer first as a power and then evaluate it.

day 0 = $1\,000\,000 \times 2^0$
1 = $1\,000\,000 \times 2^1$
(etc

day 7 = $1\,000\,000 \times 2^7$

$= 1\,000\,000(2)^7$

$\boxed{= 128\,000\,000}$

65. Rory is 16 and just invested \$1000 in a mutual fund that should grow in value by 8% per year. At this rate his money will double every 9 years. How much will his initial investment be worth when he retires at age 61? Write your answer first as a power and then evaluate it.

61 − 16 = 45 years invested.

45 ÷ 9 = 5 number of times it will double

year 0 = $1000(2)^0$
(1st ×2) 9 = $1000(2)^1$
(2nd ×2) 18 = $1000(2)^2$

$\boxed{\text{so, } \$1000(2)^5 = 32{,}000}$

it will double 5 times over 45 years.

66. The Richter scale represents a 10-fold increase in intensity for every 1 unit of magnitude on the Richter scale. That means that a Richter scale rating of 2 is ten times more intense than a Richter scale rating of 1. How much greater is a Richter scale rating of 8 compared to a Richter scale rating of 4? Write your answer first as a power and then evaluate it.

1	×10	$1 = 10^0$
2	×10	$2 = 10^1$
3	×10	$3 = 10^2$
4	×10	$4 = 10^3$
5	×10	$5 = 10^4$
6	×10	$6 = 10^5$
7	×10	$7 = 10^6$
8		$8 = 10^7$

$10^7 \div 10^3$

$\boxed{= 10^4 \\ = 10{,}000}$

Using exponents and order of operations to solve problems.

155. Balkee invested \$2000 in a mutual fund that returned 8% interest each year. The following formula can be used to determine the answer. A=\$2000$(1.08)^{23}$. How large will the investment be in 23 years?

$A = 2000(1.08)^{23}$

$A = 2000(5.87)$

$\boxed{A = \$11\,742.93}$

156. A colony of bees increases 2 fold every week. How large will the colony grow to after 20 weeks if it began with 2 bees. The following formula can be used to determine the answer. $A=2(2)^{20}$.

$A = 2(2)^{20}$

$A = 2(1048576)$

$\boxed{A = 2097152}$

157. A very nosey student asked Mr. Spray how much he charges his tenants each month for rent. Mr. Spray gladly answered, "I charge them 0.15×10^4 dollars each month." How much does he charge his tenants each month and how weird is he?

very! LOL!

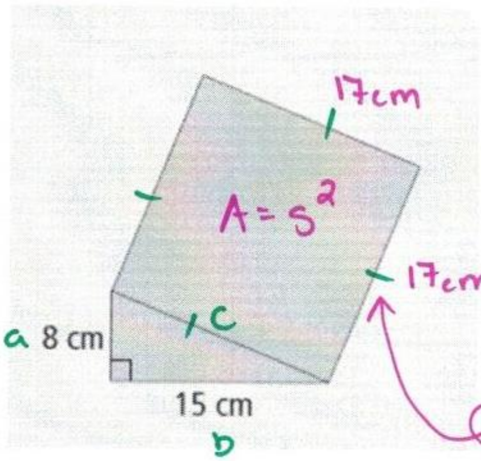
rent = 0.15×10^4
$= 0.15 \times (10\,000)$

$\boxed{\text{rent} = \$1\,500}$

...but very reasonable!

A right triangle has two shorter sides that measure 8 cm and 15 cm.

What is the area of a square attached to the hypotenuse of the right triangle?

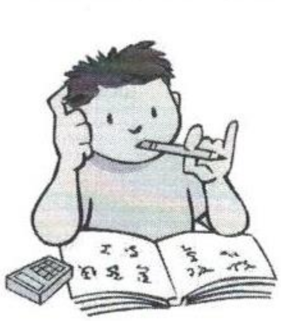


$$c = \sqrt{a^2 + b^2}$$
$$c = \sqrt{(8^2 + 15^2)}$$
$$c = \sqrt{(64 + 225)}$$
$$c = \sqrt{(289)}$$
$$\boxed{c = 17}$$

$$\therefore A = s^2$$
$$A = (17)^2$$
$$\boxed{A = 289\ cm^2}$$

"square units" when dealing in area.

ASSIGNMENT #5 Section 3.4 pg 98-99
Questions #1—6, 8-9 *10, 11



PRACTICE TEST: Chapter Review pg 101-103
Questions #1-10, 12, 14-19, 22-24

Extra (if you have time or need more practice)
Questions # 3, 5cd, 11, 13, 20, 21, 25

Extension Extra Tough BONUS:
Questions #26, 27, 28, 29

Work on in Class Tuesday!