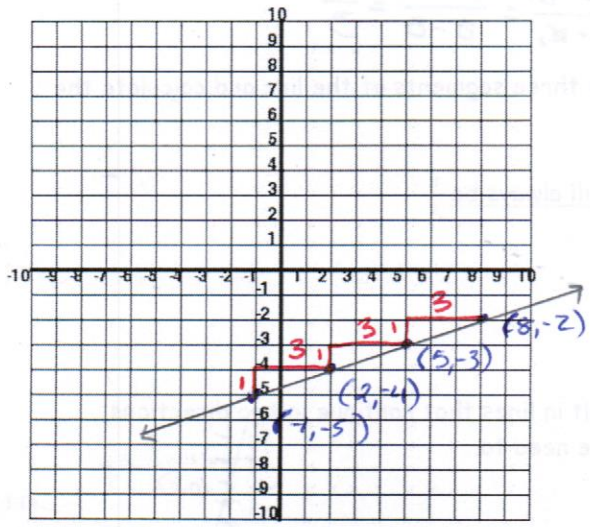
Slope of a Line

Challenge #4:

60. Plot the following points:
$(-1,-5), (2,-4), (5,-3), (8,-2)$



61. Draw a line through the points you plotted.

62. Choose three sections of the line you just plotted and find their slopes. rise / run.

Slope of section 1:

$m = \frac{\text{rise}}{\text{run}} = \frac{1}{3}$

Slope of section 2:

$m = \frac{1}{3}$

Slope of section 3:

$m = \frac{1}{3}$

63. What do you notice?

all sections have the same slope, $m = \frac{1}{3}$.

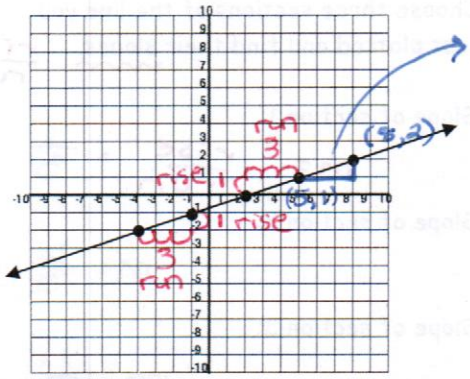
Some notes here...

if an equation shows the same slope at all points when graphed = linear

Slope of a Line

Recall from our discussion of line segments that slope can be calculated using: $m = \frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{rise}{run}$

For a straight line, the slopes of all segments on the line are equal. That is, if you find the slope of any two parts of the line, they will be equal.

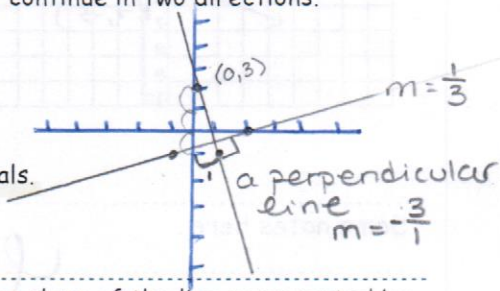


$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{8-5} = \frac{1}{3}$

Pick any three segments of the line and calculate the slope.

Slope will always be $\frac{1}{3}$.

The equations discussed earlier in this booklet result in lines that continue in two directions. Working with slope allows us to extend the line if we need to.



Remember:

- Parallel lines have equal slopes.
- Perpendicular lines have slopes that are negative reciprocals.

"Slope-Intercept Form" $y = mx + b$ (m = slope)

64. Find the slope of the line represented by the equation $y = 3x - 5$.

$m = 3$

65. Find the slope of the line represented by the equation $2x + 5y = 20$.

(0,4) (10,0) Find x and y intercepts to have 2 points.

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{0-10} = \frac{4}{-10} = -\frac{2}{5}$

$m = -\frac{2}{5}$

66. Find the slope of the line represented by the equation $y - 4 = 3(x - 5)$.

$y - 4 = 3x - 15$

$+4 \quad +4$

$y = 3x - 11$

$m = 3$

*can also manipulate equation to write in slope-int. form.

67. Find the slope of the line represented by the equation $\frac{1}{3}(x + 2) = y - 1$.

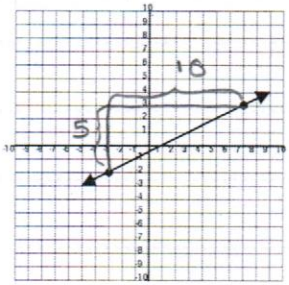
$\frac{1}{3}x + \frac{2}{3} = y - 1$

$+1 \quad +1$

$y = \frac{1}{3}x + \frac{5}{3}$

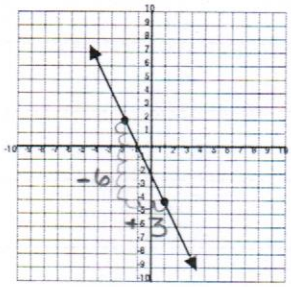
$m = \frac{1}{3}$

68. Find the slope of the line below.



Slope is $\frac{rise}{run} = \frac{5}{10} = \frac{1}{2}$

69. Find the slope of the line below.

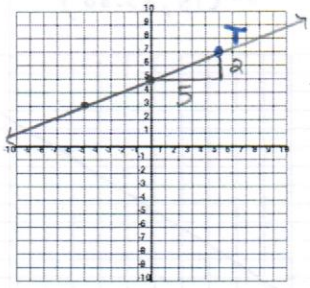


Slope is $\frac{rise}{run} = \frac{-6}{+3} = -2$

*positive slope = "uphill" = y increases with x

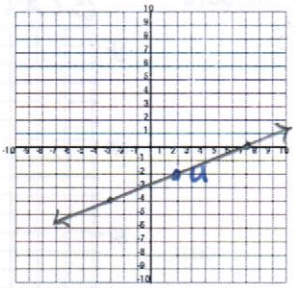
*negative slope = "downhill" = y decreases as x increases.

70. Draw a line through T(5,7) with slope $\frac{2}{5}$.



$\frac{\text{rise}}{\text{run}} = \frac{2}{5}$

71. Draw a line through U(2, -2) parallel to the line in the previous question.

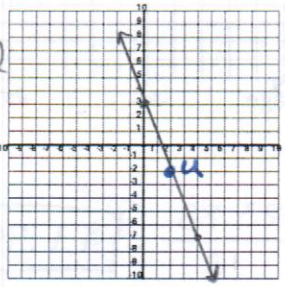


parallel lines have equal slope
$\therefore m = \frac{2}{5}$

72. Draw a line through U(2, -2) perpendicular to the line in the question above.

perpendicular lines have a neg. reciprocal slope.

$\left(\frac{2}{5}\right) \quad -\frac{5}{2} = m$



73. If you were given a triangle with its vertices drawn as coordinates on an x-y coordinate plane, how could you determine if the triangle was a right triangle?

calculate the slopes of the lines and determine if they satisfy the "negative reciprocal" rule. ∴ right angle

Do you know another way?

determine if side lengths satisfy Pythagoras's theorum $a^2+b^2=c^2$

class example *

74. The slope of a line is $\frac{3}{2}$. If the line passes through point $B(5,2)$, find the coordinates of another point.

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{3}{2} = \frac{y-2}{x-5}$

① Substitute slope + the given (x,y) point

$y = 5$

$\frac{3}{2} = \frac{(5)-2}{x-5}$

② Pick a value for x (or y)

$\frac{3}{2} = \frac{3}{x-5}$

③ Cross Multiply.

$3x - 15 = 6$

$3x = 21$

$x = 7$

④ Solve.

∴ (7,5) is also on the line. [also (1,-4)]

75. The slope of a line is -2.5. If the line passes through point $C(-1,2)$, find the coordinates of another point.

$m = \frac{y_2 - y_1}{x_2 - x_1} = -2.5 = \frac{y-2}{x-(-1)}$

$-2.5 = \frac{y-2}{x+1}$

$-2.5 = \frac{y-2}{0+1}$

$-2.5 = y-2$

$+2 \qquad +2$

$-0.5 = y$

∴ (0, -0.5)
[also (1, -3)]

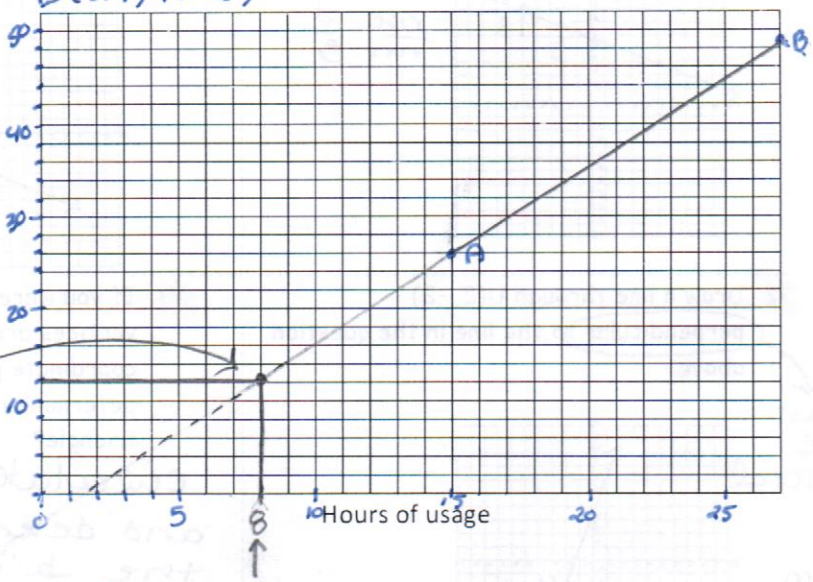
* Visualizing or drawing a graph are also acceptable ways for solving for other points on a line.

76. Julanya's internet provider charges a flat fee for the first 8 hr of access per month, plus an hourly rate for additional access. One month, 15 hr of usage cost her $25.88. The next month, 27 hr of access cost her $49.76.

a) Graph the data.

A (15, 25.88)

B (27, 49.76)



#79 find (8, y)

77. Find the hourly **rate** for access above 8 hr/month.

how much it increases each hr ∴ slope

$$\frac{49.76 - 25.88}{27 - 15} = \frac{23.88}{12} = \boxed{\$1.99/\text{hr}}$$

78. What word is synonymous with rate in this unit?

rate = slope (measure of the rate of change)

79. Find the flat fee for the first 8 hours. (Where will you find this value on the graph?)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$1.99 = \frac{49.76 - y}{27 - 8}$$

$$1.99 = \frac{49.76 - y}{19}$$

$$1.99(19) = 49.76 - y$$

$$37.81 = 49.76 - y$$

$$-49.76 \quad -49.76$$

$$-11.95 = -y$$

$$\therefore y = 11.95$$

Alternatively: work backwards

8 hrs — 15 hrs (15, 25.88)

7 hrs

? — $25.88

7 hrs × $1.99 (Flat rate /hr)

= $13.93

$25.88 − $13.93 = $11.95

*solving graphically will be an estimate only; not accurate enough.

*recall (x, y)

$m = \frac{y_2 - y_1}{x_2 - x_1}$

Find the slope of the line passing through the points:

80. (2,1) and (6,6)

$m = \frac{6-1}{6-2} = \frac{5}{4}$

81. (−5,2) and (4,2)

$m = \frac{2-2}{4-(-5)} = \frac{2-2}{4+5} = \frac{0}{9}$

$\therefore m = 0$

82. (−3,0) and (3, −4)

$m = \frac{-4-0}{3-(-3)} = \frac{-4-0}{3+3} = \frac{-4}{6}$

always simplify

$\therefore m = -\frac{2}{3}$

83. The slope of a line is -2. The line passes through (0,0) and (-3,y). Find the value of y.

$m = -2$

$\frac{-2}{1} = \frac{y-0}{-3-0}$

$\frac{-2}{1} = \frac{y}{-3}$

① Substitute

② Cross multiply

③ Solve

$-2 \times -3 = y \times 1$

$6 = y$

84. A line has a slope of 1.5. It passes through (-2,1) and (x,7). Find the value of x.

$m = 1.5 = \frac{3}{2}$

$\frac{3}{2} = \frac{7-1}{x-(-2)}$

① Sub.

$\frac{3}{2} = \frac{6}{x+2}$

② Cross Multiply

$3(x+2) = 6 \times 2$

$3x + 6 = 12$

$-6 \quad -6$

$\therefore 3x = 6$

$\div 3 \quad \div 3$

$x = 2$

85. Challenge#5: Show that (7, -1) is on the line $y = 2x - 15$

Algebraically:

substitute (7,-1) (x,y)

$y = 2x - 15$

$-1 = 2(7) - 15$

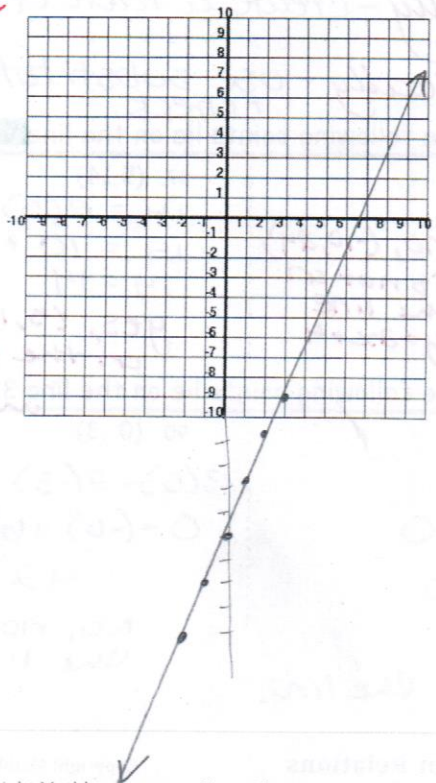
$-1 = 14 - 15$

$-1 = -1$ ← check if these (x,y) points satisfy the equation.

yes!

$\therefore$ (7,-1) is on the line $y = 2x - 15$.

Graphically:



$y = 2x - 15$

↑ y-int.

x	y	
-2	-19	$= 2(-2) - 15 = -10$
-1	-17	$= 2(-1) - 15 = -1$
0	-15	
1	-13	
2	-11	
3	-9	

* create a table of values to graph the equation

The Equation of a Line

As you have seen, equations such as $2x + 3y = 12$ or $3y = x + 9$ or $y = \frac{5}{6}x - 4$ produce straight lines when graphed. They are **linear equations**.

Linear Equations may be written in several forms:

slope y-intercept.

shows slope(m) and x, y coordinates.

Slope-Intercept Form:	$y = mx + b$	$y = 3x + 2$
Point-Slope Form:	$y_2 - y_1 = m(x_2 - x_1)$	$(y - 2) = 3(x - 0)$
General Form:	$Ax + By + C = 0$	$3x - y + 2 = 0$

Recall the *Equation of a Line Property:*

"The coordinates of every point on the line will satisfy the equation of the line."

it said this!

class example.

Eg.1. Show that (7, -1) is on the line $y = 2x - 15$

$y = 2x - 15$	If (7, -1) is on the line, it will satisfy the equation.
$(-1) = 2(7) - 15$	Substitue the ordered pair into the equation.
$-1 = 14 - 15$	Does the left side = right side?
$-1 = -1$	Yes. The point IS on the line.

2 ways to solve:
① Graphically - create a table of values
② Algebraically - use substitution + check.

Do you recall the "text box" like this on page 10?

Determine if the following points lie on the line $y = 2x + 4$ (HINT: substitution!)

86. (-10, 24)

$y = 2(x) + 4$
$24 = 2(-10) + 4$
$24 = -20 + 4$
$24 \neq -16$

no, (-10,24) is not on the line $y = 2x + 4$

87. (5,14)

$14 = 2(5) + 4$
$14 = 10 + 4$
$14 = 14$

yes, (5,14) is on the line.

88. (-7,-10)

$-10 = 2(-7) + 4$
$-10 = -14 + 4$
$-10 = -10$

yes, (-7,-10) is on the line.

Determine if the following points lie on the line $3x - 2y + 6 = 0$

89. (10, 18)

$3x - 2y + 6 = 0$
$3(10) - 2(18) + 6 = 0$
$30 - 36 + 6 = 0$
$0 = 0$

yes, (10,18) is on the line.

90. (0,-3)

$3(0) - 2(-3) + 6 = 0$
$0 - (-6) + 6 = 0$
$12 \neq 0$

No, not on the line.

91. (-6,-6)

$3(-6) - 2(-6) + 6 = 0$
$-18 - (-12) + 6 = 0$
$-18 + 12 + 6 = 0$
$0 = 0$

yes, on the line.