4- Solving by Elimination

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4) solving by elimination

Warm-Up #1: Identify the lowest common denominator for each pair of fractions.

a) $\frac{1}{3} + \frac{1}{4}$ LCD = 12

b) $\frac{2}{7} + \frac{5}{3}$ LCD = 21

Warm-Up #2: Identify the lowest common multiple for each pair of numbers.

a) 5 and 15 LCM = 15

b) 4 and 6 LCM = 12

c) 12 and 5 LCM = 60

eg. $\frac{1}{5} + \frac{1}{15}$

Warm-Up#3: Simplify each expression without the use of a calculator.

a) $-3 + (-5) = -8$

b) $-3 + 5 = 2$

c) $-3 + (+3) = 0$

d) $-2 - (-4) = 2$

e) $-2 - 4 = -6$

f) $-2 - (+2) = -4$

eg $2x + 6y = 25$ (coefficients ≠ 1)

If you **don't have a variable with a coefficient of 1** in a system of equations, substitution is difficult. ***There is another method you can use in these cases.***

You can solve a system of linear equations using the **ELIMINATION** method. ***To do this, a variable in both equations must have the same, or opposite, coefficients.*** It is often necessary to multiply one, or both, equations by a constant value to get the coefficients you need to eliminate.

=> same value opposite signs +/-

Example #1: For each linear system, write an equivalent linear system where both equations have; (i) the opposite x-coefficients and (ii) the opposite y-coefficients.

a) $\begin{cases} x - 2y = -6 \\ 3x + y = 2 \end{cases}$

Step (1) solving by Elimination

i) change x-coefficients

$(x - 2y = -6) \times (-3)$

$3x + y = 2$

$-3x + 6y = 18$

$+ 3x + 1y = 2$

$0x + 7y = 20$

ADD TO eliminate x-variable

change x or y ... not both

ii) change y-coefficients

$x - 2y = -6$

$(3x + y = 2) \times 2 \Rightarrow 6x + 2y = 4$

ADD to eliminate y-variable

$7x + 0y = -2$

b) $\begin{cases} 14x + 15y = 16 \\ 21x + 10y = -1 \end{cases}$

LCM = 42 LCM = 30

i) change x-coefficients

$(14x + 15y = 16) \times (-3)$

$(21x + 10y = -1) \times 2$

$-42x - 45y = -48$

$+ 42x + 20y = -2$

ADD to eliminate x

$0x - 25y = -50$

goal is to eliminate x

why do we do this?

$\frac{-25y}{-25} = \frac{-50}{-25}$

$y = 2$

To solve for 1 of the variables.

ii) change y-coefficients

$(14x + 15y = 16) \times (-2)$

$(21x + 10y = -1) \times (3)$

$-28x - 30y = -32$

$+ 63x + 30y = -3$

ADD TO eliminate y variable

$35x + 0 = -35$

goal is to eliminate "y"

why do we do this?

$\frac{35x}{35} = \frac{-35}{35}$

$x = -1$

$3x - 5y + 9 \Rightarrow -5y = -3x - 9 \Rightarrow y = \frac{3}{5}x + \frac{9}{5}$

Example #2: Solve each system using the elimination method.

a) $\begin{cases} 3x - 5y + 9 = 0 \\ 4x + 5y - 23 = 0 \end{cases}$

(1) Step 1: eliminate x or y variable

y-values are already - equal values - opp signs +/-

(2) Step 2: Add to eliminate

$3x - 5y + 9 = 0$

$+ 4x + 5y - 23 = 0$

$7x + 0y - 14 = 0$

$7x - 14 = 0$

(3) Solve for the variable.

$7x - 14 = 0$

$+14 \quad +14$

$\frac{7x}{7} = \frac{14}{7}$

$x = 2$

(4) Substitute into 1 of the equations & solve for "y"

$3x - 5y + 9 = 0$

$3(2) - 5y + 9 = 0$

$+5y \quad +5y$

$6 + 9 = 5y$

$\frac{15}{5} = \frac{5y}{5}$

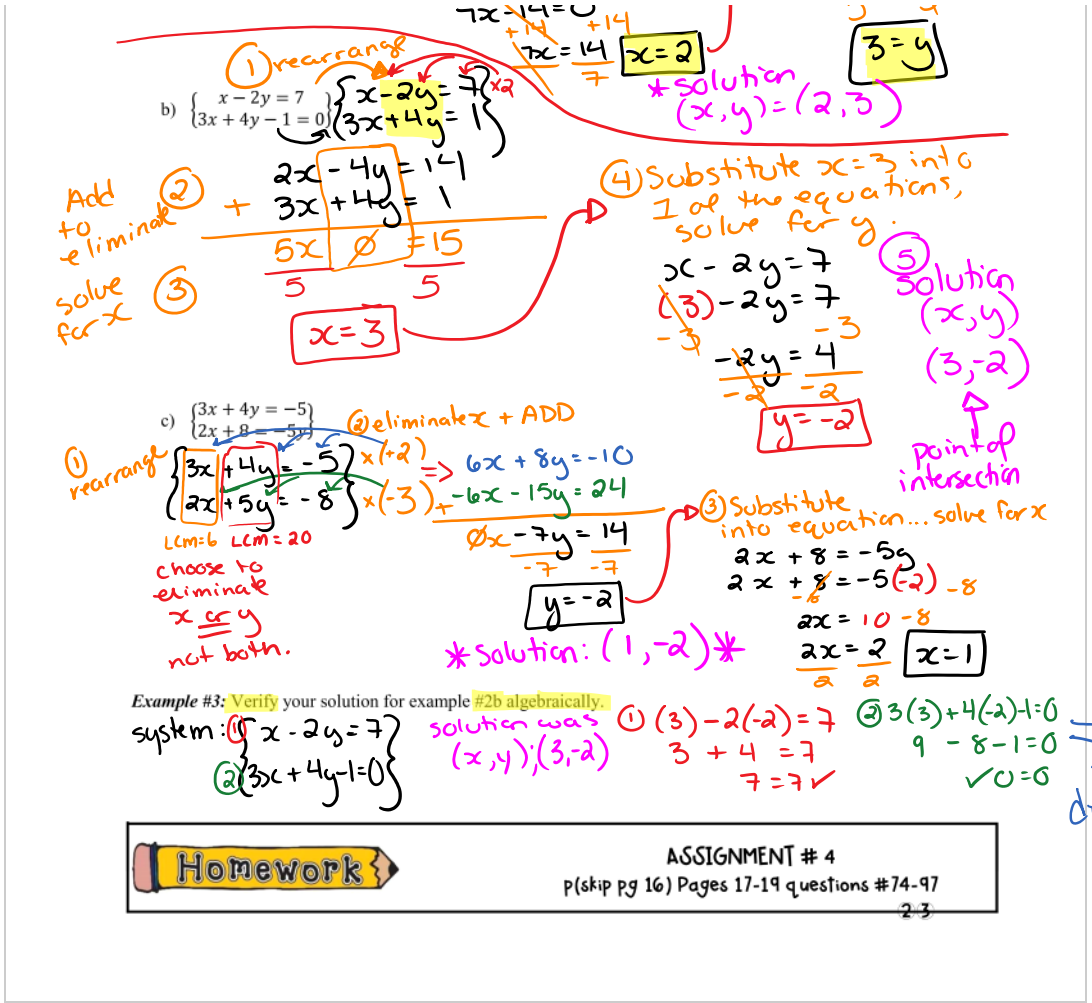
$3 = y$

(1) rearrange

$x - 2y = 7 \quad (\times 2)$

$5x - 2y = 7$

* solution


b) $\begin{cases} x-2y=7 \\ 3x+4y-1=0 \end{cases}$
① rearrange
$\begin{cases} x-2y=7 \\ 3x+4y=1 \end{cases}$ x2
Add to eliminate ②
solve for x ③
$2x-4y=14$
$+\ 3x+4y=1$
$5x \quad \emptyset \quad =15$
$5 \qquad 5$
$x=3$
④ Substitute $x=3$ into 1 of the equations, solve for y.
$x-2y=7$
$(3)-2y=7$
$-3 \quad -3$
$-2y=4$
$-2 \quad -2$
$y=-2$
⑤ solution (x,y) $(3,-2)$
point of intersection
$7x=14$
$+14 \quad +14$
$7 \quad 7$
$x=2$
$3=y$
*solution $(x,y)=(2,3)$
c) $\begin{cases} 3x+4y=-5 \\ 2x+8=-5y \end{cases}$
① rearrange
② eliminate x + ADD
$\begin{cases} 3x+4y=-5 \\ 2x+5y=-8 \end{cases}$ x(+2) x(-3)
$6x+8y=-10$
$+\ -6x-15y=24$
$\emptyset x-7y=14$
$-7 \quad -7$
$y=-2$
LCM=6 LCM=20
choose to eliminate x or y not both.
③ Substitute into equation... solve for x
$2x+8=-5y$
$2x+8=-5(-2)$
$-8 \quad -8$
$2x=10-8$
$2x=2$
$2 \quad 2$
$x=1$
solution: $(1,-2)$
Example #3: Verify your solution for example #2b algebraically.
system: $\begin{cases} x-2y=7 \\ 3x+4y-1=0 \end{cases}$
solution was $(x,y)=(3,-2)$
① $(3)-2(-2)=7$
$3+4=7$
$7=7$ ✓
② $3(3)+4(-2)-1=0$
$9-8-1=0$
$0=0$ ✓
⇒ can also verify with desmos.com/calculator
Homework
ASSIGNMENT # 4
P(skip pg 16) Pages 17-19 questions #74-97
2 3

68. Write a system of 2 linear equations for the following problem.

The sum of two numbers is 65. The first number is 17 greater than the second.

69. Find the numbers in the problem to the left.

70. Write a system of 2 linear equations for the following problem.

One number is 12 less than another number. Their sum is 102.

71. Find the numbers in the problem to the left.

72. Write a system of 2 linear equations for the following problem.

Mr. J bought a total of 12 pairs of socks. Athletic socks cost $5 per pair and dress socks cost $7 per pair. He spent $70 in total.

73. How many pairs of each type of socks did he buy?

Part 2: Solving By Elimination (Addition or Subtraction)

Challenge Questions

74. Is (3,1) a solution to the system $2x - y = 5$ and $2x - 4y = 2$?

75. Multiply each of the equations above by 2.
$2(2x - y = 5)$→ $2(2x - 4y = 2)$→

76. Is (3,1) still a solution to each of the equations above?

77. Add the two original equations together:
$2x - y = 5$
$\underline{2x - 4y = 2}$

78. Is (3,1) a solution to the new equation?

79. What conclusions can you draw about adding/subtracting equations together?

80. What conclusions can you draw about multiplying equations in a system by a constant?

81. Can you multiply the equations by different numbers without affecting the solution?

82. Graph equation ①:

① $2x + y = 8$

83. Graph equation ②:

② $y = 4x - 4$

84. Add equations ① and ②. Call this equation ③.

③_____

85. Graph equation ③.

86. Multiply ③ $\times$ 3 and call this equation ④.

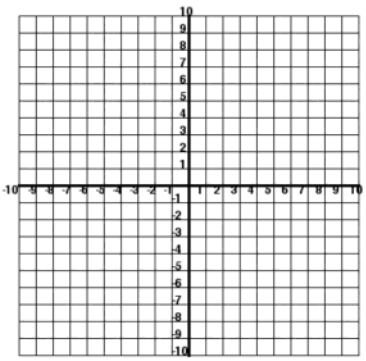
④_____

87. Graph equation ④.

88. Add ③ and ④ , call this equation ⑤.

⑤_____

89. Graph equation ⑤.



90. Describe what you see happening above.

91. Write a set of rules describing what you may do to a system of equations in order to find the solution. That is, how can you manipulate the equations without affecting the solution?

92. Add the two equations together, then solve.

$3x - 6y = 21$

$\underline{-3x - 4y = -1}$

-10y = 20

y = - 2 → 3x – 6(- 2) = 21

3x + 12 = 21

3x = 9

x = 3

Solution: (3, - 2)

93. Solve.

$2x + 3y = 18$

$2x - 3y = -6$

94. Solve.

$8x + 2y = -20$

$2x - 2y = -30$

95. Solve.

$-4t + 3s = 2$

$8t - 6s = -4$

96. Solve.

$6x - 3y = 24$

$x + y = -2$

97. Solve.

$3b - a = 1$

$-12b + 4a = -4$