**4: Solving by Elimination**

**Warm-Up #1:** Identify the lowest common denominator for each pair of fractions.

- a) 12
- b) 21
- c) 21

**Warm-Up #2:** Identify the lowest common multiple for each pair of numbers.

- a) 5 and 15: LCM = 15
- b) 4 and 6: LCM = 12
- c) 12 and 5: LCM = 60

**Warm-Up #3:** Simplify each expression without the use of a calculator.

- a) $-3 + (-5) = -8$
- b) $-3 + 5 = 2$
- c) $-3 + (-5) = -8$
- d) $-2 + (-4) = -6$
- e) $-2 - 4 = -6$
- f) $-2 + (-2) = -4$

If you don’t have a variable with a coefficient of 1 in a system of equations, substitution is difficult.

There is another method you can use in these cases.

You can solve a system of linear equations using the **elimination method.** To do this, a variable in both equations must have the same, or opposite coefficients. It is often necessary to multiply one, or both, equations by a constant value to get the coefficients you need to eliminate.

**Example #1:** For each linear system, write an equivalent linear system where both equations have: (i) the same $x$-coefficients and (ii) the opposite $y$-coefficients.

**Step 1:** solving by elimination

- i) change $x$-coefficients
  - $5x - 2y = -6$ becomes $5x - 2y = -3$
- ii) change $y$-coefficients
  - $3x + y = -2$ becomes $3x + y = 2$

**Example #2:** Solve each system using the elimination method.

- a) $3x - 5y = 9$
- b) $4x + 5y = 23$
- c) $7x + 14y = 14$
- d) $3x - 5y = 9$
- e) $5x - 7y = 1$
Example #3: Verify your solution for example 2B algebraically.

1. \( x - 2y = 7 \)
2. \( 3x + 4y = 10 \)

3. Substitute into equation... solve for \( x \)
4. \( 6x + 8y = -10 \)
5. \( 2x + y = -1 \)
6. \( x = 10 - 8 \)
7. \( x = 2 \)
8. \( x = 1 \)

Solution: \((1, -2)\)

*Solution:

\[ \frac{5x - 3y}{19} = 1 \]
\[ z = 7 \]

\[ \frac{3x + 4y}{19} = 1 \]
\[ 3x + 4y = 1 \]
\[ 3 = 3 \]

\[ \frac{9 - 8}{19} = 0 \]
\[ 0 = 0 \]
66. Write a system of 2 linear equations for the following problem.
The sum of two numbers is 65. The first number is 17 greater than the second.

69. Find the numbers in the problem to the left.

70. Write a system of 2 linear equations for the following problem.
One number is 12 less than another number. Their sum is 102.

71. Find the numbers in the problem to the left.

72. Write a system of 2 linear equations for the following problem.
Mr. I bought a total of 12 pairs of socks. Athletic socks cost $5 per pair and dress socks cost $7 per pair. He spent $70 in total.

73. How many pairs of each type of socks did he buy?
Part 2: Solving By Elimination (Addition or Subtraction)

Challenge Questions

74. Is (3,1) a solution to the system \(2x - y = 5\) and \(2x - 4y = 2\)?

75. Multiply each of the equations above by 2:
\[
\begin{align*}
2(2x - y &= 5) \\
2(2x - 4y &= 2)
\end{align*}
\]

76. Is (3,1) still a solution to each of the equations above?

77. Add the two original equations together:
\[
\begin{align*}
2x - y &= 5 \\
2x - 4y &= 2
\end{align*}
\]

78. Is (3,1) a solution to the new equation?

79. What conclusions can you draw about adding/subtracting equations together?

80. What conclusions can you draw about multiplying equations in a system by a constant?

81. Can you multiply the equations by different numbers without affecting the solution?
62. Graph equation ①.
① $2x + y = 8$
63. Graph equation ②.
② $y = 4x - 4$
64. Add equations ① and ②. Call this equation ③.
65. Graph equation ③.
66. Multiply ③ by $3$ and call this equation ④.
67. Graph equation ④.
68. Add ② and ④, call this equation ⑤.
69. Graph equation ⑤.

66. Describe what you see happening above.
Write a set of rules describing what you may do to a system of equations in order to find the solution. That is, how can you manipulate the equations without affecting the solution?

92. Add the two equations together, then solve.
\[
\begin{align*}
3x - 6y &= 21 \\
-3x - 4y &= -1
\end{align*}
\]
\[\Rightarrow \quad 3x - 6(-2) = 21 \Rightarrow 3x + 12 = 21 \Rightarrow 3x = 9 \Rightarrow x = 3\]
\[\text{Solution: (3, -2)}\]

93. Solve.
\[
\begin{align*}
2x + 3y &= 18 \\
2x - 3y &= -6
\end{align*}
\]

94. Solve.
\[
\begin{align*}
8x + 2y &= -10 \\
2x - 2y &= -30
\end{align*}
\]
\[\Rightarrow -6x + 3x = 2 \Rightarrow 8x - 6x = -4\]

95. Solve.
\[
\begin{align*}
6x - 3y &= 24 \\
x + y &= -2
\end{align*}
\]
\[\Rightarrow 3b - a = 1 \Rightarrow -12b + 4a = -4\]