4) x and y Intercepts

Warm-Up: Use the table of values method to graph the line \( y = -3x + 6 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>12</td>
</tr>
<tr>
<td>-1</td>
<td>9</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

\( y = -3(x+2) + 6 \)

\( y = -3(-1) + 6 \)

\( y = -3(0) + 6 \)

\( y = -3(1) + 6 \)

\( y = -3(2) + 6 \)

---

a) What is the x-intercept? \( x = 2 \)

The x-intercept is the point where the graph crosses the x-axis.

b) What is the y-intercept? \( y = 6 \)

The y-intercept is the point where the graph crosses the y-axis.

Notes:

- Non vertical/horizontal lines are called oblique lines.
- These lines cross both the \( x \)-axis and the \( y \)-axis.
- The point where the line crosses the \( x \)-axis is called the \( x \)-intercept.
  - Coordinates: \((x, 0)\), i.e. \( y \) is always 0.
- The point where the line crosses the \( y \)-axis is called the \( y \)-intercept.
  - Coordinates: \((0, y)\), i.e. \( x \) is always 0.

Every \( y \)-intercept has an \( x \)-coordinate of 0.

Every \( x \)-intercept has a \( y \)-coordinate of 0.
Example #1: Find the $x$ and $y$ intercepts from the following graphs:

a) \[ \begin{array}{c}
x \text{-intercept} = -3 \\
y \text{-intercept} = -7 
\end{array} \]

b) \[ \begin{array}{c}
x \text{-intercept} = 9 \\
y \text{-intercept} = -6 
\end{array} \]

Example #2: Consider the line defined by \( 2x - 3y = 6 \).

a) Determine the $x$-intercept and write the coordinates of this point:
\[
\begin{align*}
\text{at } x - \text{int } & , \ y = 0 \\
2x - 3(0) &= 6 \\
2x &= 6 \\
x &= 3
\end{align*}
\]

\[ \text{coordinates: } (3,0) \]

b) Determine the $y$-intercept and write the coordinates of this point:
\[
\begin{align*}
\text{at } y - \text{int } & , \ x = 0 \\
2(0) - 3y &= 6 \\
-3y &= 6 \\
y &= -2
\end{align*}
\]

\[ \text{coordinates: } (0,-2) \]

- One way of graphing linear relations without using a table of values is finding the $x$-intercept and $y$-intercept and connecting the two points.

c) Graph the function using the intercepts:

\[ \begin{array}{c}
x \text{-coordinates: } (3,0), (0,-2) \\
x \text{-int.} \quad y \text{-int.}
\end{array} \]
Example #3: Graph the line $2x + y = 8$

x-intercept: $y = 0$

$2x + y = 8$
$2x + 0 = 8$
$2x = 8$
$x = \frac{8}{2}$
$x = 4$

$4, 0$

y-intercept: $x = 0$

$2x + y = 8$
$2(0) + y = 8$
$y = 8$
$(0, 8)$

"b" term = y-intercept.

Example #4: Graph the line $2x + 6y = 18$

x-intercept: $y = 0$

$2x + 6y = 18$
$2x + 6(0) = 18$
$2x = 18$
$x = \frac{18}{2}$
$x = 9$

$(9, 0)$

y-intercept: $x = 0$

$2x + 6y = 18$
$2(0) + 6y = 18$
$6y = 18$
$y = \frac{18}{6}$
$y = 3$

$(0, 3)$

constant term "b" = y-int.
Example #5: Graph the line $x = -4$

x-intercept: 
$\{(x = -4)\}$

y-intercept:
$\{(x = 0)\}$
$\{(y = -4)\}$
not a true statement

$\Rightarrow \text{no y-int.}$

Example #6: Graph the line $y = 5$

x-intercept: 
$\{(y = 0)\}$

$\Rightarrow \text{no x-int.}$

y-intercept:
$\{(x = 0)\}$

only y-int = horizontal line.

---

**Homework**

Assignment #4
Pages #21-24 Questions #85-99
Intercepts

Non-vertical and non-horizontal lines are called **oblique** lines.

Oblique lines will cross both the x-axis and the y-axis.

These points are called the x-intercept and the y-intercept.

---

**Challenge Question:**
Find the intercepts for the line $y = 2x + 4$.

---

**Challenge Question:**
Find the intercepts for the line $3x + 4y - 12 = 0$. 
Finding the Intercepts from a graph.

The location where a line passes through the x-axis is called the **x-intercept**. This point will have the coordinates \((x, 0)\).

The location where a line passes through the y-axis is called the **y-intercept**. This point will have the coordinates \((0, y)\).

Consider: \(2x + 4y = 16\)

This line has an x-intercept at \((8, 0)\).
And a y-intercept at \((0, 4)\).

You may see this written as:
- x-intercept is 8.
- y-intercept is 4.

Find the x- and y-intercepts from the graph below.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>x-intercept:</td>
<td>y-intercept:</td>
</tr>
<tr>
<td>x-intercept:</td>
<td>y-intercept:</td>
</tr>
</tbody>
</table>
Finding the Intercepts from an equation.

The x-intercept will have coordinates $(x, 0)$. This means we can substitute 0 in for $y$ and solve to find the x-intercept. The y-intercept will have coordinates $(0, y)$.

Example: Find the x-intercept for $2x + 4y = 16$

- $2x + 4(0) = 16$
- $2x = 16$
- $x = 8$

Find the y-intercept: $2x + 4y = 16$

- $2(0) + 4y = 16$
- $4y = 16$
- $y = 4$

---

Calculate the x- and y-intercepts.

89. $2x + 3y = 12$

90. $3x + 5y = 30$

91. $3x - 4y + 24 = 0$

92. $4x + 5y = 10$

93. $5y = 10x$

94. $0.04x + 0.02y = 1400$
Using and Interpreting Intercepts

95. Find the intercepts and graph the line
\[ 2x + 6y = -18. \]

96. Find the intercepts and graph the line
\[ 10x - 8y = -80. \]

97. Based on the equation for the linear relation, when do you think it is most appropriate to graph the relation using intercepts?

98. The cost of a new pair of shoes at ShoeInc is reduced at a constant rate. The graph below shows the profit ShoeInc makes on each sale. In what month does ShoeInc "break even" on these shoes?

99. Use the graph below to plot the fuel consumed on Sandy's last road trip. She started out with 72 litres of fuel and drove for 2 hours. At that point she had 54 litres left. After driving another 1.5 hours she had 40.5 litres remaining.

At this rate, when will she run out of fuel?
Mixed Practice:

100. A triangle has vertices A(-2, 3), B(8, -2), and C(4, 6). Determine whether it is a right triangle.

101. P(5, 4) and Q(1, -2) are points on a line. Find the coordinates of a point, R, so that PR is perpendicular to PQ.

102. Find the value of k so that the two slopes are perpendicular.

\[ m_1 = \frac{k}{2} \] and \[ m_2 = \frac{1}{k} \]

103. Two vertices of an isosceles triangle are A(-5, 4) and B(3, 3). The third vertex is on the x-axis. What are the possible coordinates of the third vertex, C?