Applications of Right Triangle Trigonometry: Angles of Elevation & Angles of Depression

Preliminary Information: On most maps, it is customary to orient oneself relative to the direction north: for this reason, north is almost always indicated on every map. Likewise, when working with real-life trigonometry problems, it is very common to orient angles relative to a Horizontal line.

An **angle of <u>Elevation</u>** refers to the acute angle a line (or ray, segment, etc.) makes with a horizontal line, when measured <u>above</u> the horizontal (hence an angle of *elevation*).

For example, the sun's rays could form a 23° angle of elevation (above the horizon).

refers to the acute angle a line makes with a horizontal line, when measured below the horizontal (hence an angle

of depression).

An angle of Devression

Angles of depression are measured down from the horizontal.

horizontal line

Angles of **elevation** are measured **UP** from the horizontal.

For example, an airplane pilot could look down and see a feature on the ground below at a 35° angle of depression (below the horizon).

Because horizontal lines are everywhere parallel, angles of depression and elevation are numerically <u>FOUNDENT</u> because they form *alternate interior angles* of parallel lines:

angle of elevation

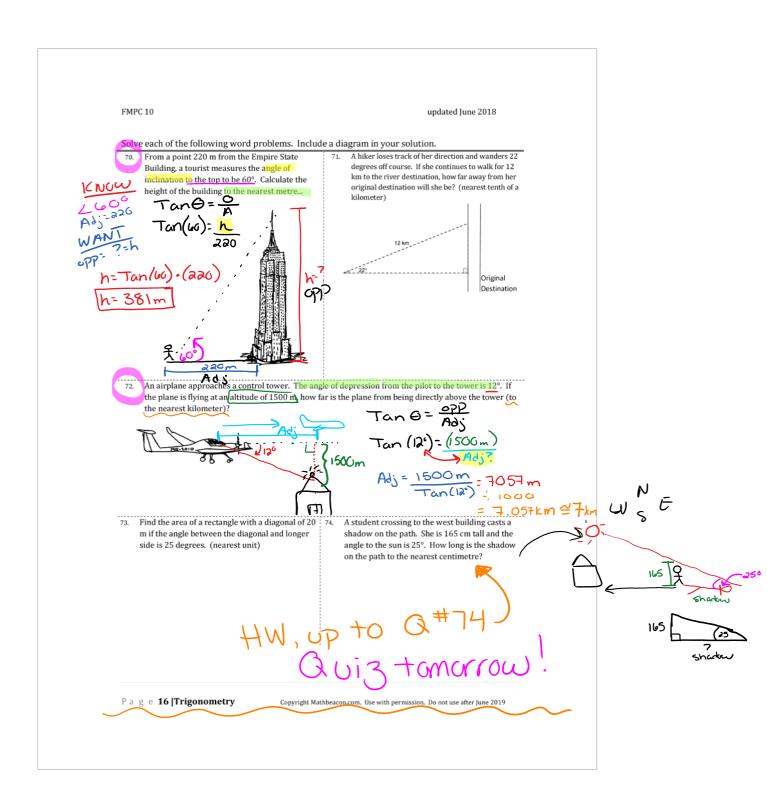
parallel lines

Students should always be encouraged to consider the following two ideas when they see either phrase mentioned in a problem:

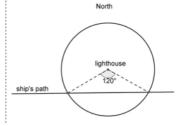
You may always draw an additional horizontal line on any diagram extending from
any point in the diagram. Just as you did in Geometry, drawing such an auxiliary line
can help to make a complex problem simpler.

The most common error students make when they encounter these terms is they mark an angle relative to a vertical line (such as an angle with a wall, building, or tree) instead of with a horizontal line.

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- 75. A radio tower is 396 feet tall. How far from the base of the tower is a technician if the angle of inclination to the top of the tower is 27°? Answer to the nearest foot.
- 76. A lighthouse attendant has a range of visibility of 24 km. A ship on the horizon passes by the lighthouse. The attendant sees the ship for a total of 120 degrees. For how many kilometers was the ship within the attendant's range of sight? (nearest tenth)



Draw a scale diagram that would $\boldsymbol{represent}$ each of the following.

77. Draw a triangle that has a the following:

 $sin\alpha = \frac{1}{5}$, hypotenuse is 10 cm long.

78. Draw a triangle that has a the following: $tan\beta = \frac{12}{5}$, hypotenuse is 26 cm long.

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79. Solve the triangle.

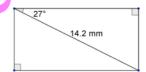


80. Solve the triangle.

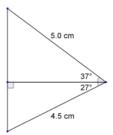


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81. Find the perimeter of the following rectangle.



82. Find the total area.



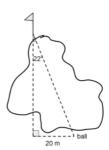
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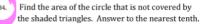
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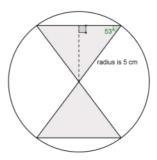
FMPC 10

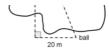
83. While golfing with his father-in-law, Mr. J hits a shot short of a pond. He walks 20 m to his left to a point directly across the pond from the hole. The angle between the two lines of sight is 22°. Find the distance from his ball to the hole to the nearest tenth of a metre.



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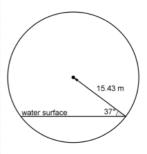






- 85. Anya lets out 125 feet of kite string at Clover Point. The wind pulls the kite string tight at an angle of 55° to the ground. Approximate the height of the kite to the nearest foot.
- 86. The radius of a circular tunnel in Shanghai is 15.43 m. During a flood, a worker in the water at the side of the tunnel measured an angle to the centre to be 37°. Find the depth of the water at its deepest point. (The water surface forms a chord across the tunnel.)

What assumptions did you make?



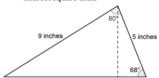
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Solve the following problems involving triangles.

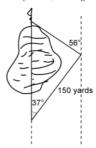
87. Find the area of the triangle below to the nearest square inch.



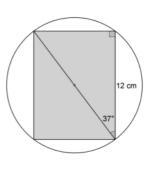
88. At 11:00 in the morning, the angle of elevation to the sun 58°. A tree in the school yard casts a shadow of 56 m. How tall is the tree to the nearest metre?

89. Tucker has two choices to get his ball to the hole at the 17th at Cordova Bay, go around the lake or go over it. He decides to go around the lake as shown on the diagram. How much farther does he have to hit the ball going around the lake instead of going straight over it? Answer to the nearest yard.

(Careful, not a right triangle shown.)



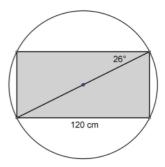
 Find the area of the circle that is not covered by the shaded rectangle to the nearest square unit.



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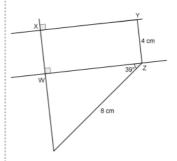
91. Find the area of the circle not covered by the shaded ${\bf rectangle}$ to the nearest 100 cm.



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92. Sandra stands at the midpoint between two buildings and measures the angles of elevation to their tops to be 14° and 18°. If the two buildings are 80 metres apart, what is the difference in their heights? Answer to the nearest metre.

- 93. Find the length of AG to the nearest tenth of a millimetre.
- 238) G 21.3 mm
- 94. Find the area of rectangle WXYZ to the nearest square unit.



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Solving for Angles

Inverse Functions

For example, if we are doing the "inverse of $sin\theta$ "...we are trying to FIND the angle, when we are GIVEN both side lengths.

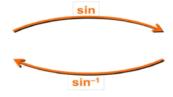


The inverse of sin

 $\sin \theta = 0.5$, what is the value of θ ?

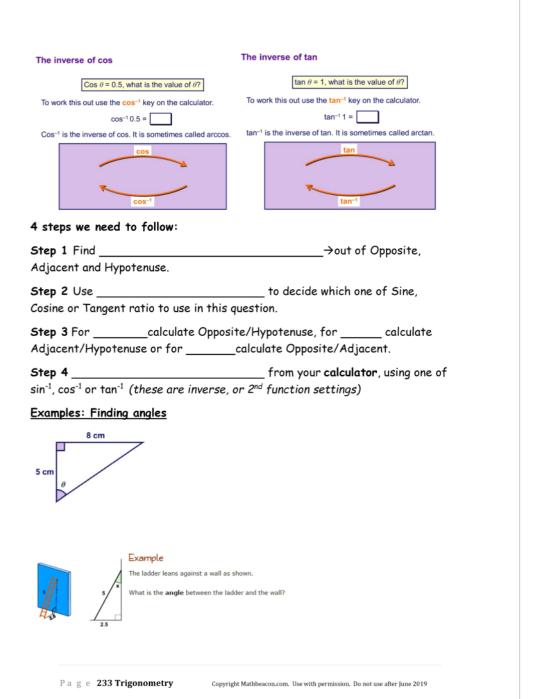
To work this out use the sin-1 key on the calculator.

sin-1 is the inverse of sin. It is sometimes called arcsin.



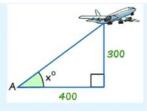
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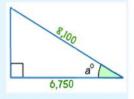
Example

Find the angle of elevation of the plane from point $\boldsymbol{\mathsf{A}}$ on the ground.



Example

Find the size of angle ao



Example: Find the angle "a"

We know

- The distance down is 18.88 m.
- The cable's length is 30 m.

And we want to know the angle "a"



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Finding Angles Using the Three Ratios

Recall:

The three primary trig. ratios:

Tangent Ratio:

 $\tan \theta = \frac{length\ of\ side\ opposite\ \theta}{length\ of\ side\ adjacent\ \theta}$

Sine Ratio: $sin\theta = \frac{length\ of\ side\ opposite\ \theta}{length\ of\ hypotenuse}$

Cosine Ratio: $cos\theta = \frac{length\ of\ side\ adjacent\ \theta}{length\ of\ hypotenuse}$

Unless otherwise stated, calculate the measure of angles to the nearest tenth of a degree.

Eg. 42.8°

The stored values in your calculator allow you to find angles using the ratios.

The magic of sin^{-1} , cos^{-1} , and tan^{-1} .

The "inverse trigonometric functions".

These functions convert the stored ratios in your calculator to the angle.

Challenge

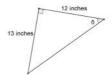
Challenge

95. Find the measure of angle A in a right triangle if tanA = 1.000.

96. Find the measure angle B in a right triangle if $sinB = \frac{1}{1}$

Challenge

97. What ratio would you use to find the measure of the indicated angle?



Find the measure of the indicated angle.

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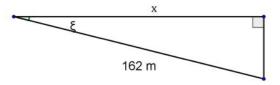
Use the Inverse functions to find the indicated angle to the nearest tenth.

98. Find the measure angle A in a right triangle if tanA=1.000. Use the tan-1 button. Type: tan-1(1.000)=A A = 45°	99. Find the measure angle B in a right triangle if $sinB=0.5000$. Use the sin^{-1} button. Type: $sin^{-1}(1 \div 2) = B$ B = 30 °	100. What ratio would you use to find the measure of the indicated angle? Use the tangent ratio: $tan\delta=\frac{13}{12}$ 12 inches Find the measure of the indicated angle. Type: $tan^{-1}(13\div 12)=\delta$ $\delta=47.3^{\circ}$
101. Find angle A, if $sinA = 0.2654$.	102. Find angle B, if $cosB = \frac{5}{7}.$	103. Find angle Q, if $tanQ = \frac{15}{\lambda}.$
104. Find angle T, if $sinT = \frac{15}{11}.$	105. Find angle D, if $cosD = \frac{11}{10}.$	106. Find angle U, if tanU = 2.6784.
107. In a right triangle, one acute angle has sine ratio of 0.5. Find the sine ratio of the other acute angle.	108. In a right triangle, one angle has cosine ratio Find the sine ratio of t other acute angle.	of $\frac{1}{\sqrt{l}}$. angle has cosine ratio of $\frac{1}{l}$.
110. Which of the three trigonon ratios (sine, cosine, tangent have a value greater than 1) can previous questi	angle and use it to explain your answer to the on.

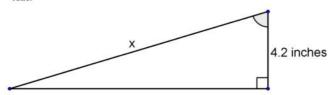
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- 112. Draw a right triangle with an acute angle that has an adjacent side equal in length to the opposite side. Find the cosine ratio for that angle. (Round your answer to 3 decimals.)
- 113. Draw a right triangle with an acute angle that has a hypotenuse 50% longer than the adjacent side. Find the cosine ratio for that angle.

114. Use a protractor to measure the indicated angle. Then determine the length of side x using the cosine ratio.



115. Use a protractor to measure the indicated angle. Then determine the length of side x using the cosine ratio.

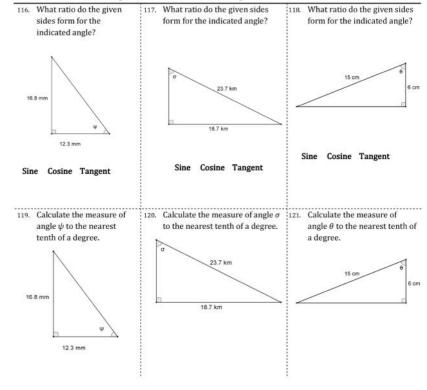


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Working with the ratios to find angles.

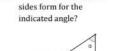
Have a plan...

- Choose the correct ratio {sine, cosine, or tangent}.
 Fill in the known side lengths into your chosen ratio.
- Use the "inverse trig. function" to convert ratio → angle.



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Working with the ratios to find angles.

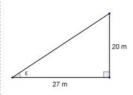


122. What ratio do the given



Sine Cosine Tangent

123. What ratio do the given sides form for the indicated angle?

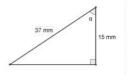


Sine Cosine Tangent

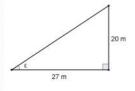
124. What ratio do the given sides form for the indicated angle?



Sine Cosine Tangent



126. Calculate the measure of angle arepsilon to the nearest tenth of a degree.



127. Calculate the measure of angle θ to the nearest tenth of a degree.



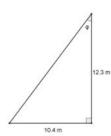
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FMPC 10 updated June 2018 Find the measure of the indicated angle. Round answers to the nearest tenth of a degree. 129. 152 m 12.3 m

131.

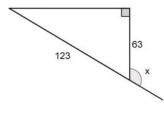
130. 1.75 mm 2.23 mm

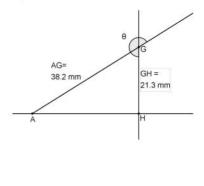
104 miles



27.2 yards 16 yards

132. Find the measure of angle x to the nearest tenth of a degree. 133. Find the measure of angle θ to the nearest degree.





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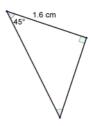
FMPC 10 updated June 2018 Solve the following triangles. Calculate answers to the nearest tenth. \\ 136. 137. 180 m

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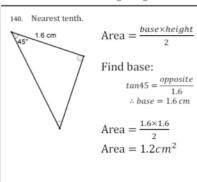


139. Challenge.

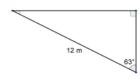
Find the Area of the following triangle to the nearest tenth of a square unit.



Find the area of the following triangles. Units for each question are indicated.



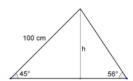
141. Nearest square metre.



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142. Nearest hundred square centimetres.



143. Nearest square foot.



- 144. A triangle has side lengths of 8 cm, 7cm and 12 cm. Find the area of the triangle if the angle between the 8 cm and 12 cm side is 34°.

 Answer to the nearest square cm.

 145. A triangle has side lengths of 10 km, 23 km and 32 km. The angle opposite the 10 km side is 9.2°
 Find the area of the triangle. Answer to the nearest square km.

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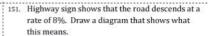
Applications of trigonometry.

- 146. A kite stuck in a nearby tree. A child standing 25 m from the base of a tree pulls the string tight. If the tree is 30 m tall, approximately how far is the kite from the child to the nearest metre?
- 147. A surveyor measures the angle of elevation to the top of a building to be 23°. If the surveyor is 1345 feet from the base of the building, how tall is the building to the nearest foot?

- 148. From the top of a 20 m cliff above a road, the angle of depression to two approaching cars is 25° and 40° respectively. How far apart are the cars to the nearest metre?
- 149. Two hot air balloons float above the ocean at a height of 1000 feet. From a sailboat an observer measures the angle of elevation to one balloon is 60° and to the other balloon is 50°. [both balloons are on the same bearing from the observer] How far apart are the balloons to the nearest foot?

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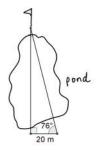
150. Two boys on opposite sides of the tree below measure the angle of elevation to the top of the tree. If the tree is 175 feet tall, how many feet apart are the boys?





If a 3 km section of straight road descends at this grade, what is the drop in elevation?

152. While golfing with his father-in-law, Mr. J hits a shot short of a pond. The flag (hole) is directly across the pond from his ball. He paces 20 m to the right of his ball and measures the angle back to the hole to be 76°. How far is the ball from the hole to the nearest metre? 153. A hiker leaves base camp travelling due north at 5 km/h. After two hours, she turns and travels east. Three hours later, she sprains her ankle. At what bearing would a rescue team need to travel to reach the injured hiker? How far away is she from base camp? (nearest tenth)



North E

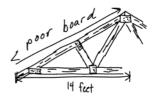
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154. A student approaches a large Sequoia tree outside the entrance to the school and wonders how tall the tree is. He paces 150 metres from the base of the tree and measures the angle of elevation to the top of the tree to be 35°. Find the height of the tree to the nearest metre.



8

155. A homeowner wants to cut a new board to replace a decaying roof truss. He can measure the horizontal distance and the angle of inclination but needs to know how long to cut the board. The horizontal distance is 14 feet and the angle of inclination is 24° . Find the distance to the nearest tenth of a foot.



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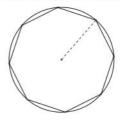
156. An engineer is constructing a Ferris wheel for a downtown park. There are 16 passenger carts and the radius of the wheel is 10 metres. How far apart are the passenger carts to the nearest hundredth of a metro?



157. Find the area of the circle to the nearest square centimetre. $[A = \pi r^1]$

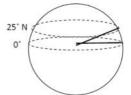


158. Find the perimeter of the octagon inscribed in a circle of radius 8 cm. (Nearest cm)

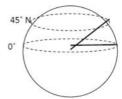


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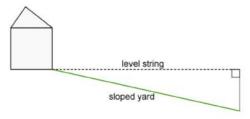
159. Find the length of the 25° line of latitude. The Earth's radius is 6380 km. Answer to the nearest km.



160. Find the length of the 45° line of latitude. The Earth's radius is 6380 km. Answer to the nearest km.

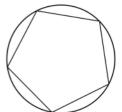


161. Mr. Teespré's backyard slopes away from his house towards the beach. The instructions for his new lawnmower state that the mower should not be used if the slope is greater than 15°. Being a trigonometry specialist, he extends a level string 125 feet from the base of his house. From that point, he measures that the distance along the ground back to his house is 130 m. Is his yard too steep for this mower?



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162. A regular pentagon is inscribed in a circle of radius 10 cm. Calculate the perimeter of the pentagon. Answer to the nearest cm.



163. A regular decagon (10 sides) is inscribed inside a circle of radius 8 cm. Find the perimeter of the decagon. Answer to the nearest cm.

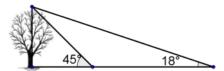
164. Find the area of the octagon inscribed in a circle of radius 8 cm. Answer to the nearest square cm.



165. A regular hexagon is inscribed in a circle with a radius 18 cm. What would be the side length of the hexagon? Answer to the nearest cm.

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166. From a point 15 m from the base of a tree, a woman found the angle of inclination to the top of the tree to be 45°. Her sister found the angle to be 18° from a point farther away from the base of the tree. How far away are the two women away from each other? (nearest tenth of a metre)



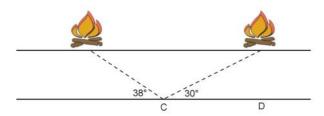
More word problems using right triangles:

- Draw a diagram.
- Fill in known values.
- Let a variable represent unknown(s)
 Choose an appropriate strategy to solve for the unknown(s).
 Interpret the problem.
- 167. Solve the triangle given the following.

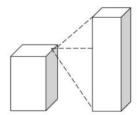
Solve the to ΔXYZ x = 9 cm $\angle Y = 90^{\circ}$ $\angle Z = 36^{\circ}$

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168. A firefighter is walking along the river at point C when she spots two fires on the opposite river bank. She measures the angles below and paces a distance of 300 m from point C to point D. Point D is directly across the river from one of the fires. How far apart are the fires to the nearest metre?

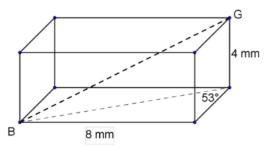


169. Anya stands on top of a building in downtown Victoria. From her position, the angle of elevation to the top of an adjacent building is 47° . The angle of depression to the base of the building is 62° . She is told that the buildings are 45 m apart. Based on this information, what is the height of the taller building to the nearest metre?



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 $170. \quad \text{Find the length of diagonal BG in the rectangular prism. Answer to the nearest tenth of a millimeter.}$



171. The line of sight from an inflatable boat to the top of an oil derrick is 24 degrees. If the derrick is 45 m tall, how far is the boat from its base? (nearest tenth)



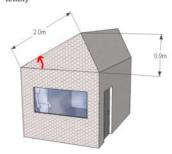


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172. A pilot on a level path knows she should descend at an angle of 3 degrees to maintain comfort and safety. If she is flying at an altitude of 12 000 feet, how many miles from the runway should she begin her descent? 173. An aircraft ascends after takeoff at an angle of 22 degrees. What will be the altitude of the aircraft after it flies at that angle for 1200 m? (nearest metre) 174. A hamster scurries up a ramp at a speed of 1.5 m/s. The ramp is inclined at an angle of 18 degrees. How many metres above the ground will the hamster be after 30 seconds?
173. An aircraft ascends after takeoff at an angle of 22 degrees. What will be the altitude of the aircraft after it flies at that angle for 1200 m? (nearest metre) 174. A hamster scurries up a ramp at a speed of 1.5 m/s. The ramp is inclined at an angle of 18 degrees. How many metres above the ground
m/s. The ramp is inclined at an angle of 18 degrees. How many metres above the ground

175. Anya travels down a zip line at 25 km/h. The angle of descent of the zip line is 11 degrees. How many vertical metres has she fallen after 3 minutes?

- 176. The Earth's radius is 6380 km. A) Find the length of the 35° latitude to the nearest 10 km.
 - B) What assumptions did you make?
- 177. Find the angle of inclination at the back of the roof. The "rise" of the roof is 0.9 m. (nearest tenth)

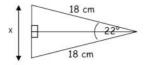


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178. A ladder should make an angle of 72° with the ground for maximum safety. If the ladder is 4 m long, how far should it reach up the wall? (nearest tenth)

179. The angle of elevation to the top of a tree, measured on a 1.5 m transit from a distance of 30 m, is 15° . Find the height of the tree. (nearest tenth)

180. Find the value of 'x'.



181. Mr. J has developed the ideal ice cream cone. The cone has a slant height of 13 cm and a diameter of 7.8 cm. Find the angle that the curved surface makes with the diameter.



182. Mr. J continues to work on his isolated surf hut. Below is two-thirds of a roof truss he wants to complete. Find the length of wood he must cut (nearest tenth) to complete the truss. The long side is 8.2 m and the short side is 6.8 m. The angle between them is 35°.

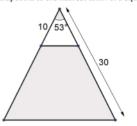


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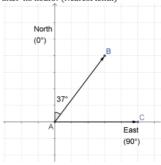


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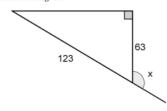
183. Both triangles (large and smaller inset) are isosceles. Find the area of the shaded trapezoid to the nearest tenth of a square unit.



184. From a fire station in central BC, Georgia travels on a bearing of 37° at 6 km/h. Shelby leaves the station at the same time travelling due east at 5 km/h. How far apart are they after 4.5 hours? (Nearest tenth)



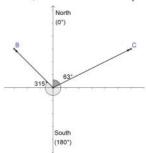
185. Find the measure of angle x to the nearest tenth of a degree.



i

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186. At 9:00 am, a ship leaves port traveling at 30 km/h on a bearing of 63°. At the same time, another ship leaves port on a bearing of 315° at a speed of 19 km/h. When the boats stop after two hours, how far east is the boat at point C?



Draw an accurate diagram to answer each of the following questions. \\

187.	In $\triangle QRS$, $\angle QSR = 90^{\circ}$, $QR = 12 cm$ and $QS = 1880$	In $\triangle TUV$, $\triangle TVU = 90^{\circ}$, $TU = 115 m$ and $TV =$
	10 cm. Find the measure of ∠QRS	99 m. Find the measure of $\angle UTV$

189. In $\triangle DEF$, $\triangle DFE = 90^{\circ}$, $DE = 12 \ cm$ and $\triangle DEF = 30^{\circ}$. Find the length of FE.

190. In $\triangle ABC$, $\triangle ACB = 90^{\circ}$, $BC = 5 \ cm$ and $\triangle ABC = 12^{\circ}$. Find the length of AC.

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Answers:

1.	25.0	
2.	18.9	
3.	24.0	
4.	8.0	
5.	84.0	
6.	54.4	
7.		







11.
$$tan\theta = \frac{3}{4}$$
$$sin\theta = \frac{3}{5}$$
$$cos\theta = \frac{4}{5}$$

12.
$$sin\theta = \frac{1}{0}$$
 $cos\theta = \frac{4}{0}$
 $tan\theta = \frac{3}{1}$

13.
$$sin\theta = \frac{\lambda}{17}$$

$$cos\theta = \frac{15}{17}$$

$$tan\theta = \frac{\lambda}{15}$$

14.
$$sin\theta = \frac{1}{\sqrt{\Gamma}}$$

$$cos\theta = \frac{1}{\sqrt{\Gamma}}$$
$$tan\theta = 1$$

15.	$sin\theta = \frac{4}{5}$
16.	$cos\theta = \frac{1.4}{17.5}$
17.	$tan\theta = \frac{5.0}{16}$

- Tangent Tangent Sine 0.5000 2.7475 19.
- 21. 22.
- 0.8192 24. 25. 0.6691 1.0000
- 0.5000
- A right triangle with an acute angle of 45° is an isosceles triangle with equal legs therefore opp adj will always equal 1, tangent 45 will always equal 1.
- 28. Sine is a ratio of opposite to hypotenuse. If the sine ratio is $\frac{1}{l}$ it means the hypotenuse is twice as long as the opposite side.
- 30. x = 2431. x = 2.232. x = 25
- 33. x = 1.5434. x = 15
- x = 10 cm y = 5.3Answered on page. 36. 37.
- Answered on page. 261.8 miles 39.
- 39. 261.8 mites40. w = 5.5 feet41. x = 17.042. t = 7.9 cm43. t = 6.4 mm
- 44. q = 374.0 km45. w = 2.3 cm46. y = 3.3 m
- 47. v = 13.6 m48. r = 47.1 miles49. d = 22.2 cm
- x = 56.7 mm51. $z = 159.2 \, mm$

- 52. z = 67.2 inches 53. 12.7 km, 6.5 km 54. x = 22.6 mm
- 55. x = 7.6
- Opposite and adjacent Not directly. The tangent 57. ratio does not involve the hypotenuse. Yes, the sine ratio
- involves the hypotenuse.
- 16 cm and 12 cm 3.6 m and 7.1 m 59.
- Not possible, the hypotenuse would need to be shorter than the adjacent side to have a cosine ratio greater than
- 62. Not possible, like the answer above, a sine ratio cannot be greater than 1.

3.
$$\cos J = \frac{\delta x}{\delta c} = \frac{11}{11\sqrt{\Gamma}} = \frac{1}{\sqrt{\Gamma}}$$

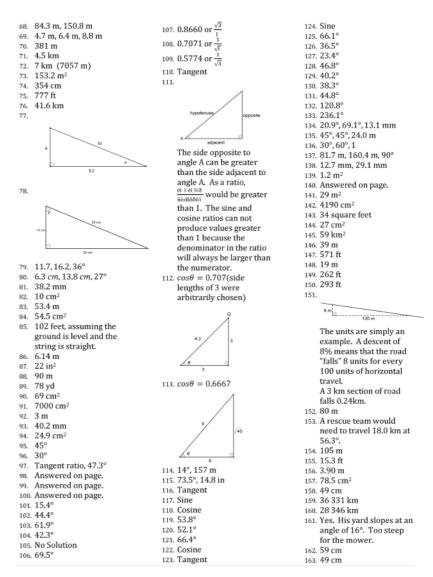


64. $tanP = \frac{\dot{e}\dot{e}}{\dot{e}\dot{e}} = \frac{I}{\dot{A}}$



- Answer will vary. But you will need to use the given side and angle to find another side length. Then choose to find another side or remaining angles. AB = 11.9, AC = 11.3
- 67. 16.6 in, 20.8 in

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164. 181 cm ²	173. 450 m	182. 4.7 m
165. 18 cm	174. 14 m	183. 319.6 square units
166. 31.2 m	175. 239 m	184. 22.4 km
167. 6.5 cm, 11.1 cm, 54°	176. 32 840 km	185. 120.8°
168. 522 m	177. 26.7°	186. 80 km
169. 133 m	178. 3.8 m	187. ∠QRS = 56.4°
170. 10.8 mm	179. 9.5 m	188. ∠UTV = 30.6°
171. 101.1 m	180. 6.9 cm	189. 10.4 cm
172. 43 miles	181. 72.5°	190. 1.1 cm

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