5.3 Graphing Linear Equations

November 26, 2018   8:01 PM

5.3 GRAPHING LINEAR RELATIONS

Name:_____   Block_____

Review

Relation – *"relationship" between 2 variables*
if its __Linear__ => constant rate of change

Example: *(n)number of T-shirts sold for (P)profit*

A) GRAPHING LINEAR EQUATIONS: METHOD 1 - TABLE OF VALUES   *→(x,y)*

You can **graph a linear relation** by building a *table of values* and graphing the *ordered pairs* from the table. It helps to know the "rate of change" *(or the slope)*. *"The pattern"*   *coordinates*

Example #1: Crothall Car Rental charges customers $20 per day, plus $0.10 for each kilometre driven.   *initial/flat fee*   *$0.10/km*

a)   Complete the table of values.

b)   Write an equation for cost:

$$C = 0.10n + 20$$

(x,y)   rate of change is multiplied by the variable   initial fee added

The rate of change is "per" unit. (should show both units)

Number of kilometres driven(n)	Total Cost (C)	
0	20	(0,20)
100	30	(100,30)
200	40	(200,40)
300	50	(300,50)
400	60	(400,60)

C=0.10(100)+20
C=0.10(200)+20
C=0.10(300)+20

Arrow means - shows) the pattern(rate of change) continues

d) Graph the relation using your table of values.
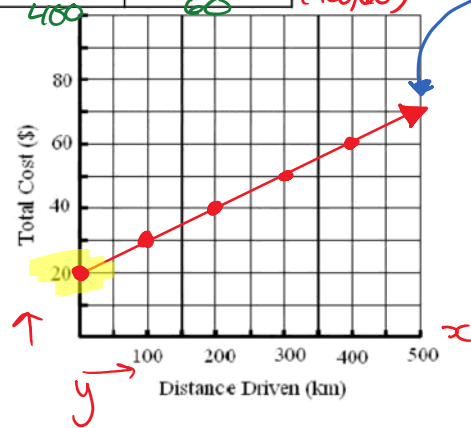
n | C
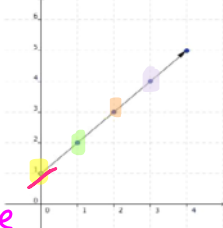
x-values   y-values
(x,y) coordinate

x-values   ↑
(x,y) coordinate

Total Cost ($) — 80, 60, 40, 20

Distance Driven (km) — 100, 200, 300, 400, 500

x
y

PRACTICE

Complete the table of values from the given linear relation.

154.

Example →

A. Complete the table of values.

x	y
0	1
1	2
2	3
3	4

+1 ( ) +1
+1 ( ) +1
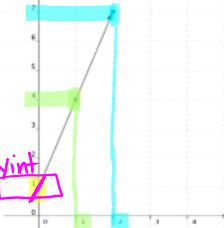
B. Write an equation to represent the table of values?
Y is one more than x. Therefore $y=x+1$

C. How can the equation be used to determine where the line crosses the y-axis (y-intercept)?
The 1 from y=x+1 is where it the graph crosses the y-axis.

when x=0

155.

y int

A. Complete the table of values. *y int is when x=0*

x	y
0	1
1	4
2	7
3	10

+1 ( ) +3
+1 ( ) +3
+1 ( ) +3

B. Write an equation to represent the table of values?
$y = 3x + 1$ *rate of change*

C. How can the equation be used to determine the y-intercept?
y-int = 1
a) rate of change 3 ÷ 1 = 3

156.
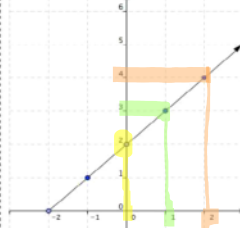
A. Complete the table of values.

x	y
0	2
1	3
2	4
3	5

+1 ( ) +1
+1 ( ) +1
+1 ( ) +1

rate of change $1 ÷ 1 = 1$ *how y changes / how x changes*

B. Write an equation to represent the table of values?
$y = 1x + 2$

C. How can the equation be used to determine the rate of change?
= the number in front of x

Complete the following table of values: *use the formula provided, and substitute x-values*

1) $y = 6x$ $y = 6x$

x	y	
2	12	$y=6(2)$
7	42	$y=6(7)$
-2	-12	$y=6(-2)$
-8	48	$y=6(-8)$
0	0	$y=6(0)$

x =

2) $y = -4x$

x	y	
1	-4	$y=-4(1)$
5	-20	$y=-4(5)$
8		
-8		
0		

5) $y = -5x + 8$

x	y
-5	33
-8	48
4	-12
0	8
2	-2

6) $y = -\frac{1}{8}x - 2$

x	y
6	
-5	
-3	
1	
7	

y-axis / "1"
x - axis
"1" / I -12
① ⑤ 2
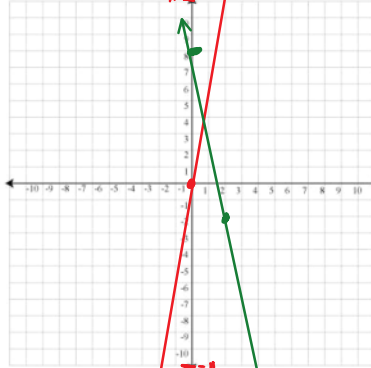
5) $y = -5(-5) + 8 = 33$
$y = -5(-8) + 8 = 48$
$y = -5(4) + 8 = -12$
$y = -5(0) + 8 = 8$
$y = -5(2) + 8 = -2$

"Line" → $y = mx + b$ ← *y-intercept*
rate of change

There is the same x-distance ← → and y-distance ↑↓ between all points

Remember!

Linear Relation
- A linear relation when graphed forms a straight line.
- Or a straight line can be drawn through every point of the graph.
- A linear relation has a constant rate of change.

rate of change = 1
1 ÷ 1 = 1

Which of the following are linear relations?

80. $y = 1x + 3$

x	y
-2	1
-1	2
0	3
1	4
2	5

+1 ( ) +1

Rate of change?
= 1

81. $y = 2x - 1$

x	y
-2	-5
-1	-3
0	-1
1	1
2	3

+1 ( ) +2
+1 ( ) +2

Rate of change?
= 2   *2 ÷ 1 = 2*

82. $y = x^2$

x	y
-2	4
1	1
0	0
-1	1
-2	4

-3 / -1 / +1 / +3

Rate of change?
NOT constant

83. $y = x^3$

x	y
2	8
1	1
0	0
-1	-1
-2	8

-7 / -1 / -1 / +9

Rate of change?
NOT constant

84. $y = \sqrt{x}$

x	y
4	2
1	1
0	0
-1	Ø
-2	Ø

-3 / -1 / -1 / -1   *do not exist*

Rate of change?
NOT constant

NOT constant   NOT constant   NOT constant

$x \xrightarrow{}$

constant rate
of change
⟹ Linear
Relation

Linear
Relation

NOT
Linear

NOT
Linear

NOT
Linear

"Rate of Change" and "the Slope of a line" mean the same thing

B) SLOPE OF A LINE ⟹ symbol "m" = slope

Determine the "Rate of Change" for the following table of values:

x	y
5	5
7	3
9	1
11	−1

+2 → −2
+2 → −2
+2 → −2

rate of change $= \dfrac{y}{x} = \dfrac{-2}{+2}$

$= -1$

Slope $= \dfrac{\text{how } y \text{ changes}}{\text{how } x \text{ changes.}}$

$y = -2$
$x = +2$

arrows
show the
pattern

(x,y) coordinates:

(5,5), (7,3), (9,1), (11,-1)

y - axis

x - axis
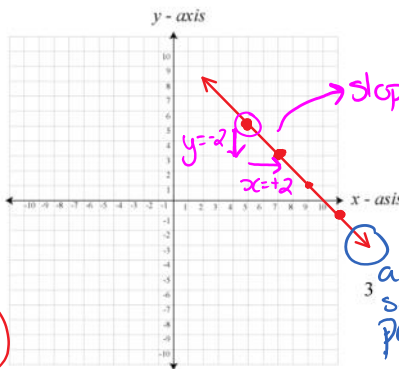
Name:

Slope (vertical change over horizontal change) is represented by the letter "m."

$$m = \frac{\text{"rise"}}{\text{"run"}}$$

Δ = "delta" = Greek "change in"

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

(x, y)

The slope of a line can be determined from a table, by **counting** units on a coordinate plane, or by **subtracting** coordinates.

The slope of a horizontal line is **zero**.

$$\frac{\Delta y}{\Delta x} = \frac{0}{x}$$

↳ any number

The slope of a vertical line is "undefined".

$$\frac{\Delta y}{\Delta x} = \frac{y}{0} \xleftarrow{\text{any number}} = \text{ERROR!}$$
∴ by zero X

Remember:
↑ UP and RIGHT are **Positive** movements;
↓ DOWN and LEFT are **Negative** movements.

Slope represents the **rate of change**.
Slope should be written as a **fraction** in **simplest form**

eg. $\frac{4}{1}$ ← rise ← run
$\frac{2}{3}$
$\frac{5}{5} = \frac{1}{1}$

HOW STEEP?
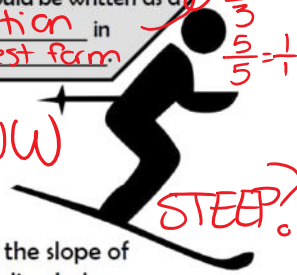
Find the slope of each line below.

$m = \frac{\Delta y}{\Delta x} = \frac{3}{2}$ Ⓐ

start here
+3
+2

Ⓑ start
(2, 4)
$m = \frac{\Delta y}{\Delta x} = \frac{-6}{2} = -3 = \frac{-3}{1} = -3$
-6

Ⓒ
(4, -2)
start
+2
+4
+2
$m = \frac{\Delta y}{\Delta x} = \frac{+2}{+4} = \frac{1}{2}$

Find the slope between the two points.

1. $(3, -2)$ and $(4, 4)$
$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-2)}{4 - 3} = \frac{6}{1} = 6$

2. $(6, 0)$ and $(-8, -1)$
$m = \frac{(-1) - 0}{(-8) - 6} = \frac{-1}{-14} = \frac{1}{14}$ ← Δy ← Δx

Plot a line that starts at the origin and has a slope of **-3**. Label it "a."
$(0, 0)$
$\frac{\Delta y}{\text{"rise"}} \Rightarrow \frac{-3}{1} \leftarrow \frac{\Delta x}{\text{"run"}}$

Plot a line that starts at $(0, 4)$ and has a slope of $\frac{-3}{4}$. Label it "b."
Δy down 3
Δx right 4

Ⓐ
+3 4 up 3
-3 down 3
+1

© Copyright 2015 Math Giraffe
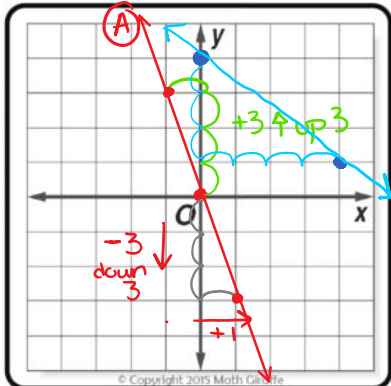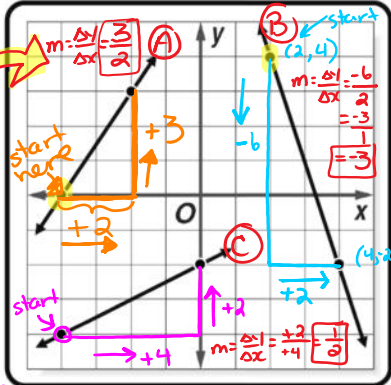
Steeper slopes have greater **value**.

STEEP

tRY-it

(A) (-4,4)
$m = \frac{-4}{+1} \frac{rise}{run}$

(B) (0,2)
$m = \frac{-1}{+2} \frac{rise}{run}$

A

-4

-2 +1

B

-1

+2

slope = $-\frac{1}{2}$ less steep.

⊖ negative slopes.

O

x

slope = -4 = steeper
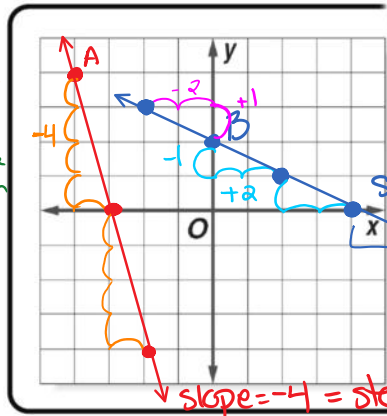
Graph four different lines, all with different negative slopes. Show each slope and compare steepness.

slopes will be represented with fractions with a greater

(ignore ± sign)

| value |

less steep = lower value

Sketch a sample (or a few) of each type of slope. Add a skier if you want! It may help you remember the direction and whether the values are increasing or decreasing.

sketch it

positive
slope
"upstairs"

negative
slope
"downstairs"

zero
slope
horizontal
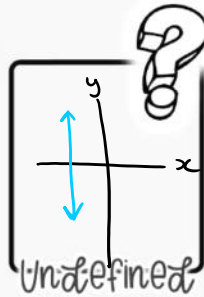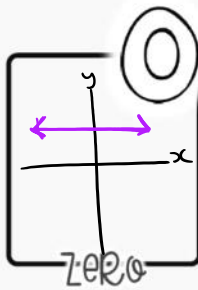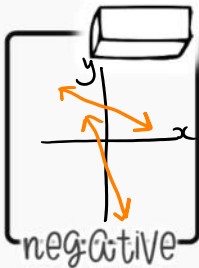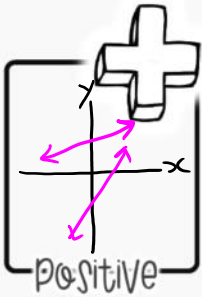
undefined
slope
vertical

order from steepest to least steep: 1/3, 3, 3/2, 3/4

steepest ————→ least steep
3 , $\frac{3}{2}$, $\frac{3}{4}$, $\frac{1}{3}$

$\frac{rise}{run}$ = $\frac{1}{3}$, $\frac{3}{1}$, $\frac{3}{2}$, $\frac{3}{4}$

Name_____   Date_____

EXTREME SPORTS

FINDING SLOPE

"slope"
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
(x, y)

> HW

Directions: Find the slope of the line given a pair of points, a table, a graph or a verbal situation. Write your answer in the box below the problem number.

1	$x_1\ y_1\quad x_2\ y_2$ $(20, -3), (19, -20)$ $\frac{-20-(-3)}{19-20}$
A	A. ⟨17⟩ B. $\frac{1}{17}$ C. -17 D. $-\frac{1}{17}$ $=\frac{-17}{-1}=17$

2	$(-5, 11), (0, 7)$
C	A. $-\frac{5}{4}$ B. $\frac{5}{4}$ C. $-\frac{4}{5}$ D. $\frac{4}{5}$

3	$(-20, 9), (14, 9)$
D	A. undefined B. 3 C. -3 D. 0

4	$(12, -1), (-2, -3)$
B	A. -7 B. $\frac{1}{7}$ C. $-\frac{1}{7}$ D. 7

5	$(-3, 15), (9, 17)$
B	A. 6 B. $\frac{1}{6}$ C. $-\frac{1}{6}$ D. -6

6	$(7, -11), (16, 7)$
D	A. $\frac{1}{2}$ B. $-\frac{1}{2}$ C. -2 D. 2

7	$(-1, -16), (-1, 11)$
A	A. undefined B. 1 C. -1 D. 0

8	$(-2, -7), (3, -10)$
B	A. -3 B. $-\frac{3}{5}$ C. $\frac{3}{5}$ D. 5

9	Table of values $x_1\ x_2$
$\frac{3}{4}$		x	-8	0	4	8	 	y	-7	-1	2	5	 $y_1\ y_2$

10	
$\boxed{-2}$		x	-1	0	1	2	 	y	19	17	15	13	

11	
	x	y	
	0	6	
	5	5	$\boxed{-\frac{1}{5}}$
	10	4	
	15	3	

12	
	x	y	
	-6	-27	
	-5	-23	$\boxed{-4}$
	-1	-7	
	0	-3	

13

"rise" +1
"run" +4

$m = \dfrac{rise}{run} = \dfrac{+1}{+4}$

$m = \dfrac{1}{4}$

14

= undefined

15

= -2

16

$= \dfrac{-4}{5}$
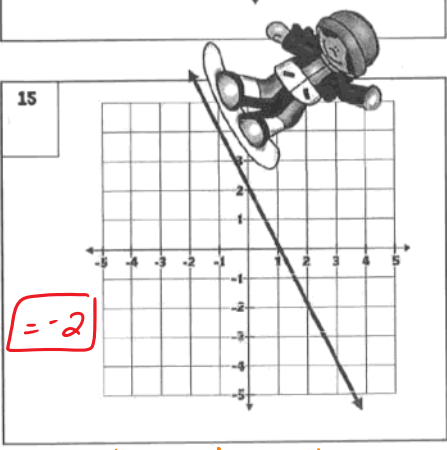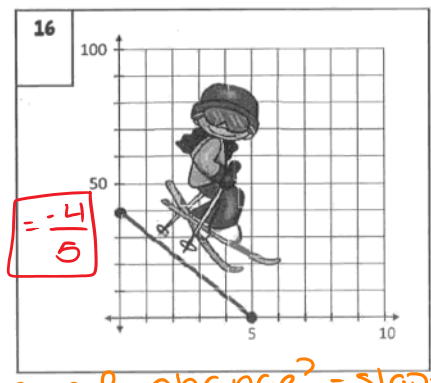
"what is the rate of change? = slope"

17 The water level in Noah's water tank is 72 inches high. Noah begins to drain a water tank by opening a valve. The water drains at the speed of 5 inches per minute.

$m = \dfrac{5 in}{min}$

18 A pet store is selling puppies for $50 each plus a $14.99 transfer fee.

$ \$\; $64.99/puppy

$ (graph: \$ vs \#puppies) $

19 A small bookstore is selling copies of the book To Kill a Mockingbird for $6.99 each plus $0.99 for shipping and handling.

$7.98/book

20 To rent movies from the store, a person has to pay an annual membership fee of $20 plus $2.50 for each movie rented.

$2.50/movie

Line → how is x related to y?

Example #2: Graph the linear relation $y = 2x - 4$ using a table of values.

e) In this example, the __y-value__ depends on the __x-value__

Dependent Variable (Vertical Axis)	Independent Variable (Horizontal Axis)

Table of Values ↓

$y = 2x - 4$

x	y
-1	-6
0	-4
1	-2
2	0
3	2

$y = 2(-1) - 4 = -6$
$y = 2(0) - 4 = -4$
$y = 2(1) - 4 = -2$
$y = 2(2) - 4 = 0$
$y = 2(3) - 4 = 2$
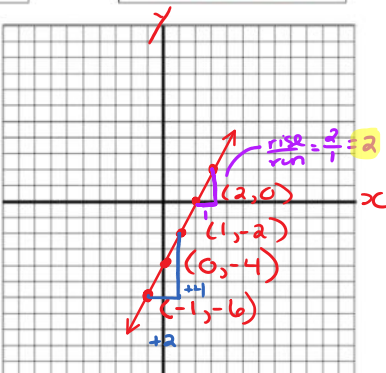
(x, y) coordinates ⟹

what is the slope?

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$\dfrac{\text{rise}}{\text{run}} = \dfrac{2}{1} = 2$

$(2, 0)$
$(1, -2)$
$(0, -4)$
$(-1, -6)$
+4
+2

$\dfrac{\Delta y}{\Delta x} = \dfrac{\text{rise}}{\text{run}} = \dfrac{4}{2} = \dfrac{2}{1} = 2$

HW **PRACTICE** ✏️

Graph $y = 6 - 3x$ using a table of values.

$y = 6 - 3x$

x	y
-2	12
-1	9
0	6
1	3
2	0

choose your own values

$m = -3$
$= \dfrac{-3}{1}$

$y = 6$
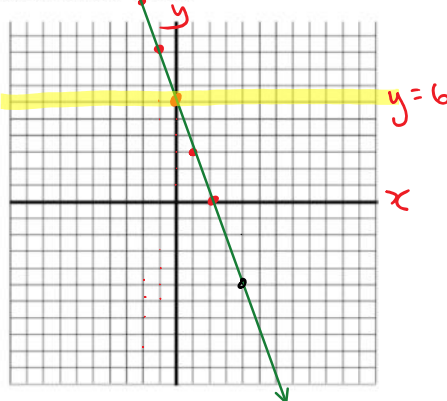
Using your graph estimate the value of y if x = 4: __-6__   value of x if y = 3: __x = 1__

8

Example #3: The Reynolds student council is planning to hold a dance.
The profit in dollars is four times the number of students who attend, minus $200 for the cost of the DJ.

$4x \rightarrow n$ — 200

a) Write an equation that relates the profit (P) to the number of students (n) who attend.

$$P = 4n - 200$$

b) What is the lowest value of *n* that we can include in the table of values?
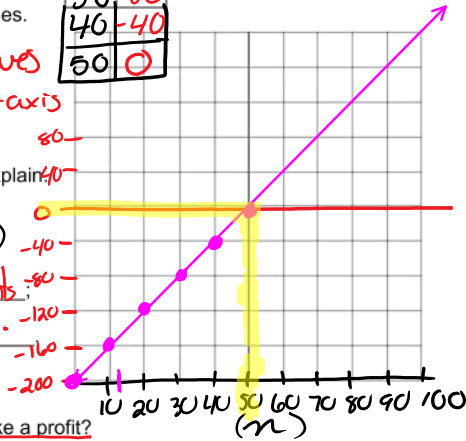
0

c) Create a table of values for this relation

n	P
0	-200
10	-160
20	-120
30	-80
40	-40
50	0

$P = 4(10) - 200 = -160$
$P = 4(20) - 200 = -120$

d) Graph the relation using your table of values.

* when you have - values create another "0" x-axis

e) Does is make sense to join the points? Explain

discrete data
- no joing dots
- no 1/2 people

(P)

f) The independent variable is # of students ;
(x-axis)

the dependent variable is Profit.
(y-axis)

g) How many students have to attend to make a profit?

51

Where did you look to find this?

interpolate on graph.

or table of values.

h) How would the graph be different if the DJ was free?

There would be no ⊖ y-values
(start) at zero.

⑨

HW: complete up to p.9.

C) GRAPHING HORIZONTAL & VERTICAL LINES
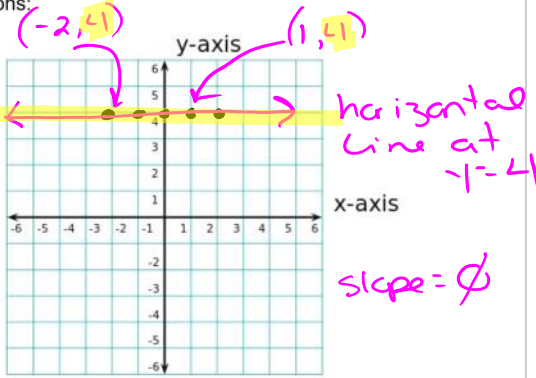
Create a graph for each of the following relations:

a) $y = 4$

same ⊖
y-intercept
same ⊕

x	y
-2	4
-1	4
0	4
1	4
2	4

All of y-values = 4 for ANY x-value

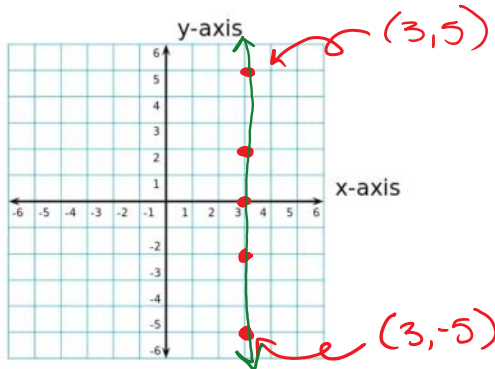$(-2, 4)$ y-axis $(1, 4)$

horizontal line at y = 4

x-axis

slope = ∅

The equation of a **horizontal line** has the form ___$y = b$___ . where "b" = y-intercept.

b) $x = 3$

all x-values = 3 for ANY y-value

x	y
3	-5
3	-2
3	0
3	2
3	5

y-axis $(3, 5)$

x-axis

$(3, -5)$

The equation of a **vertical line** has the form ___$x = a$___ .

"a" = the x-intercept.

※ NO y-intercept
※ NO slope
        ↳ "undefined"
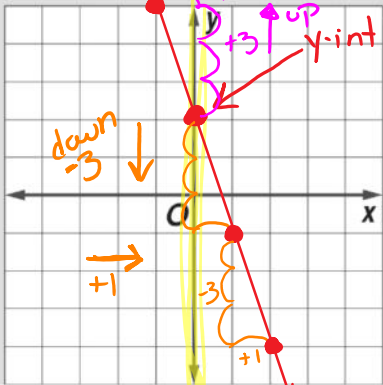
10

Practice Using Slope-Intercept Form

Write an equation for each line in slope-intercept form.

$$y = \boxed{m}x + \boxed{b}$$
slope   y-int

Left graph:

slope = $\frac{rise}{run}$

$= \frac{-2}{+6} = -\frac{1}{3}$

$y = \boxed{\frac{1}{3}}x - \boxed{1}$

slope   y-int

y-int = -1

rise -2

run +6

Right graph:

$y = \boxed{6}x - \boxed{3}$

slope = $\frac{6}{1} = \boxed{6}$

$\frac{+6}{+1} = \boxed{6}$

+6

+1

y-int = -3

© Copyright 2016 Math Giraffe

Graph each line.

Left graph:

y-int = +4

down -1

→ +5

run

$y = -x + 4$

means -1x

slope = -1 = $\frac{-1}{1} = \frac{rise}{run}$

Right graph:

+4

+1

-1

-4

$y = \frac{1}{4}x - 1$

slope = $\frac{1}{4} = \frac{rise}{run}$

y-int.

Identify the slope and y-intercept for each.

A. $4 - 2x = 8$

B. $7 = y - x$

C.

x	-2	2	6
y	5	-3	-11

D.   the line that passes through (3, 3) and (-6, 0)

Name:

Special Cases

NO y-int.
$y = 2x$
(vertical line)

No slope
$y = 4$
∅ slope
Horizontal Line

Extra Work

$3y + 5 = 6x$

D) GRAPHING: METHOD 2-SLOPE INTERCEPT FORM

You can graph a linear relation represented using the *equation of the line* in **SLOPE-INTERCEPT FORM**:

$$y = mx + b$$

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$
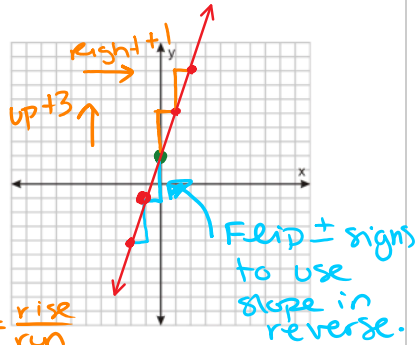
y-intercept.

Example #5:
Without using a table of values graph the following relation:

$$y = 3x + 2$$

a) What is the fixed term? (y-intercept) +2
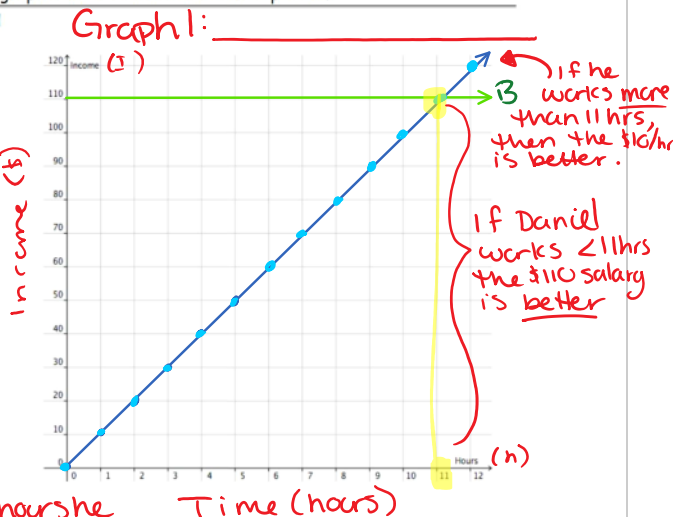
b) What is the rate of change? (slope) = 3

*remember when slope is a whole number... write it over 1 ex. $3 = \frac{3}{1} = \frac{\text{rise}}{\text{run}}$

right +1 →
up +3 ↑

FLIP ± signs to use slope in reverse.

Challenge #9: Write an equation, graph a linear relation and solve a problem.

200. Daniel works at a restaurant and currently makes **$10/h.** The general manager has just asked him if he would like to take a salary job for **$110 per day.**

A. Write an equation to represent income in terms of **hourly pay.**

$$I = 10h$$

B. Write an equation to represent income in terms of **salary.**

$$I = 110$$

C. Graph a linear relation that compares the two income options.

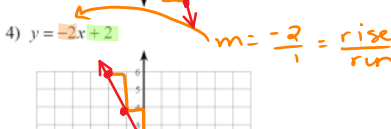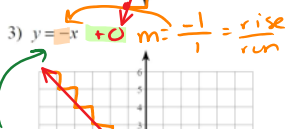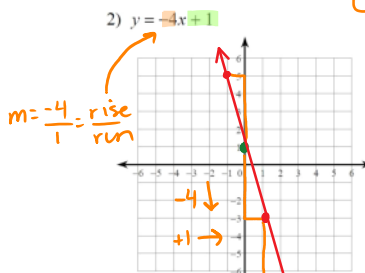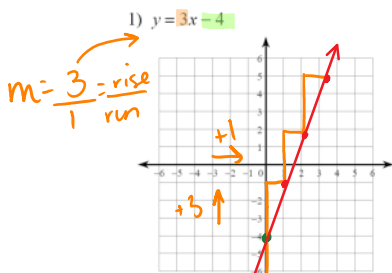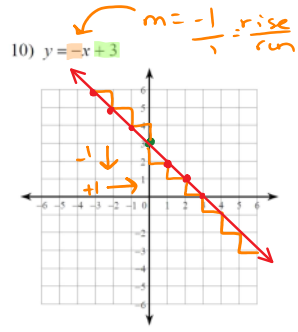D. He decides against the salary position. Was this wise? Explain.

Depends how many hours he typically works (see graph notes)

Graph 1:

If he works more than 11 hrs, then the $10/hr is better.

If Daniel works <11hrs the $110 salary is better

Income ($)
Time (hours)

13

HW: Challenge #9 + Practice Page 14+15

PRACTICE

shown with green dot.

Sketch a graph of each line by identifying the **y-intercept** & **using the slope**:

↳ shown in orange

1) $y = 3x - 4$

$m = \frac{3}{1} = \frac{\text{rise}}{\text{run}}$
+1 →
+3 ↑

2) $y = -4x + 1$

$m = \frac{-4}{1} = \frac{\text{rise}}{\text{run}}$
-4 ↓
+1 →

3) $y = -x + 0$ $m = \frac{-1}{1} = \frac{\text{rise}}{\text{run}}$

4) $y = -2x + 2$ $m = \frac{-2}{1} = \frac{\text{rise}}{\text{run}}$

no y-int.
in equation
means
y-int=0

$-1 \downarrow$

$+1 \rightarrow$

$m = \dfrac{-1}{1} = \dfrac{rise}{run}$

$-2 \downarrow$

$+1 \rightarrow$

9) $y = \dfrac{1}{2}x + 3$ $m = \dfrac{1}{2} = \dfrac{rise}{run}$

$-1 \downarrow$

$+2 \rightarrow$

10) $y = -x + 3$ $m = \dfrac{-1}{1} = \dfrac{rise}{run}$

$-1 \downarrow$

$+1 \rightarrow$

14
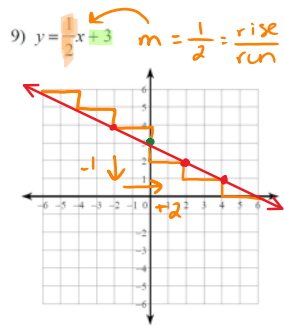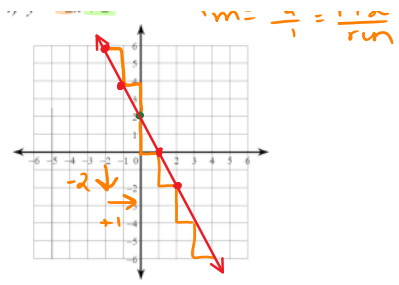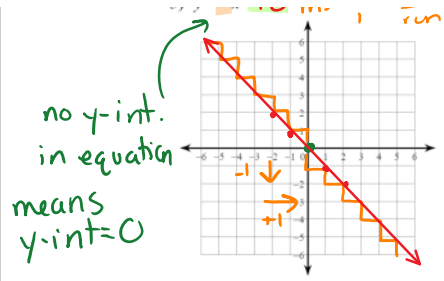
PRACTICE
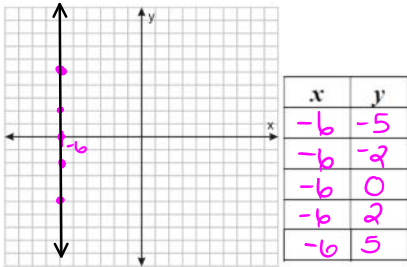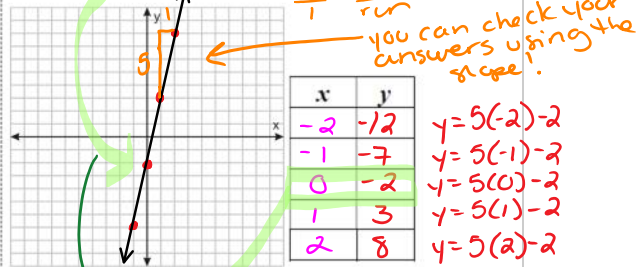
Graph the following relations:
a. using the properties of $y = mx + b$.
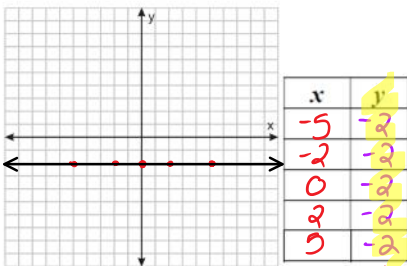b. then check your points with a table of values.
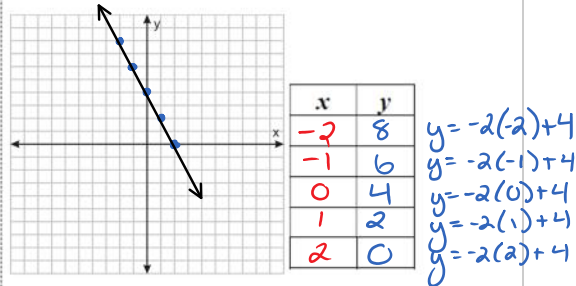
A. $x = -6$ *(vertical line)*

x	y
-6	-5
-6	-2
-6	0
-6	2
-6	5

B. $y = 5x - 2$

remember: use $\emptyset$ always then use a couple $\ominus$ and a couple $\oplus$...any numbers!!

$m = \dfrac{5}{1} = \dfrac{rise}{run}$

you can check your answers using the slope!

x	y	
-2	-12	$y = 5(-2)-2$
-1	-7	$y = 5(-1)-2$
0	-2	$y = 5(0)-2$
1	3	$y = 5(1)-2$
2	8	$y = 5(2)-2$

use the formula to check your answers in table of values + the graph!

C. $y = -2$ *(horizontal line)*

x	y
-5	-2
-2	-2
0	-2
2	-2
5	-2

for any x-value!

D. $y = -2x + 4$

x	y	
-2	8	$y = -2(-2)+4$
-1	6	$y = -2(-1)+4$
0	4	$y = -2(0)+4$
1	2	$y = -2(1)+4$
2	0	$y = -2(2)+4$

Homework

Assignment #5.3 pg 185-189

Required	Extra Practice	Extension
1, 2, 3, 4ace, 5ace, 7, 8ab, 9, 10, 11, 12, 15, 18	4bdf, 5bdf, 6, 8cd, 13, 14, 16, 17, 19	20, 21, 22, 23

15