4 Angle of Elevation & Depression

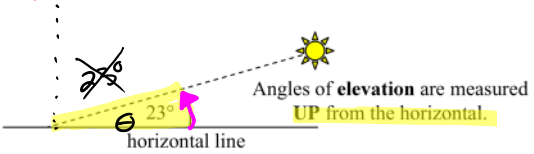
September 24, 2018 9:22 PM

Applications of Right Triangle Trigonometry:
*Angles of **Elevation** & Angles of **Depression***

Preliminary Information: On most maps, it is customary to orient oneself relative to the direction north: for this reason, north is almost always indicated on every map. Likewise, when working with real-life trigonometry problems, it is very common to orient angles relative to a Horizontal line.

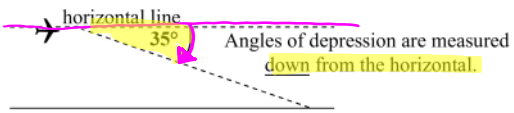
(inclination)

An **angle of** Elevation refers to the acute angle a line (or ray, segment, etc.) makes with a horizontal line, when measured above the horizontal (hence an angle of *elevation*).



For example, the sun's rays could form a 23° angle of elevation *(above the horizon)*.

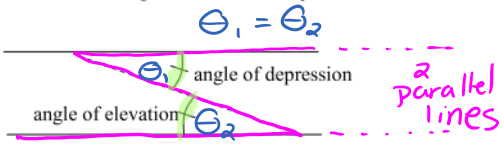
An angle of Depression refers to the acute angle a line makes with a horizontal line, when measured below the horizontal (hence an angle of depression).



For example, an airplane pilot could look down and see a feature on the ground below at a 35° angle of depression (below the horizon).

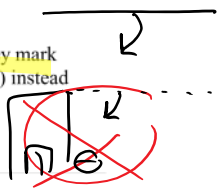
Angles of elevation and depression typically have their vertex at the point where an observer is positioned. In the previous example, notice that the vertex of the 35° angle is located at the pilot's location.

Because horizontal lines are everywhere parallel, **angles of depression and elevation** are numerically EQUIVALENT because they form ***alternate interior angles*** of parallel lines:



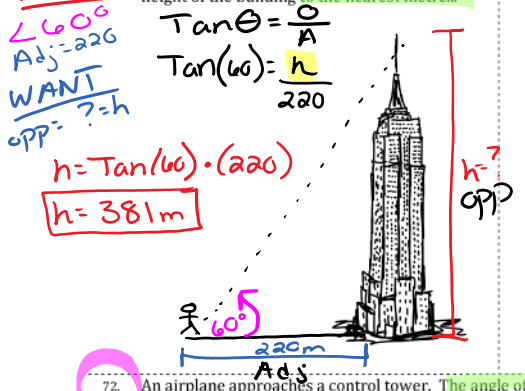
Students should always be encouraged to consider the following two ideas when they see either phrase mentioned in a problem:

- **You may** always ***draw an additional horizontal line on any diagram*** extending from any point in the diagram. Just as you did in Geometry, drawing such an *auxiliary line* can help to make a complex problem simpler.
- **The most common error** students make when they encounter these terms is they mark an angle relative to a vertical line (such as an angle with a wall, building, or tree) instead of with a horizontal line.

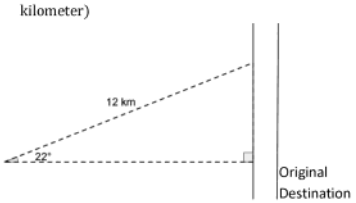


Solve each of the following word problems. Include a diagram in your solution.

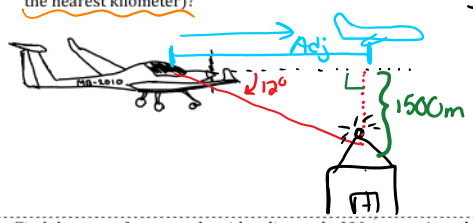
70. From a point 220 m from the Empire State Building, a tourist measures the angle of inclination to the top to be 60°. Calculate the height of the building to the nearest metre...



71. A hiker loses track of her direction and wanders 22 degrees off course. If she continues to walk for 12 km to the river destination, how far away from her original destination will she be? (nearest tenth of a kilometer)



72. An airplane approaches a control tower. The angle of depression from the pilot to the tower is 12°. If the plane is flying at an altitude of 1500 m, how far is the plane from being directly above the tower (to the nearest kilometer)?



$\tan\theta = \frac{opp}{Adj}$

$\tan(12°) = \frac{1500\,m}{Adj?}$

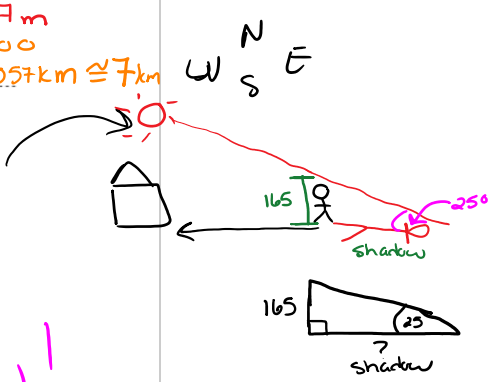
$Adj = \frac{1500\,m}{\tan(12°)} = 7057\,m$

$\div 1000$

$= 7.057\,km \cong 7\,km$

73. Find the area of a rectangle with a diagonal of 20 m if the angle between the diagonal and longer side is 25 degrees. (nearest unit)

74. A student crossing to the west building casts a shadow on the path. She is 165 cm tall and the angle to the sun is 25°. How long is the shadow on the path to the nearest centimetre?

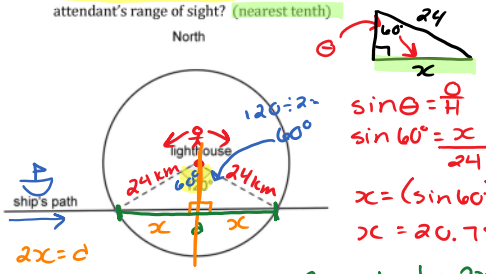


HW, up to Q #74

Quiz tomorrow!

75. A radio tower is 396 feet tall. How far from the base of the tower is a technician if the angle of inclination to the top of the tower is 27°? Answer to the nearest foot.

76. A lighthouse attendant has a range of visibility of 24 km. A ship on the horizon passes by the lighthouse. The attendant sees the ship for a total of 120 degrees. For how many kilometers was the ship within the attendant's range of sight? (nearest tenth)



$\sin\theta = \frac{O}{H}$

$\sin 60° = \frac{x}{24}$

$x = (\sin 60°)(24)$

$x = 20.78$

range of sight = $d = 2x$

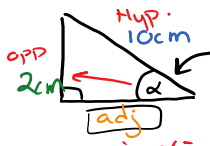
$d = 2(20.78)$

$d = 41.6$ km

Draw a scale diagram that would **represent** each of the following.

77. Draw a triangle that has a the following: $\sin\theta = \frac{O}{H}$

$\sin\alpha = \frac{1}{5}$, hypotenuse is 10 cm long.



$\sin\alpha = \frac{1}{5} = \frac{O}{H} = \frac{?}{10cm}$

$\sin\alpha = \frac{1}{5} \overset{\times 2}{=} \frac{opp}{10}$ $\quad opp = 2$

$a^2 + b^2 = c^2$

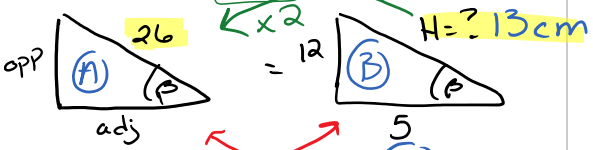
$a = \sqrt{c^2 - b^2}$

$a = \sqrt{10^2 - 2^2}$ $\quad$ adj. $= 9.2$ cm

78. Draw a triangle that has a the following:

$\tan\beta = \frac{12}{5}$, hypotenuse is 26 cm long.

$\tan\beta = \frac{12}{5} = \frac{Opp}{Adj}$



opp = 24

adj = 10

×scale factor

B $c = \sqrt{a^2 + b^2}$

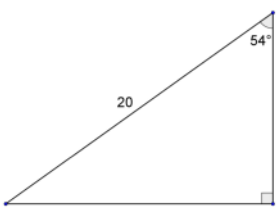
$c = \sqrt{5^2 + 12^2}$

$c = 13$

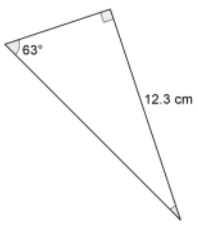
$\tan\beta = \frac{O}{A} = \frac{24}{10}$ $\quad \times 2$ $\quad \tan\beta = \frac{12}{5}$

HW: up to Q# 94 + Quiz corrections

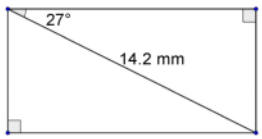
79. Solve the triangle.



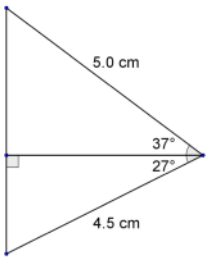
80. Solve the triangle.



81. Find the perimeter of the following rectangle.



82. Find the total area.

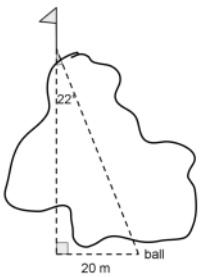


Lesson 4 (after Quiz 1)

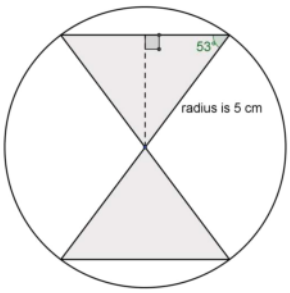
FMPC 10

updated June 2018

83. While golfing with his father-in-law, Mr. J hits a shot short of a pond. He walks 20 m to his left to a point directly across the pond from the hole. The angle between the two lines of sight is 22°. Find the distance from his ball to the hole to the nearest tenth of a metre.



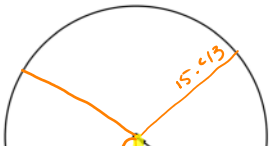
84. Find the area of the circle that is not covered by the shaded triangles. Answer to the nearest tenth.



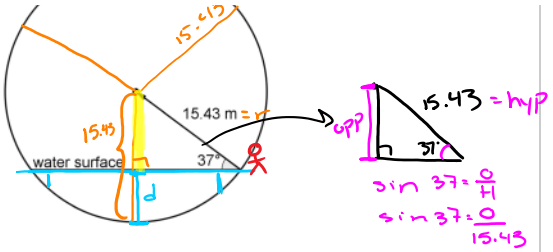
85. Anya lets out 125 feet of kite string at Clover Point. The wind pulls the kite string tight at an angle of 55° to the ground. Approximate the height of the kite to the nearest foot .

What assumptions did you make?

86. The radius of a circular tunnel in Shanghai is 15.43 m. During a flood, a worker in the water at the side of the tunnel measured an angle to the centre to be 37°. Find the depth of the water at its deepest point. (The water surface forms a chord across the tunnel.)



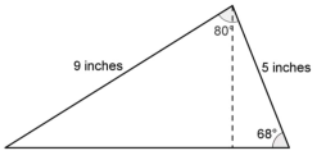
What assumptions did you make?



FMPC 10 updated June 2018

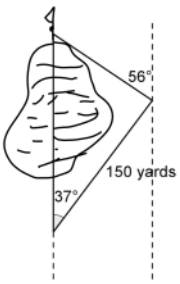
Solve the following problems involving triangles.

87. Find the area of the triangle below to the nearest square inch.

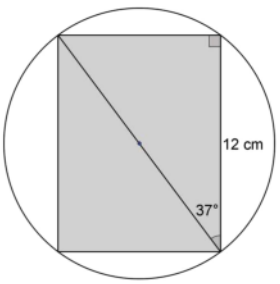


88. At 11:00 in the morning, the angle of elevation to the sun 58°. A tree in the school yard casts a shadow of 56 m. How tall is the tree to the nearest metre?

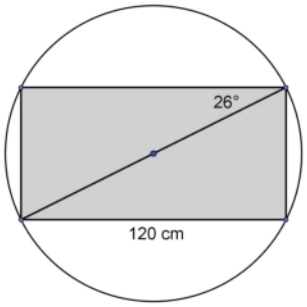
89. Tucker has two choices to get his ball to the hole at the 17th at Cordova Bay, go around the lake or go over it. He decides to go around the lake as shown on the diagram. How much farther does he have to hit the ball going around the lake instead of going straight over it? Answer to the nearest yard.
(Careful, not a right triangle shown.)



90. Find the area of the circle that is not covered by the shaded rectangle to the nearest square unit.

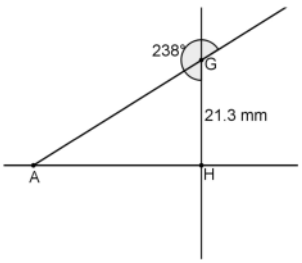


91. Find the area of the circle not covered by the shaded **rectangle** to the nearest 100 cm.

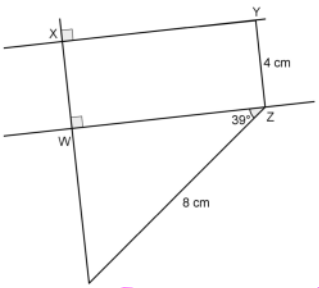


92. Sandra stands at the midpoint between two buildings and measures the angles of elevation to their tops to be 14° and 18°. If the two buildings are 80 metres apart, what is the difference in their heights? Answer to the nearest metre.

93. Find the length of AG to the nearest tenth of a millimetre.



94. Find the area of rectangle WXYZ to the nearest square unit.



Complete up to here Oct 1st H.W.