6.3 SOLVING MULTI-STEP LINEAR EQUATIONS

Name: Key Block ____

In this section we are going to look at ***more complicated examples*** of linear equations that involve using ***multiple steps*** and ***all the techniques we've learnt so far*** to find a solution.

The aim of the game:
Use the algebraic techniques we have been working with in this unit to **isolate the variable on <u>one side</u>** of the equation and the **numeric terms on the <u>other side</u>** of the equation

A) VARIABLES ON BOTH SIDES

Many multi-step equations contain variables on BOTH sides.

So how do we decide which to isolate? COLLECT LIKE TERMS! → x, m whatever the variable is.

Move ***all variables to the same side*** and all <u>*constant (numbers only) terms to the other side*</u>!

EXAMPLE:

a) $3m + 3 = 7m + 12$

$-7m \quad -7m$

$-4m + 3 = 12$

$-3 \quad -3$

$\frac{-4m}{-4} = \frac{9}{-4}$

$m = -\frac{9}{4} = -2.25$

Check: $3(-2.25)+3 = 7(-2.25)+12$

$-3.75 = -3.75$ ✓

b) $-5m + 20 = -7m - 15$

$+5m \quad +5m$

$20 = -2m - 15$

$+15 \quad +15$

$\frac{35}{-2} = \frac{-2m}{-2}$

$-17.5 = m$

Give students ~5mins to make a start on Practice Q... check around to see how they are doing... can go over a few together... but this is part of HW. (all Practice) questions

PRACTICE

120. Solve. $5m+1=3m-7$

$-1 \quad -1$

$5m = 3m - 8$

$-3m \quad -3m$

$\frac{2m}{2} = \frac{-8}{2}$

$m = -4$

121. Solve. $13m+5=11m-7$

$-11m \quad -11m$

$2m + 5 = -7$

$-5 \quad -5$

$\frac{2m}{2} = \frac{-12}{2}$

$m = -6$

124. Solve. $2m+3=-7m-15$

$+7m \quad +7m$

$9m + 3 = -15$

$-3 \quad -3$

$\frac{9m}{9} = \frac{-18}{9}$

$m = -2$

122. Solve. $2m+10=7m-15$

$-7m \quad -7m$

$-5m + 10 = -15$

$-10 \quad -10$

$\frac{-5m}{-5} = \frac{-25}{-5}$

$m = 5$

123. Solve. $-3m+18=6m-6$

$+3m \quad +3m$

$18 = 9m - 6$

$+6 \quad +6$

$\frac{24}{9} = \frac{9m}{9}$

$m = \frac{24}{9} = \frac{8}{3}$

125. Solve. $2m+20=-7m-15$

$+7m \quad +7m$

$9m + 20 = -15$

$-20 \quad -20$

$\frac{9m}{9} = \frac{-35}{9}$

$m = -\frac{35}{9}$

B) BRACKETS, FRACTIONS VARIABLES ON BOTH SIDES...OH MY!

Now that the equations are getting more complex, it may helpful to review these steps.

- Eliminate Fractions by multiplying both sides by the common denominator.
- Eliminate brackets by Expanding.
- Collect Like Terms on each side of the equal sign.
- Get variables to same side by Subtracting or Adding variables to each side.
- Get constants to same side by Subtracting or Adding constants to each side.
- Isolate the variable by Dividing both sides by the coefficient.

EXAMPLE:

① Expand Brackets

a) $2(6w+2) = 4w - 3$

$12w + 4 = 4w - 3$

$-4 \quad -4$

$12w = 4w - 7$

$-4w \quad -4w$

$\frac{8w}{8} = \frac{7}{8}$

$w = \frac{7}{8}$

b) $\frac{2}{3} + \frac{5}{6}c = \frac{1}{3}c - \frac{1}{6}$ ×6 (multiply by the LCD! (lowest common denominator) to remove the fractions)

$\frac{12}{3} + \frac{30}{6}c = \frac{6}{3}c - \frac{6}{6}$

(12÷3) (divide top by bottom)

$4 + 5c = 2c - 1$

c) $\frac{2(m-1)}{1} + \frac{5m}{2} = \frac{2}{3}(m+3)$ ×6 LCD = 6

$12(m-1) + \frac{30m}{2} = \frac{12}{3}(m+3)$ ÷ and simplify fractions

$12(m-1) + 15m = 4(m+3)$ expand brackets

$12m - 12 + 15m = 4m + 12$

$-4m \quad -4m$

$12m + 15m - 4m - 12 = 12$

collect like terms $+12 \quad +12$

$\frac{23m}{23} = \frac{24}{23}$

$m = \frac{24}{23}$

PRACTICE

(same as before... students TRY these on own)

⊖ · ⊖ = ⊕ watch the signs! remember the rules!

142. Solve. $4(m-1)-6m=-10(2m-1)-1$

$4m - 4 - 6m = -20m + 20 - 1$

$+20m$

$4m - 6m + 20m - 4 = +20 - 1$

$+4 \quad +4$

combine like terms

$\frac{18m}{18} = \frac{23}{18}$

$m = \frac{23}{18} = 1.28$

143. Solve. $3(m-1)+6m=5(2m-1)+1$

$3m - 3 + 6m = 10m - 5 + 1$

$-10m \quad -10m$

$3m + 6m - 10m - 3 = -5 + 1$

collect. $+3 \quad +3$

$\frac{-1m}{-1} = \frac{-1}{-1}$ watch signs! * $\frac{\ominus}{\ominus} \div = \oplus$

$m = 1$

158. $\frac{5m}{2} + \frac{m}{3} = \frac{1}{2}m + 5$ ×6 multiply by LCD

$\frac{30m}{2} + \frac{6m}{3} = \frac{6m}{2} + 30$ ÷ divide fractions.

$15m + 2m = 3m + 30$

$-3m \quad -3m$

$\frac{14m}{14} = \frac{30}{14}$

$m = \frac{30}{14} = \frac{15}{7} = 2.14$

160. $m - \frac{m}{3} = \frac{1}{4}m + 4$ ×12 (LCD of 3 and 4)

$12m - \frac{12}{3}m = \frac{12}{4}m + 48$ ÷ divide fractions

$12m - 4m = 3m + 48$

$-3m \quad -3m$

$\frac{5m}{5} = \frac{48}{5}$

$m = \frac{48}{5} = 9.6$

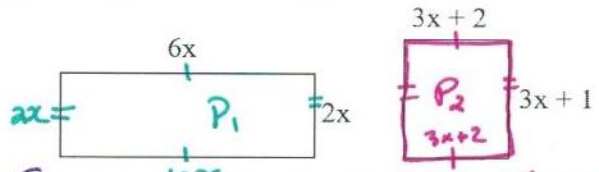
C) MULTI-STEP LINEAR EQUATION WORD PROBLEMS

Example 1:

Distance around... adding up all sides.

The rectangles pictured have the ***same perimeter.***

a) Determine the value of x that makes this true.



Step ① Find Perimeter: $P_1 = 6x + 6x + 2x + 2x$ or $P_1 = 2(6x) + 2(2x)$

$P_2 = 2(3x+2) + 2(3x+1)$

$P_1 = P_2$

Step ② Solve for x: expand brackets

$2(6x) + 2(2x) = 2(3x+2) + 2(3x+1)$

$12x + 4x = 6x + 4 + 6x + 2$ *combine like terms!

$16x = 12x + 6$

$-12x \quad -12x$

$\frac{4x}{4} = \frac{6}{4}$

$x = 1.5$

b) What are the dimensions of each rectangle? (substitution)

$P_1 = 6x + 6x + 2x + 2x$

$P_1 = 6(1.5) + 6(1.5) + 2(1.5) + 2(1.5)$

$9 + 9 + 3 + 3$

$P_1 = 24$

$P_2 = 2(3x+2) + 2(3x+1)$

$= 2(3(1.5)+2) + 2(3(1.5)+1)$

$= 2(4.5+2) + 2(4.5+1)$

$P_2 = 2(6.5) + 2(5.5)$

$P_2 = 13 + 11$

$P_2 = 24$

BEDMAS ↳ solve in brackets 1st!

Example 2:

Ted and Wayne are both travelling across British Columbia. Ted drives at an average speed of 90 km/h. Wayne left 30 minutes later and drove at an average speed of 100 km/h.

a) How long did it take for Wayne to catch up with Ted?

① Let t = time in min after Ted left.

② convert speed $\frac{km}{h}$ to $\frac{km}{min}$ by ÷ by 60 Ted $90 \div 60 = 1.5$ km/min or $\frac{90}{60}$

Wayne $100 \div 60 = 1.6\overline{6}$ km/min ↳ repeating decimal... better to leave as fraction $\frac{100}{60}$ km/min

③ Teds distance = $\frac{90}{60}t$ Waynes distance $\frac{100}{60}(t-30)$

④ want to find "t" when their distances are equal... so ⟹

Teds distance = Waynes distance

⑤ solve ⟹ $\left(\frac{90}{60}t = \frac{100}{60}(t-30)\right) \times 60$

$90t = 100(t-30)$

$90t = 100t - 3000$

$-100t \quad -100t$

$\frac{-10t}{-10} = \frac{-3000}{-10}$

$t = 300$ min. $\div 60 = 5$ hrs

b) How far have they driven to this point?

(substitute $t = 300$ min $= 5$ hrs)

Ted's distance = $\frac{90}{60}(300) = 450$ km

$\left[\frac{90 \cdot 300}{60} = 450\text{km}\right]$

Waynes distance = $\frac{100}{60}(300-30)$

$= \frac{100}{60}(270)$

$\frac{100 \cdot 270}{60} = 450$ km

if their distances are equal (as they should be) you've just checked your work ✓

Homework	Required	Extra Practice	Extension
Assignment #6.3 231 - 235 (Textbook HW)	#1, 2, 4ace, 5acef, 6ace, 7, 10abc, 11, 13, 14, 15, 16, 17, 18, 20, 26a	#3, 4bdf, 5bd, 6bd, 8, 9, 10d, 12, 19, 21, 22, 26b	23, 24, 25