Often in scientific investigations, we are interested in measuring how the value of some property changes as we vary something that affects it. We call the value that responds to the variation the dependent variable, while the other value is the independent variable.

For example, we might want to measure the extension of a spring as we attach different masses to it. In this case, the extension would be the dependent variable, and the mass would be the independent variable.

Notice that the amount of extension depends on the mass loaded and not the other way around. The variable "time" is nearly always independent.

The series of paired measurements collected during such an investigation is quantitative data. It is usually arranged in a data table. Tables of data should indicate the unit of measurement at the top of each column. The information in such a table becomes even more useful if it is presented in the form of a graph.

The independent data is plotted on the x-axis. A graph reveals many data points not listed in a data table.

Once a graph is drawn, it can be used to find a mathematical relationship (equation) that indicates how the variable quantities depend on each other.

The first step to determining the relationship is to calculate the slope "m" for the line of best fit.

First, the constant is determined by finding the change in y over the change in x (Δy/Δx or the "rise over the run"). Then substitution of the y and x variable names and the calculated value for m, including its units, into the general equation y = mx + b. The result will be an equation that describes the relationship represented by our data.

y = mx + b is the general form for the equation of a straight line relationship where:

m represents the slope m = \( \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} \)

In scientific relationships, the slope includes units and represents the constant that relates two variables.

For this reason, it is sometimes represented by a k.

The three most common types of graphic relationships are shown below in Figure 1.2.2.

**Figure 1.2.2 Three common types of graphic relationships**
**Sample Problem — Determination of a Relationship from Data**

Find the relationship for the graphed data below:

![Graph of Distance vs Time](image)

1. **What to Think about**
   - Determine the constant of proportionality (the slope) for the straight line. To do this, select two points on the line of best fit. These should be points whose values are easy to determine on both axes. *Do not use data points to determine the constant.*
   - Determine the change in $y$ ($\Delta y$) and the change in $x$ ($\Delta x$) including the units. The constant is $\Delta y / \Delta x$.

2. **How to Do It**
   - The relationship is determined by subbing in the variable names and the constant into the general equation, $y = Kx + b$.
   
   Often, a straight line graph passes through the origin, in which case, $y = Kx$.

   \[
   m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{20 m}{17 s} = 1.18 \frac{m}{s}
   \]

**Practice Problem — Determination of a Relationship from Data**

Examine the following graphs. What type of relationship does each represent?

(a) [Graph of Volume vs Length](image)

- Exponential: As $x$ increases, $y$ increases more.

(b) [Graph of Voltage vs Current](image)

- Inverse: As current $\uparrow$, voltage $\downarrow$

(c) [Graph of Distance vs Time](image)

- Direct (linear): As time increases, distance increases proportionally.
Activity: Graphing Relationships

Question
Can you produce a graph given a set of experimental data?

Background
A beaker full of water is placed on a hotplate and heated over a period of time. The temperature is recorded at regular intervals. The following data was collected.

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
</tr>
<tr>
<td>38</td>
<td>4</td>
</tr>
<tr>
<td>46</td>
<td>6</td>
</tr>
<tr>
<td>54</td>
<td>8</td>
</tr>
<tr>
<td>62</td>
<td>10</td>
</tr>
<tr>
<td>70</td>
<td>12</td>
</tr>
</tbody>
</table>

Procedure
1. Use the grid above to plot a graph of temperature against time. (Time goes on the x-axis.)

Results and Discussion
1. What type of relationship was studied during this investigation?

   \[ y = mx + b \]

   \[ y = (4 \, ^\circ C/\text{min})x + 22 \]

   \[ y \text{ is direct linear relationship} \]

2. What is the constant (be sure to include the units)?

   \[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{80 - 22}{12 - 0} = \frac{58}{12} = 4.833 \, ^\circ C/\text{min} \]

3. What temperature was reached at 5 minutes?

   \[ y = (4 \, ^\circ C)(5) + 22 = 42 ^\circ C \]

4. Use the graph to determine the relationship between temperature and time.

   Temperature increases with time. (proportional relationship)

5. How long would it take the temperature to reach 80°C?

   \[ 80 ^\circ C = (4 \, ^\circ C)(x) + 22 \]

   \[ \frac{80 - 22}{4} = x = 14.5 \, \text{min} \]

6. What does the y-intercept represent?

   the initial temp. of the water

7. Give a source of error that might cause your graph to vary from that expected.

   - Hotplate may not heat evenly - alter temperature
   - Temperature readings could be erroneous due to lack of precision in reading instruments.
Use the grid provided to plot graphs of mass against volume for a series of metal pieces with the given volumes. Plot all three graphs on the same set of axes with the independent variable (volume in this case) on the x-axis. Use a different colour for each graph.

<table>
<thead>
<tr>
<th>Volume (mL)</th>
<th>Copper (g)</th>
<th>Aluminum (g)</th>
<th>Platinum (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>17.4</td>
<td>5.4</td>
<td>42.9</td>
</tr>
<tr>
<td>8.0</td>
<td>71.7</td>
<td>21.6</td>
<td>171.6</td>
</tr>
<tr>
<td>12.0</td>
<td>107.5</td>
<td>32.4</td>
<td>257.4</td>
</tr>
<tr>
<td>15.0</td>
<td>134.4</td>
<td>40.5</td>
<td>321.8</td>
</tr>
<tr>
<td>19.0</td>
<td>170.2</td>
<td>51.3</td>
<td>407.6</td>
</tr>
</tbody>
</table>

Graph: Density of common metals

- Copper
- Aluminum
- Platinum

(a) Determine the constant for each metal.

\[ k = \frac{\text{mass}}{\text{volume}} = \frac{g}{\text{mL}} \]

\[ Cu = \frac{35.86}{4\text{mL}} = 8.95 \approx 9.09/\text{mL} \]

\[ Pt = \frac{236}{11\text{mL}} = 21.5\text{g/mL} \]

\[ Al = \frac{29.79}{11\text{mL}} = 2.70\text{g/mL} \]

(b) The constant represents each metal's density. Which metal is most dense?

Platinum is the most dense metal.
2. Two different liquids (water & acetic acid) were heated at a constant rate. The data is:

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Water Temperature (°C)</th>
<th>Acetic Acid Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20.0</td>
<td>20.0</td>
</tr>
<tr>
<td>1</td>
<td>21.6</td>
<td>23.1</td>
</tr>
<tr>
<td>2</td>
<td>23.0</td>
<td>26.1</td>
</tr>
<tr>
<td>3</td>
<td>24.5</td>
<td>29.0</td>
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<tr>
<td>4</td>
<td>25.8</td>
<td>33.0</td>
</tr>
<tr>
<td>5</td>
<td>27.3</td>
<td>35.8</td>
</tr>
<tr>
<td>6</td>
<td>29.0</td>
<td>38.8</td>
</tr>
<tr>
<td>7</td>
<td>30.6</td>
<td>41.1</td>
</tr>
<tr>
<td>8</td>
<td>32.0</td>
<td>44.0</td>
</tr>
<tr>
<td>9</td>
<td>33.5</td>
<td>47.2</td>
</tr>
<tr>
<td>10</td>
<td>34.9</td>
<td>49.9</td>
</tr>
</tbody>
</table>

![Graph showing temperature vs. time for water and acetic acid](image-url)