5 Solving for Angles

September 27, 2018 4:43 PM

Solving for Unknown ...

Solving for Angles

Inverse Functions

$$\theta = \cos^{-1}(x) \quad \Leftrightarrow \quad x = \cos(\theta)$$

$$\theta = \sin^{-1}(x) \quad \Leftrightarrow \quad x = \sin(\theta)$$

$$\theta = \tan^{-1}(x) \quad \Leftrightarrow \quad x = \tan(\theta)$$

For example, if we are doing the "inverse of sinθ"…we are trying to FIND the angle, when we are GIVEN both side lengths.

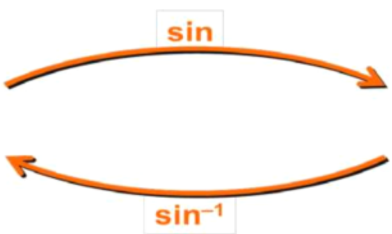
The inverse of sin

sin θ = 0.5, what is the value of θ?

To work this out use the **sin⁻¹** key on the calculator.

$\sin^{-1} 0.5 = \boxed{}$

$\sin^{-1}$ is the inverse of sin. It is sometimes called arcsin.



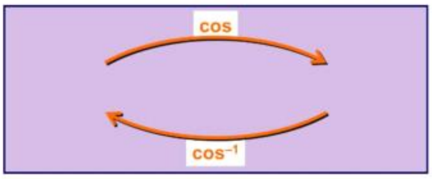
The inverse of cos

$\boxed{\text{Cos } \theta = 0.5\text{, what is the value of } \theta?}$

To work this out use the **cos^{-1}** key on the calculator.

$\cos^{-1} 0.5 = \boxed{}$

Cos^{-1} is the inverse of cos. It is sometimes called arccos.



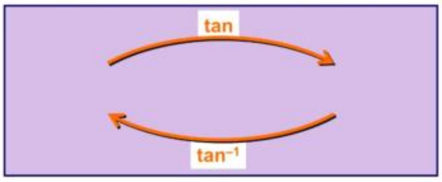
The inverse of tan

$\boxed{\tan \theta = 1\text{, what is the value of } \theta?}$

To work this out use the **tan^{-1}** key on the calculator.

$\tan^{-1} 1 = \boxed{}$

tan^{-1} is the inverse of tan. It is sometimes called arctan.



4 steps we need to follow:

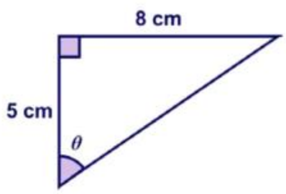
Step 1 Find ________________________________→out of Opposite, Adjacent and Hypotenuse.

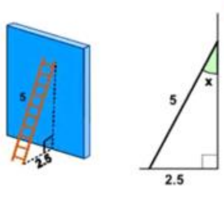
Step 2 Use __________________________ to decide which one of Sine, Cosine or Tangent ratio to use in this question.

Step 3 For ________calculate Opposite/Hypotenuse, for ______ calculate Adjacent/Hypotenuse or for _______calculate Opposite/Adjacent.

Step 4 ______________________________ from your **calculator**, using one of $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$ *(these are inverse, or 2nd function settings)*

<u>**Examples: Finding angles**</u>





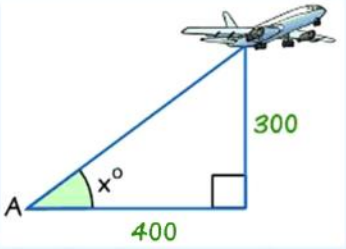
Example

The ladder leans against a wall as shown.

What is the **angle** between the ladder and the wall?

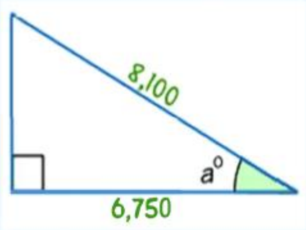
Example

Find the angle of elevation of the plane from point A on the ground.



Example

Find the size of angle a°

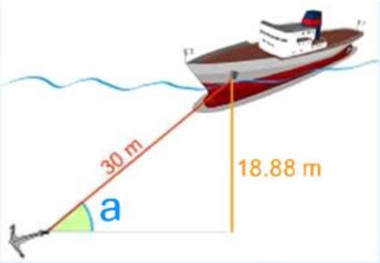


Example: Find the angle "a"

We know

- The distance down is 18.88 m.
- The cable's length is 30 m.

And we want to know the angle "a"



Finding Angles Using the Three Ratios

Recall:

Unless otherwise stated, calculate the measure of angles to the nearest tenth of a degree.

Eg. 42.8°

The three primary trig. ratios:

Tangent Ratio: $\tan\theta = \frac{\text{length of side opposite } \theta}{\text{length of side adjacent } \theta}$

Sine Ratio: $sin\theta = \frac{\text{length of side opposite } \theta}{\text{length of hypotenuse}}$

Cosine Ratio: $cos\theta = \frac{\text{length of side adjacent } \theta}{\text{length of hypotenuse}}$

The stored values in your calculator allow you to find angles using the ratios.

The magic of sin^{-1}, cos^{-1}, and tan^{-1}.

The "inverse trigonometric functions". These functions convert the stored ratios in your calculator to the angle.

Challenge

95. Find the measure of angle A in a right triangle if $tanA = 1.000$.

"angle"

$\angle A = tan^{-1}(1.000)$

$\angle A = 45°$

Challenge

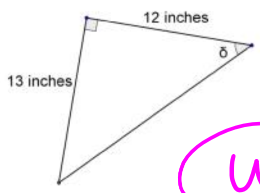
96. Find the measure angle B in a right triangle if $sinB = \frac{1}{2}$

$\angle B = sin^{-1}(1 \div 2)$

$\angle B: 30°$

Challenge

97. What ratio would you use to find the measure of the indicated angle?



Find the measure of the indicated angle.

Working up to Q # 127

Use the Inverse functions to find the indicated angle to the nearest tenth.

98. Find the measure angle A in a right triangle if tanA=1.000.

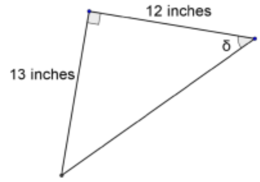
Use the $\tan^{-1}$ button.
Type:
$\tan^{-1}(1.000)=A$
$A = 45\,°$

99. Find the measure angle B in a right triangle if sinB=0.5000.

Use the $\sin^{-1}$ button.
Type:
$sin^{-1}(1 \div 2)=B$
$B = 30\,°$

100. What ratio would you use to find the measure of the indicated angle?

Use the tangent ratio: $tan\delta = \frac{13}{12}$



Find the measure of the indicated angle.

Type:
$\tan^{-1}(13 \div 12)= \delta$
$\delta = 47.3\,°$

101. Find angle A, if $sinA = 0.2654$.

102. Find angle B, if $cosB = \frac{5}{7}$.

103. Find angle Q, if $tanQ = \frac{15}{8}$.

104. Find angle T, if $sinT = \frac{15}{22}$

105. Find angle D, if $cosD = \frac{11}{10} = \frac{A}{H}$

the adj side cannot be longer than H

106. Find angle U, if $tanU = 2.6784$.

107. In a right triangle, one acute angle has sine ratio of 0.5. Find the sine ratio of the other acute angle.

108. In a right triangle, one acute angle has cosine ratio of $\frac{1}{\sqrt{2}}$. Find the sine ratio of the other acute angle.

109. In a right triangle, one acute angle has cosine ratio of $\frac{1}{2}$. Find the tangent ratio of the other acute angle.

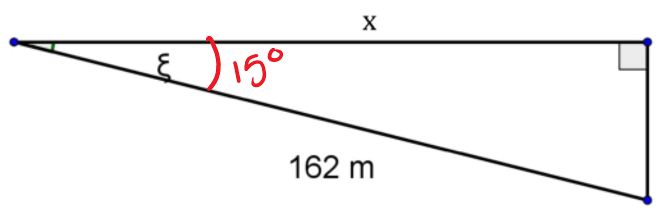
110. Which of the three trigonometric ratios (sine, cosine, tangent) can have a value greater than 1 ?

111. Draw a right triangle and use it to explain your answer to the previous question.

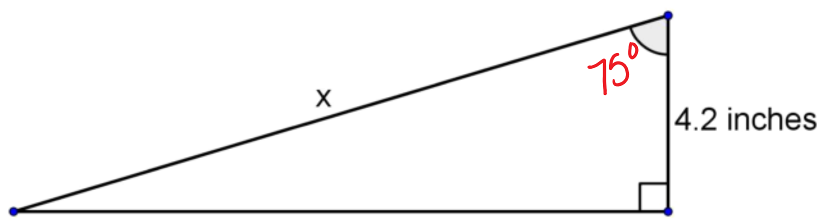
112. Draw a right triangle with an acute angle that has an adjacent side equal in length to the opposite side. Find the cosine ratio for that angle. (Round your answer to 3 decimals.)

113. Draw a right triangle with an acute angle that has a hypotenuse 50% longer than the adjacent side. Find the cosine ratio for that angle.

114. Use a protractor to measure the indicated angle. Then determine the length of side x using the cosine ratio.



115. Use a protractor to measure the indicated angle. Then determine the length of side x using the cosine ratio.

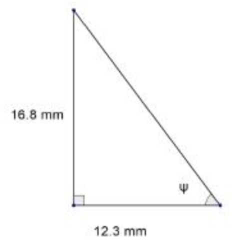


Working with the ratios to find angles.

Have a plan…

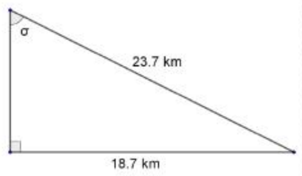
1. **Choose the correct ratio {sine, cosine, or tangent}.**
2. **Fill in the known side lengths into your chosen ratio.**
3. **Use the "inverse trig. function" to convert ratio → angle.**

116. What ratio do the given sides form for the indicated angle?



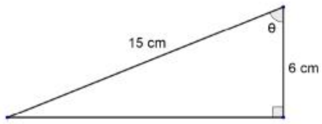
Sine Cosine Tangent

117. What ratio do the given sides form for the indicated angle?



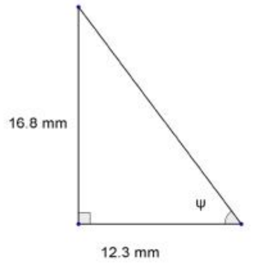
Sine Cosine Tangent

118. What ratio do the given sides form for the indicated angle?

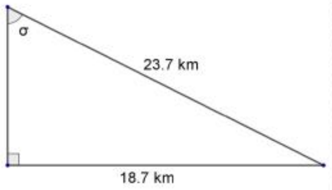


Sine Cosine Tangent

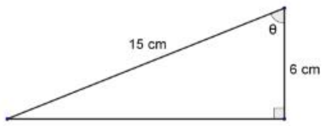
119. Calculate the measure of angle ψ to the nearest tenth of a degree.



120. Calculate the measure of angle σ to the nearest tenth of a degree.

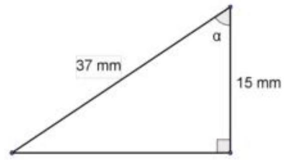


121. Calculate the measure of angle θ to the nearest tenth of a degree.



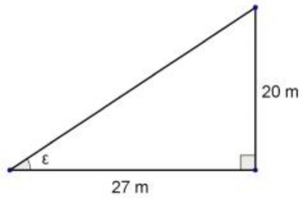
Working with the ratios to find angles.

122. What ratio do the given sides form for the indicated angle?



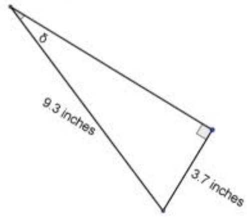
Sine Cosine Tangent

123. What ratio do the given sides form for the indicated angle?



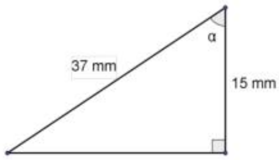
Sine Cosine Tangent

124. What ratio do the given sides form for the indicated angle?

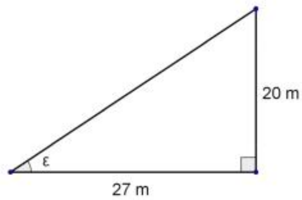


Sine Cosine Tangent

125. Calculate the measure of angle ∝ to the nearest tenth of a degree.



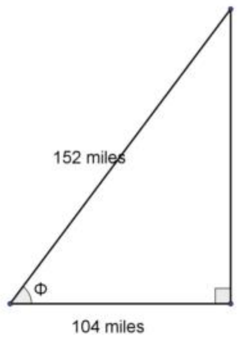
126. Calculate the measure of angle ε to the nearest tenth of a degree.



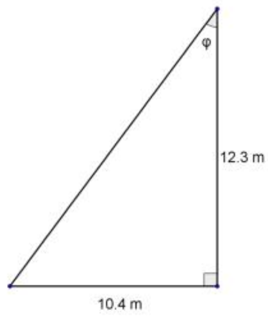
127. Calculate the measure of angle θ to the nearest tenth of a degree.

Find the measure of the indicated angle. Round answers to the nearest tenth of a degree.

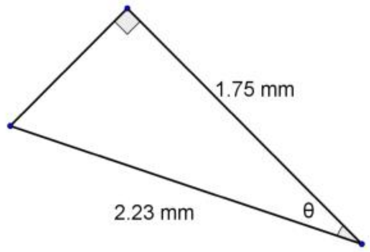
128.



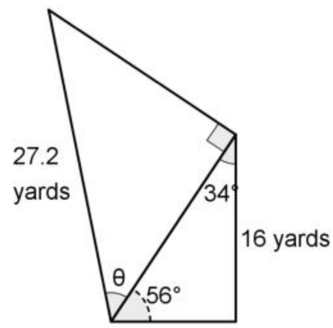
129.



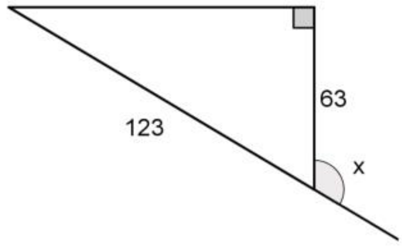
130.



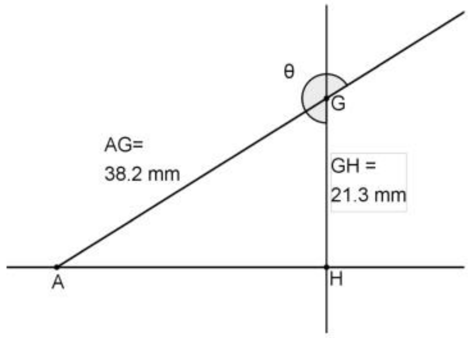
131.



132. Find the measure of angle x to the nearest tenth of a degree..

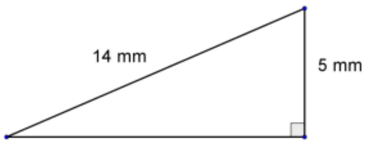


133. Find the measure of angle θ to the nearest degree.

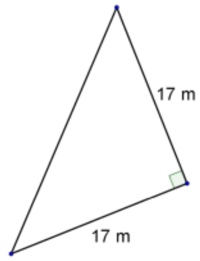


Solve the following triangles. Calculate answers to the nearest tenth.

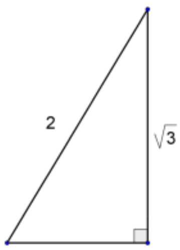
134.



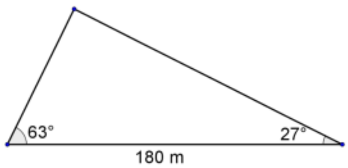
135.



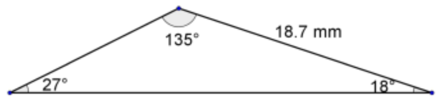
136.



137.

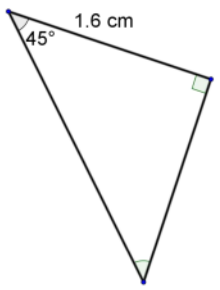


138.



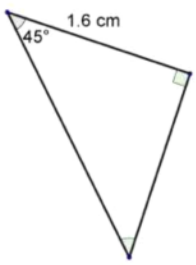
139. **Challenge.**

Find the Area of the following triangle to the nearest tenth of a square unit.



Find the area of the following triangles. Units for each question are indicated.

140. Nearest tenth.



$$\text{Area} = \frac{base \times height}{2}$$

Find base:

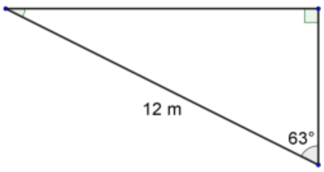
$$tan45 = \frac{opposite}{1.6}$$

$$\therefore base = 1.6\ cm$$

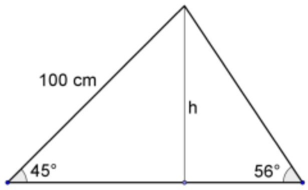
$$\text{Area} = \frac{1.6 \times 1.6}{2}$$

$$\text{Area} = 1.2cm^2$$

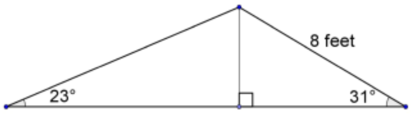
141. Nearest square metre.



142. Nearest hundred square centimetres.



143. Nearest square foot.



144. A triangle has side lengths of 8 cm, 7cm and 12 cm. Find the area of the triangle if the angle between the 8 cm and 12 cm side is 34°. Answer to the nearest square cm.

145. A triangle has side lengths of 10 km, 23 km and 32 km. The angle opposite the 10 km side is 9.2° Find the area of the triangle. Answer to the nearest square km.

Applications of trigonometry.

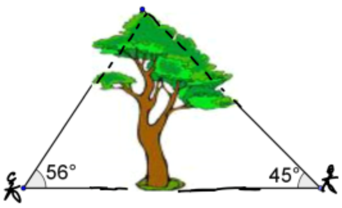
146. A kite stuck in a nearby tree. A child standing 25 m from the base of a tree pulls the string tight. If the tree is 30 m tall, approximately how far is the kite from the child to the nearest metre?

147. A surveyor measures the angle of elevation to the top of a building to be 23°. If the surveyor is 1345 feet from the base of the building, how tall is the building to the nearest foot?

148. From the top of a 20 m cliff above a road, the angle of depression to two approaching cars is 25° and 40° respectively. How far apart are the cars to the nearest metre?

149. Two hot air balloons float above the ocean at a height of 1000 feet. From a sailboat an observer measures the angle of elevation to one balloon is 60° and to the other balloon is 50°. [both balloons are on the same bearing from the observer] How far apart are the balloons to the nearest foot?

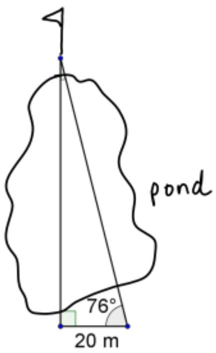
150. Two boys on opposite sides of the tree below measure the angle of elevation to the top of the tree. If the tree is 175 feet tall, how many feet apart are the boys?



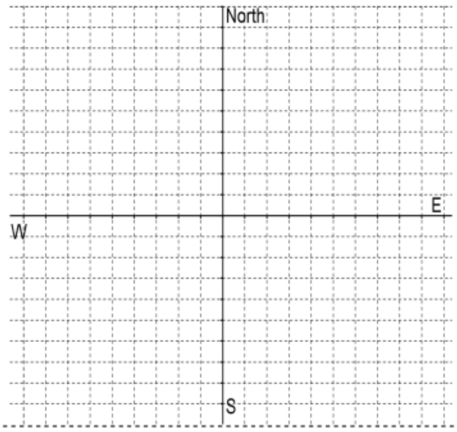
151. Highway sign shows that the road descends at a rate of 8%. Draw a diagram that shows what this means.

If a 3 km section of straight road descends at this grade, what is the drop in elevation?

152. While golfing with his father-in-law, Mr. J hits a shot short of a pond. The flag (hole) is directly across the pond from his ball. He paces 20 m to the right of his ball and measures the angle back to the hole to be 76°. How far is the ball from the hole to the nearest metre?

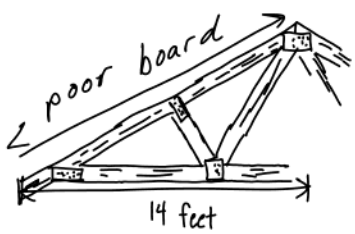


153. A hiker leaves base camp travelling due north at 5 km/h. After two hours, she turns and travels east. Three hours later, she sprains her ankle. At what bearing would a rescue team need to travel to reach the injured hiker? How far away is she from base camp? (nearest tenth)

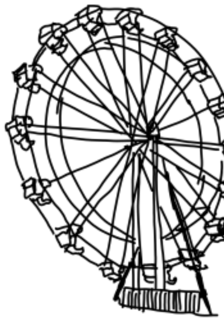


154. A student approaches a large Sequoia tree outside the entrance to the school and wonders how tall the tree is. He paces 150 metres from the base of the tree and measures the angle of elevation to the top of the tree to be 35°. Find the height of the tree to the nearest metre.

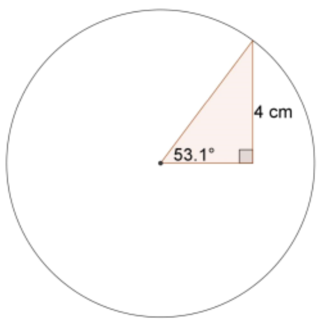
155. A homeowner wants to cut a new board to replace a decaying roof truss. He can measure the horizontal distance and the angle of inclination but needs to know how long to cut the board. The horizontal distance is 14 feet and the angle of inclination is 24°. Find the distance to the nearest tenth of a foot.



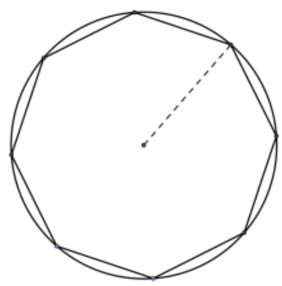
156. An engineer is constructing a Ferris wheel for a downtown park. There are 16 passenger carts and the radius of the wheel is 10 metres. How far apart are the passenger carts to the nearest hundredth of a metre?



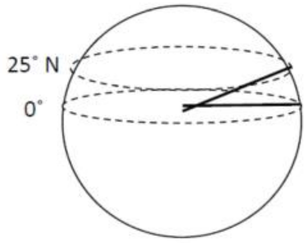
157. Find the area of the circle to the nearest square centimetre. $[A = \pi r^1]$



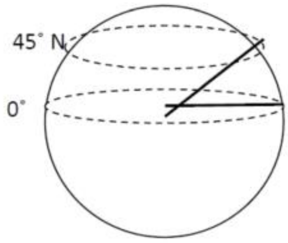
158. Find the perimeter of the octagon inscribed in a circle of radius 8 cm. (Nearest cm)



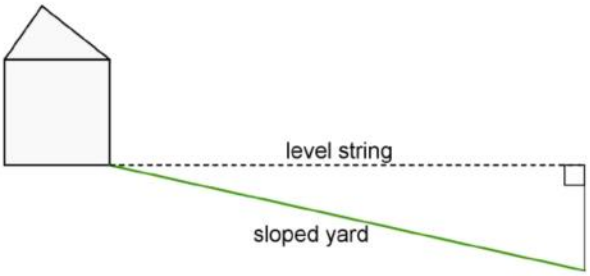
159. Find the length of the 25° line of latitude. The Earth's radius is 6380 km. Answer to the nearest km.



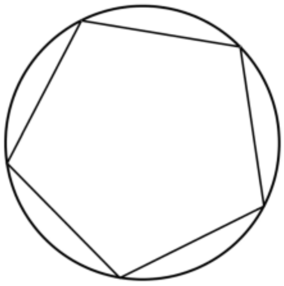
160. Find the length of the 45° line of latitude. The Earth's radius is 6380 km. Answer to the nearest km.



161. Mr. Teespré 's backyard slopes away from his house towards the beach. The instructions for his new lawnmower state that the mower should not be used if the slope is greater than 15°. Being a trigonometry specialist, he extends a level string 125 feet from the base of his house. From that point, he measures that the distance along the ground back to his house is 130 m. Is his yard too steep for this mower?

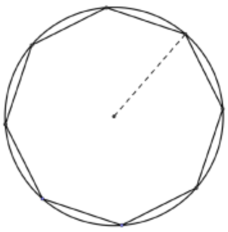


162. A regular pentagon is inscribed in a circle of radius 10 cm. Calculate the perimeter of the pentagon. Answer to the nearest cm.



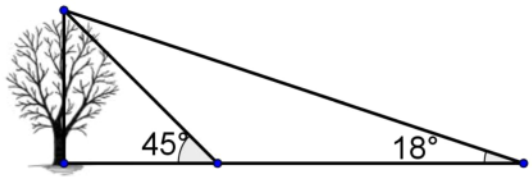
163. A regular decagon (10 sides) is inscribed inside a circle of radius 8 cm. Find the perimeter of the decagon. Answer to the nearest cm.

164. Find the area of the octagon inscribed in a circle of radius 8 cm. Answer to the nearest square cm.



165. A regular hexagon is inscribed in a circle with a radius 18 cm. What would be the side length of the hexagon? Answer to the nearest cm.

166. From a point 15 m from the base of a tree, a woman found the angle of inclination to the top of the tree to be 45°. Her sister found the angle to be 18° from a point farther away from the base of the tree. How far away are the two women away from each other?
(nearest tenth of a metre)

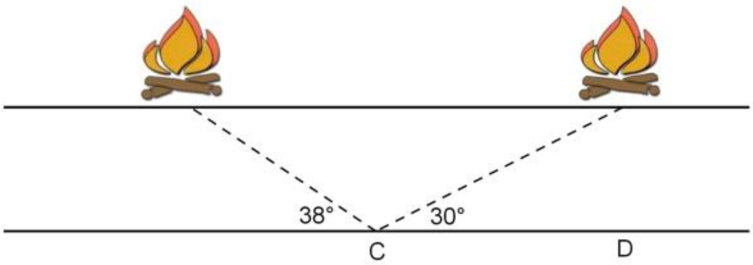


More word problems using right triangles:

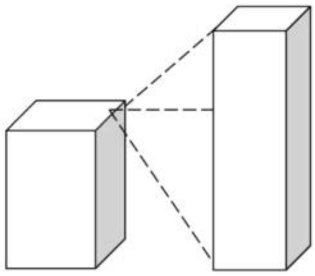
- Draw a diagram.
- Fill in known values.
- Let a variable represent unknown(s)
- Choose an appropriate strategy to solve for the unknown(s).
- Interpret the problem.

167. Solve the triangle given the following.
ΔXYZ
$x = 9\ cm$
$\angle Y = 90°$
$\angle Z = 36°$

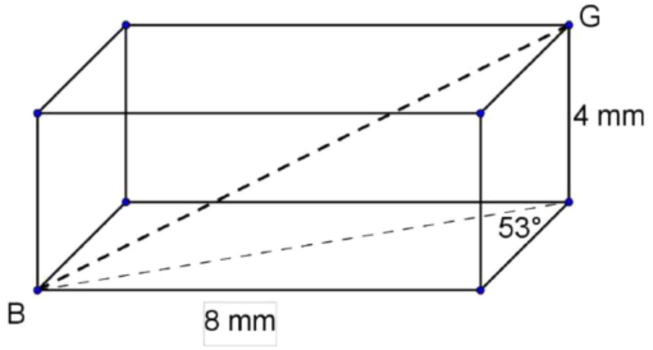
168. A firefighter is walking along the river at point C when she spots two fires on the opposite river bank. She measures the angles below and paces a distance of 300 m from point C to point D. Point D is directly across the river from one of the fires. How far apart are the fires to the nearest metre?



169. Anya stands on top of a building in downtown Victoria. From her position, the angle of elevation to the top of an adjacent building is 47°. The angle of depression to the base of the building is 62°. She is told that the buildings are 45 m apart. Based on this information, what is the height of the taller building to the nearest metre?



170. Find the length of diagonal BG in the rectangular prism. Answer to the nearest tenth of a millimeter.



171. The line of sight from an inflatable boat to the top of an oil derrick is 24 degrees. If the derrick is 45 m tall, how far is the boat from its base? (nearest tenth)

172. A pilot on a level path knows she should descend at an angle of 3 degrees to maintain comfort and safety. If she is flying at an altitude of 12 000 feet, how many miles from the runway should she begin her descent?

173. An aircraft ascends after takeoff at an angle of 22 degrees. What will be the altitude of the aircraft after it flies at that angle for 1200 m? (nearest metre)

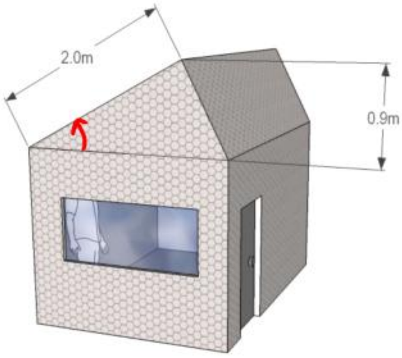
174. A hamster scurries up a ramp at a speed of 1.5 m/s. The ramp is inclined at an angle of 18 degrees. How many metres above the ground will the hamster be after 30 seconds?

175. Anya travels down a zip line at 25 km/h. The angle of descent of the zip line is 11 degrees. How many vertical metres has she fallen after 3 minutes?

176. The Earth's radius is 6380 km.
   A) Find the length of the 35° latitude to the nearest 10 km.

   B) What assumptions did you make?

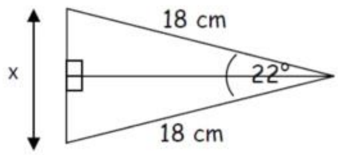
177. Find the angle of inclination at the back of the roof. The "rise" of the roof is 0.9 m. (nearest tenth)



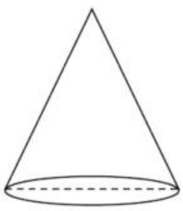
178. A ladder should make an angle of 72° with the ground for maximum safety. If the ladder is 4 m long, how far should it reach up the wall? (nearest tenth)

179. The angle of elevation to the top of a tree, measured on a 1.5 m transit from a distance of 30 m, is 15°. Find the height of the tree. (nearest tenth)

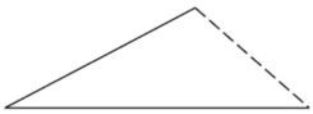
180. Find the value of 'x'.



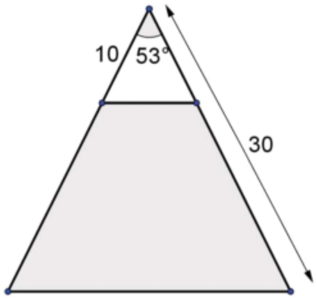
181. Mr. J has developed the ideal ice cream cone. The cone has a slant height of 13 cm and a diameter of 7.8 cm. Find the angle that the curved surface makes with the diameter.



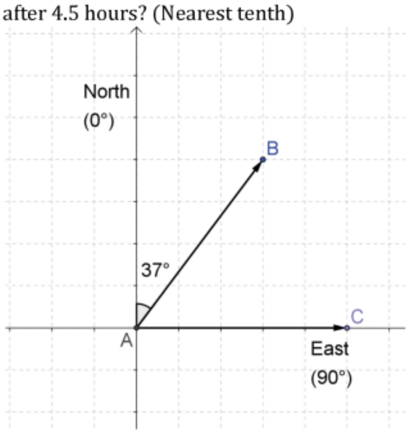
182. Mr. J continues to work on his isolated surf hut. Below is two-thirds of a roof truss he wants to complete. Find the length of wood he must cut (nearest tenth) to complete the truss. The long side is 8.2 m and the short side is 6.8 m. The angle between them is 35°.



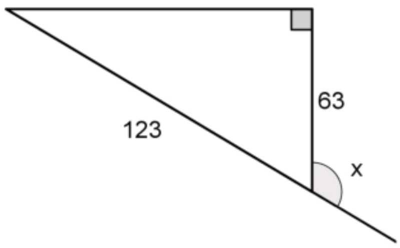
183. Both triangles (large and smaller inset) are isosceles. Find the area of the shaded trapezoid to the nearest tenth of a square unit.



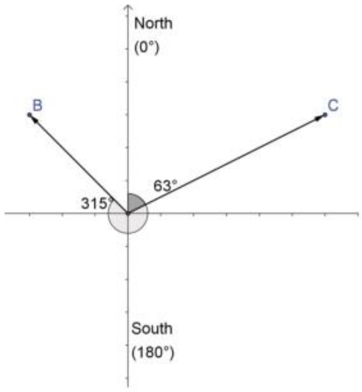
184. From a fire station in central BC, Georgia travels on a bearing of 37° at 6 km/h. Shelby leaves the station at the same time travelling due east at 5 km/h. How far apart are they after 4.5 hours? (Nearest tenth)



185. Find the measure of angle x to the nearest tenth of a degree.



186. At 9:00 am, a ship leaves port traveling at 30 km/h on a bearing of 63°. At the same time, another ship leaves port on a bearing of 315° at a speed of 19 km/h. When the boats stop after two hours, how far east is the boat at point C?



Draw an accurate diagram to answer each of the following questions.

187. In $\triangle QRS$, $\angle QSR = 90°$, $QR = 12\ cm$ and $QS = 10\ cm$. Find the measure of $\angle QRS$

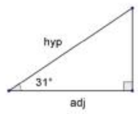
188. In $\triangle TUV$, $\angle TVU = 90°$, $TU = 115\ m$ and $TV = 99\ m$. Find the measure of $\angle UTV$

189. In $\triangle DEF$, $\angle DFE = 90°$, $DE = 12\ cm$ and $\angle DEF = 30°$. Find the length of FE.

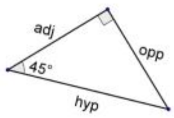
190. In $\triangle ABC$, $\angle ACB = 90°$, $BC = 5\ cm$ and $\angle ABC = 12°$. Find the length of AC.

Answers:

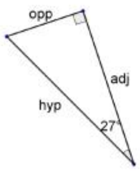
1. 25.0
2. 18.9
3. 24.0
4. 8.0
5. 84.0
6. 54.4
7.



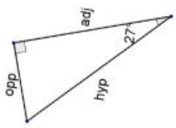
8.



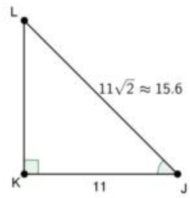
9.



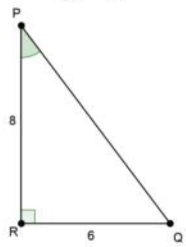
10.



11. $tan\theta = \frac{3}{4}$
 $sin\theta = \frac{3}{5}$
 $cos\theta = \frac{4}{5}$
12. $sin\theta = \frac{Î}{Û}$
 $cos\theta = \frac{4}{Û}$
 $tan\theta = \frac{3}{Ï}$
13. $sin\theta = \frac{À}{17}$
 $cos\theta = \frac{15}{17}$
 $tan\theta = \frac{À}{15}$
14. $sin\theta = \frac{1}{\sqrt{Γ}}$
 $cos\theta = \frac{1}{\sqrt{Γ}}$
 $tan\theta = 1$
15. $sin\theta = \frac{4}{5}$
16. $cos\theta = \frac{Î.4}{17.5}$
17. $tan\theta = \frac{5.Û}{16}$
18. Tangent
19. Tangent
20. Sine
21. 0.5000
22. 2.7475
23. 0.8192
24. 0.6691
25. 1.0000
26. 0.5000
27. A right triangle with an acute angle of 45° is an isosceles triangle with equal legs therefore $\frac{opp}{adj}$ will always equal 1, tangent 45 will always equal 1.
28. Sine is a ratio of opposite to hypotenuse. If the sine ratio is $\frac{1}{Ï}$ it means the hypotenuse is twice as long as the opposite side.
29. $x = 3$
30. $x = 24$
31. $x = 2.2$
32. $x = 25$
33. $x = 1.54$
34. $x = 15$
35. $x = 10\ cm$
36. $y = 5.3$
37. Answered on page.
38. Answered on page.
39. $261.8\ miles$
40. $w = 5.5\ feet$
41. $x = 17.0$
42. $t = 7.9\ cm$
43. $t = 6.4\ mm$
44. $q = 374.0\ km$
45. $w = 2.3\ cm$
46. $y = 3.3\ m$
47. $v = 13.6\ m$
48. $r = 47.1\ miles$
49. $d = 22.2\ cm$
50. $x = 56.7\ mm$
51. $z = 159.2\ mm$
52. $z = 67.2\ inches$
53. $12.7\ km$, $6.5\ km$
54. $x = 22.6\ mm$
55. $x = 7.6$
56. Opposite and adjacent
57. Not directly. The tangent ratio does not involve the hypotenuse.
58. Yes, the sine ratio involves the hypotenuse.
59. 16 cm and 12 cm
60. 3.6 m and 7.1 m
61. Not possible, the hypotenuse would need to be shorter than the adjacent side to have a cosine ratio greater than 1.
62. Not possible, like the answer above, a sine ratio cannot be greater than 1.
63. $\cos J = \frac{âæ}{âç} = \frac{11}{11\sqrt{Γ}} = \frac{1}{\sqrt{Γ}}$

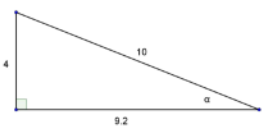


64. $tanP = \frac{èé}{êé} = \frac{Ï}{À}$

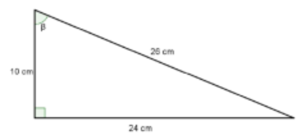


65. Answer will vary. But you will need to use the given side and angle to find another side length. Then choose to find another side or remaining angles.
66. AB = 11.9, AC = 11.3
67. 16.6 in, 20.8 in

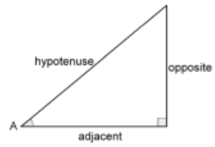
68. 84.3 m, 150.8 m
69. 4.7 m, 6.4 m, 8.8 m
70. 381 m
71. 4.5 km
72. 7 km (7057 m)
73. 153.2 m^2
74. 354 cm
75. 777 ft
76. 41.6 km
77.



78.

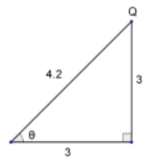


79. 11.7, 16.2, 36°
80. 6.3 *cm*, 13.8 *cm*, 27°
81. 38.2 mm
82. 10 cm^2
83. 53.4 m
84. 54.5 cm^2
85. 102 feet, assuming the ground is level and the string is straight.
86. 6.14 m
87. 22 in^2
88. 90 m
89. 78 yd
90. 69 cm^2
91. 7000 cm^2
92. 3 m
93. 40.2 mm
94. 24.9 cm^2
95. 45°
96. 30°
97. Tangent ratio, 47.3°
98. Answered on page.
99. Answered on page.
100. Answered on page.
101. 15.4°
102. 44.4°
103. 61.9°
104. 42.3°
105. No Solution
106. 69.5°
107. 0.8660 or $\frac{\sqrt{3}}{1}$
108. 0.7071 or $\frac{1}{\sqrt{1}}$
109. 0.5774 or $\frac{1}{\sqrt{3}}$
110. Tangent
111.

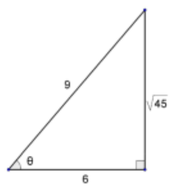


The side opposite to angle A can be greater than the side adjacent to angle A. As a ratio, $\frac{\text{ëì ì ëí îîð}}{\text{ñòđñôðõï}}$ would be greater than 1. The sine and cosine ratios can not produce values greater than 1 because the denominator in the ratio will always be larger than the numerator.

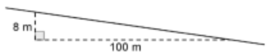
112. $cos\theta = 0.707$(side lengths of 3 were arbitrarily chosen)



113. $cos\theta = 0.6667$



114. 14°, 157 m
115. 73.5°, 14.8 in
116. Tangent
117. Sine
118. Cosine
119. 53.8°
120. 52.1°
121. 66.4°
122. Cosine
123. Tangent
124. Sine
125. 66.1°
126. 36.5°
127. 23.4°
128. 46.8°
129. 40.2°
130. 38.3°
131. 44.8°
132. 120.8°
133. 236.1°
134. 20.9°, 69.1°, 13.1 mm
135. 45°, 45°, 24.0 m
136. 30°, 60°, 1
137. 81.7 m, 160.4 m, 90°
138. 12.7 mm, 29.1 mm
139. 1.2 m^2
140. Answered on page.
141. 29 m^2
142. 4190 cm^2
143. 34 square feet
144. 27 cm^2
145. 59 km^2
146. 39 m
147. 571 ft
148. 19 m
149. 262 ft
150. 293 ft
151.



The units are simply an example. A descent of 8% means that the road "falls" 8 units for every 100 units of horizontal travel.
A 3 km section of road falls 0.24km.

152. 80 m
153. A rescue team would need to travel 18.0 km at 56.3°.
154. 105 m
155. 15.3 ft
156. 3.90 m
157. 78.5 cm^2
158. 49 cm
159. 36 331 km
160. 28 346 km
161. Yes. His yard slopes at an angle of 16°. Too steep for the mower.
162. 59 cm
163. 49 cm

164. 181 cm^2
165. 18 cm
166. 31.2 m
167. 6.5 cm, 11.1 cm, 54°
168. 522 m
169. 133 m
170. 10.8 mm
171. 101.1 m
172. 43 miles
173. 450 m
174. 14 m
175. 239 m
176. 32 840 km
177. 26.7°
178. 3.8 m
179. 9.5 m
180. 6.9 cm
181. 72.5°
182. 4.7 m
183. 319.6 square units
184. 22.4 km
185. 120.8°
186. 80 km
187. $\angle QRS = 56.4°$
188. $\angle UTV = 30.6°$
189. 10.4 cm
190. 1.1 cm