159. (-2, -4) and (0, 6)

$m = \frac{-4-6}{-2-0} = \frac{-10}{-2} = 5$

$y-(-4) = 5(x-(-2))$

$y+4 = 5(x+2)$

$y+4 = 5x+10$ $-y-4$

$-y-4$

$\therefore 5x - y + 6 = 0$

Explain your reasoning

① Find slope

② use 1 ordered pair (-2, -4) to put in Point Slope form

③ Convert to general form by expanding.

Write the equation of the line that passes through the following two points in general form.

160. (-5, -8) and (-7, -9)

$m = \frac{-9-(-8)}{-7-(-5)} = \frac{-1}{-2} = \frac{1}{2}$

$y-(-8) = \frac{1}{2}(x-(-5))$

$y+8 = \frac{1}{2}(x+5)$

$2x\left(y+8 = \frac{1}{2}x + \frac{5}{2}\right)$

$2y+16 = x+5$

$0 = x-2y-11$

161. (-1, -2) and (3,0)

$m = \frac{-2-0}{-1-3} = \frac{-2}{-4} = \frac{1}{2}$

$y-0 = \frac{1}{2}(x-3)$

$2x\left(y = \frac{1}{2}x - \frac{3}{2}\right)$

$2y = x-3$

$x-2y-3 = 0$

162. (0,4) and (5, 0)

$m = \frac{0-4}{5-0} = \frac{-4}{5}$

$y-0 = \frac{-4}{5}(x-5)$

$5x\left(y = \frac{-4}{5}x + 4\right)$

$5y = -4x+20$

$4x+5y-20 = 0$

class example

163. (8, -7) and (-6, -7)

$m = \frac{-7-(-7)}{-6-8} = \frac{0}{14} = 0$

$y-(-7) = 0(x-8)$

$\therefore y+7 = 0$

* 164. $\left(\frac{2}{3}, \frac{1}{4}\right)$ and $\left(\frac{1}{3}, \frac{1}{3}\right)$ *

$m = \frac{\frac{1}{3}-\frac{1}{4}}{\frac{1}{3}-\frac{2}{3}} = \frac{\frac{4}{12}-\frac{3}{12}}{-\frac{1}{3}} = \frac{\frac{1}{12}}{-\frac{1}{3}}$

$m = \frac{1}{12} \div \left(\frac{-1}{3}\right)$ ↑ reciprocal

$= \frac{1}{12} \times \frac{-3}{1} = \frac{-3}{12} = \frac{-1}{4}$

$y-\frac{1}{3} = \frac{-1}{4}(x-\frac{1}{3})$

$12x\left(y-\frac{1}{3} = -\frac{1}{4}x + \frac{1}{12}\right)$

$12y-\frac{12}{3} = -\frac{12}{4}x+1$

$12y-4 = -3x+1$

$3x+12y-5 = 0$

165. (0.3, 0.4) and (0.5, 0.7)

$m = \frac{0.7-0.4}{0.5-0.3} = \frac{0.3}{0.2} = \frac{\frac{3}{10}}{\frac{2}{10}}$

$m = \frac{3}{10} \div \left(\frac{2}{10}\right) = \frac{3}{10} \times \frac{10}{2} = \frac{30}{20} = \frac{3}{2}$

$y-\frac{4}{10} = \frac{3}{2}(x-\frac{3}{10})$

$\left(y-\frac{4}{10} = \frac{3}{2}x - \frac{9}{20}\right) \times 20$

$20y-\frac{80}{10} = \frac{60}{2}x-9$

$20y-8 = 30x-9$

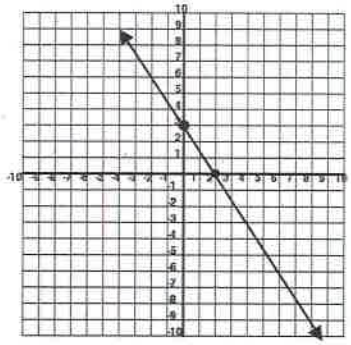
$30x-20y-1 = 0$

Working With Linear Equations:

- Be able to convert equations between general form and slope-intercept form.
- Be able to graph equations given to you in either form.
- Be able to make comparisons based on parallel and perpendicular lines.

Eg.1. Graph the line $3x + 2y - 6 = 0$.

Your Options:
1) use intercepts
2) make a table of values — ✗ time consuming
3) convert to slope-intercept form — ✓ useful; but more steps.



I chose **option 1** because this equation allows for easy calculations to find both intercepts.

$3(0) + 2y - 6 = 0 \quad 2y - 6 = 0 \quad 2y = 6 \quad y = 3$
The y-intercept is 3.

$3x + 2(0) - 6 = 0 \quad 3x - 6 = 0 \quad 3x = 6 \quad x = 2$
The x-intercept is 2.

Plot the two points & draw the line through them.

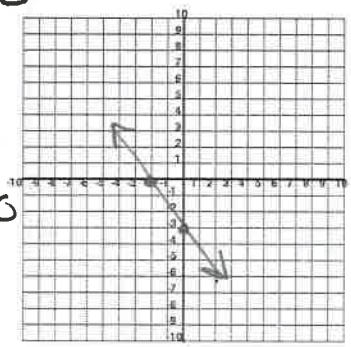
My second choice would have been option 3, conversion to slope-intercept form.
$3x + 2y - 6 = 0 \quad 2y = -3x + 6 \quad y = \frac{-3}{2}x + 3$

Plot the y-intercept then use the slope to plot another point, draw a line through the two points.

In general form $y\text{-int} = \frac{-C}{B}$

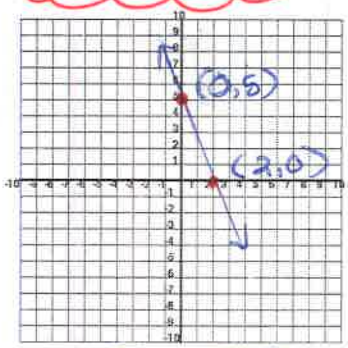
Graph the lines represented by each of the following equations. Use any method.

166. $3x + 2y + 6 = 0$



$3(0)+2y+6=0$
$\frac{2y}{2} = \frac{-6}{2}$
$y = -3$
$3x+2(0)+6=0$
$\frac{3x}{3} = \frac{-6}{3}$
$x = -2$

167. $5x + 2y - 10 = 0$

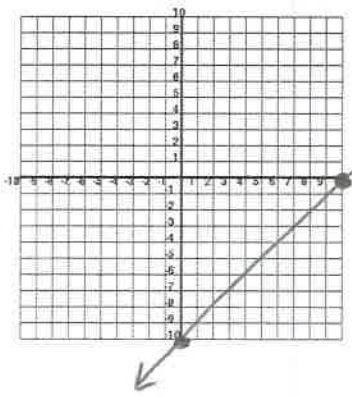


also $y\text{-int} = \frac{-C}{B}$
$\frac{-(-10)}{2} = \frac{10}{2} = 5$
$(0,5)$

$5(0)+2y-10=0$
$2y-10=0$
$+10 \quad +10$
$\frac{2y}{2} = \frac{10}{2} \quad y=5$

$5x+2(0)-10=0$
$5x-10=0$
$+10 \quad +10$
$\frac{5x}{5} = \frac{10}{5} \quad x=2$

168. $x - y = 10$



$(0)-y=10$
$\therefore y=-10$
$x-0=10$
$\therefore x=10$

Graph the lines represented by each of the following equations. Use any method.

169. $3x + 2y - 4 = 0$

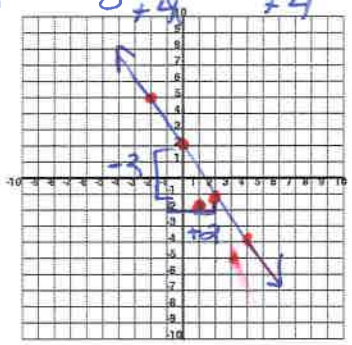
$-3x \quad -3x$

$2y - 4 = -3x$

$+4 \quad +4$

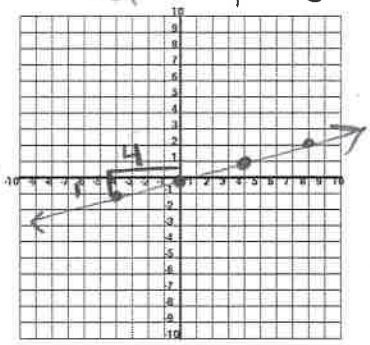
$\frac{2y}{2} = \frac{-3x+4}{2}$

$y = \frac{-3}{2}x + 2$

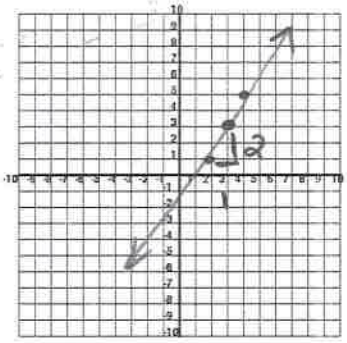


171. $x - 4y = 0$

$\frac{-4y}{-4} = \frac{-x}{-4} \therefore y = \frac{1}{4}x$



172. $2(x-3) = y - 3$ ↑ point-slope form



170. Explain your strategy:

isolate y to get in slope-intercept form. +2 is the y-int. Use $\frac{-3}{2}$ slope to plot other points

Explain your strategy:

y int = 0

$m = \frac{1}{4}$

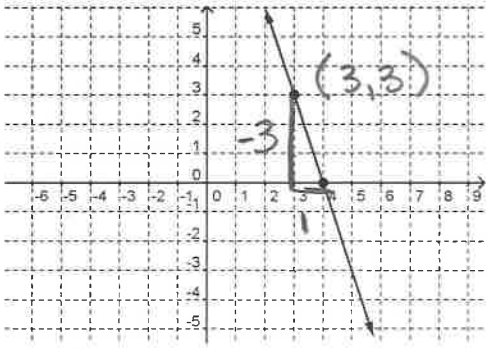
Explain your strategy:

$(x, y) = (3, 3)$

$m = 2$

Match the following graphs to their corresponding equations. Choose the best match.

173.



$m = \frac{-3}{1}$

a) $3y + 3 = 0$

b) $3x - y - 12 = 0$

c) $3x + y - 12 = 0$

d) None of the above

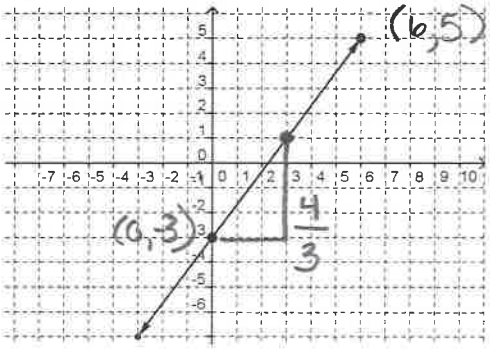
$y = -3x + 12$

$y - 3 = -3(x - 3)$

(x, y)

$(3, 3)$

174.



a) $4x - 3y + 9 = 0$

b) $3x - 4y + 9 = 0$

c) $3x + 4y - 9 = 0$

d) None of the above

$m = \frac{4}{3}$

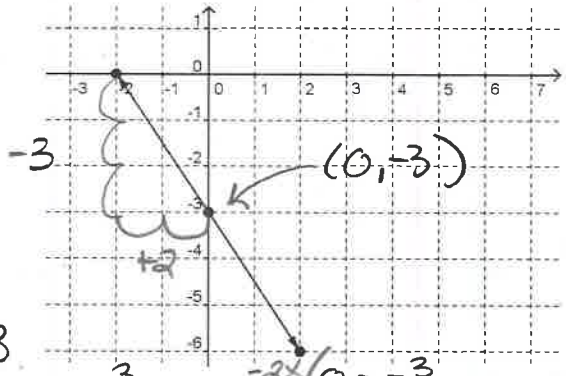
$\therefore y = \frac{4}{3}x - 3$

$-y \quad -y$

$\times 3 \quad 0 = \frac{4}{3}x - y - 3$

$0 = 4x - 3y - 9$

175. Which equation on the right represents the graph below?



a) $2x - 3y + 6 = 0$

b) $3x - 2y + 6 = 0$

c) $3x + 2y + 6 = 0$

d) None of the above

176. Which of the following equations represents the word statement "each element of the range is equal to one less than double an element in the domain."

a. $2x - y - 1 = 0$
b. $x - 2y = -1$
c. $2x + y + 1 = 0$

177. Which of the following equations represents the word statement "each element of the range is equal to two more than one third an element in the domain."

a. $3x - y = 6$
b. $x - 3y = -6$
c. $x + 3y + 6 = 0$

178. Which of the following equations represents the word statement "triple each element of the range is equal to one less than double an element in the domain."

a. $2x - 3y = -1$
b. $2x - 3y = 1$
c. $2x + 3y = 1$

179. Write a "word statement" to describe the following equation.

$y = 3x - 2$

range is equal to two less than three times the domain.

180. Write a "word statement" to describe the following equation.

$2x + 4y - 8 = 0$

range is 2 more than neg. ½ of domain.

181. Write a "word statement" to describe the following equation.

$3x - 5y = 20$

Range is 4 less than 3/4 an element of the domain

182. Which of the following equations represent the same line as $y = 3x - 2$?

Circle all that apply.

a. $3x = y + 2$
b. $3x - y - 2 = 0$
c. $y - 3x = -2$
d. none

183. Which of the following equations represent the same line as $5x - 2y + 10 = 0$?

Circle all that apply.

e. $y = \frac{5}{2}x + 5$
f. $\frac{2}{5}(x - 4) = y - 15$
g. $x = \frac{2}{5}y - 2$
h. none

184. Which of the following equations represent the same line as $y - 4 = 2(x + 1)$?

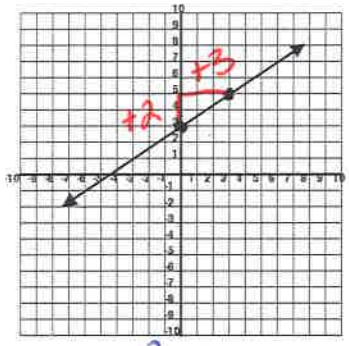
Circle all that apply.

i. $2x - y + 6 = 0$
j. $y = 2x + 6$
k. $2x + y = 6$
l. none

m b

Find the slope and y-intercept, write the equation in slope-intercept form, then in general form.

185.



$\frac{rise}{run} = \frac{2}{3}$

m $\frac{2}{3}$ b $+3$

slope-intercept form $y = \frac{2}{3}x + 3$

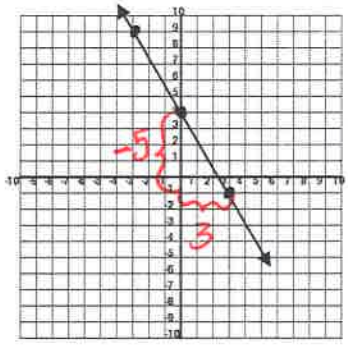
general form $2x - 3y + 9 = 0$

$3\left(y = \frac{2}{3}x + 3\right)$

$3y = 2x + 9$

$0 = 2x - 3y + 9$

186.



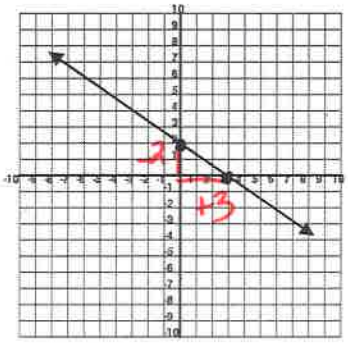
m $\frac{-5}{3}$ b 4

slope-intercept form $y = \frac{-5}{3}x + 4$ $\times 3$

$3y = -5x + 12$

general form $5x + 3y - 12 = 0$

187.



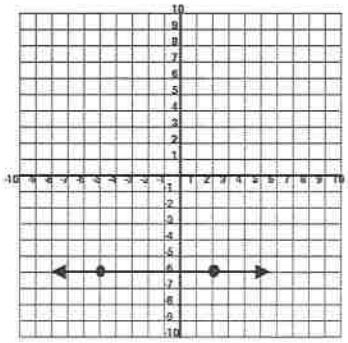
m $\frac{-2}{3}$ b 2

slope-intercept form $y = \frac{-2}{3}x + 2$ $\times 3$

$3y = -2x + 6$

general form $2x + 3y - 6 = 0$

188.



m 0 b -6

slope-intercept form $y = -6$

general form $y + 6 = 0$

Parallel and Perpendicular Lines

Recall:

- Parallel lines have equal slopes.
- Perpendicular lines have slopes that are negative reciprocals.

eg. $m = \frac{1}{3}$ ↑ $\perp$ $= \frac{-3}{1}$ ↓

For each line below, state the slope of a line that would be (a) parallel (b) perpendicular.

189. $y = 3x - 5$

a) // parallel $m = 3$

b) $\perp$ perpendicular $m = -\frac{1}{3}$

190. $y - 5 = -\frac{2}{3}x$

a) // $m = -\frac{2}{3}$

b) $\perp$ $m = \frac{3}{2}$

191. $5x - 3y = 14$

a) // $m = \frac{5}{3}$

b) $\perp$ $m = -\frac{3}{5}$

$\frac{-3y}{-3} = \frac{-5x + 14}{-3}$

$y = \frac{5}{3}x - \frac{14}{3}$

192. CHALLENGE.

Write the equation of the line parallel to $5x - 8y + 12 = 0$ and through the point (-2,3).

m = same

① $5x - 8y + 12 = 0$

$\frac{5x + 12}{8} = \frac{8y}{8}$

$y = \frac{5}{8}x + \frac{12}{8}$

now we know slope

② use same slope, but sub in different point (-2,3) for the "other line"

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{5}{8} = \frac{y - 3}{x - (-2)}$

$5(x+2) = 8(y-3)$ → Point-Slope Form

$5x + 10 = 8y - 24$

$5x - 8y + 10 + 24 = 0$

$5x - 8y + 34 = 0$ → General Form

$\frac{-8y}{-8} = \frac{-5x - 34}{-8}$

$y = \frac{5}{8}x + \frac{34}{8}$ → slope intercept

← Help with challenge #192.

SOLUTION to Q. 192.
Write the equation of the line parallel to $5x - 8y + 12 = 0$ and through the point (-2,3).

**Parallel means same slope. So we need to find slope of $5x - 8y + 12 = 0$.

$5x - 8y + 12 = 0$ $-8y = -5x - 12$ $y = \frac{5}{8}x + \frac{12}{8}$	Convert to slope intercept form. This gives us the slope. $m = \frac{5}{8}$
	Use the slope, $m = \frac{5}{8}$ and the point (-2,3) to write the equation.
$m = \frac{y_2 - y_1}{x_2 - x_1}$	Fill in what you know. $m = \frac{5}{8}$. Substitute point (-2,3)
$\frac{5}{8} = \frac{y-3}{x--2}$	Cross-Multiply.
$5(x + 2) = 8(y - 3)$ $5x + 10 = 8y - 24$	Simplify.
$5x - 8y + 34 = 0$	General Form
$y = \frac{5}{8}x + \frac{17}{4}$	Slope-Intercept Form

// = same slope.

193. Write the equation of the line parallel to $4x - 6y + 12 = 0$ and through the point (5,7).

Explain your reasoning

① Find slope (m)

② Use slope formula + cross multiply subing-in points (5,7)

③ Convert to General Form

④ Convert to Slope-Intercept form

① $4x - 6y + 12 = 0$

$\frac{4x + 12}{6} = \frac{6y}{6}$

$\frac{4}{6}x + 2 = y \therefore y = \frac{2}{3}x + 2$

$m = \frac{2}{3}$

② $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2}{3} = \frac{y-7}{x-5}$

$2(x-5) = 3(y-7)$ Point slope form

③ $2x - 10 = 3y - 21$

$2x - 3y + 11 = 0$ General Form

$-3y = -2x - 11$

$\therefore y = \frac{2}{3}x + \frac{11}{3}$ equation of the parallel line through the point (5,7)

Eg.2. Write the equation of the line perpendicular to $3x + 2y - 4 = 0$ and through the point (2,3).

Perpendicular means slopes are negative reciprocals.

Step 1: Find the slope of $3x + 2y - 4 = 0$.

$3x + 2y - 4 = 0$	Convert to slope-intercept form.
$2y = -3x + 4$	
$y = \frac{-3}{2}x - \frac{4}{2}$	This line has a slope, $m = \frac{-3}{2}$.

Negative reciprocal!

The perpendicular line will have a slope of $m = \frac{2}{3}$

Use: $m = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{2}{3} = \frac{y-3}{x-2}$	Fill in what you know. $m = \frac{2}{3}$. Substitute point (2,3)
$2(x-2) = 3(y-3)$	Cross-Multiply.
$2x - 4 = 3y - 9$	Simplify.
$2x - 3y + 5 = 0$	General Form
$y = \frac{2}{3}x + \frac{5}{3}$	Slope-Intercept Form

$Ax + By + C = 0$ $\therefore m = \frac{-A}{B}$

How "General Form" can be useful!!

194. Write the equation of the line perpendicular to $4x + 3y - 24 = 0$ (A B C) and through the point (1,4).

neg. reciprocal ↳ recall $m = \frac{-A}{B}$

$\therefore m = -\frac{4}{3}$

So a ⊥ slope ↳ $\therefore m = \frac{3}{4}$

$\frac{3}{4} = \frac{y-4}{x-1}$

$3(x-1) = 4(y-4)$

$3x - 3 = 4y - 16$

$3x - 4y + 13 = 0$

$\therefore \frac{-4y}{-4} = \frac{-3x - 13}{-4}$

$\therefore y = \frac{3}{4}x + \frac{13}{4}$

Eg.3. Write an equation for the line through C(2,4) that is perpendicular to the line through A(1,2) and B(4,8).

First find slope AB. $m = \frac{8-2}{4-1} = \frac{6}{3} = 2$ Therefore, the perpendicular line has slope, $m = \frac{-1}{2}$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Fill in what you know: $m = \frac{-1}{2}$. & substitute point (2,4)

$\frac{-1}{2} = \frac{y-4}{x-2}$ Cross-Multiply.

$-1(x-2) = 2(y-4)$ Simplify.

$-x + 2 = 2y - 8$

$x + 2y - 10 = 0$ General Form

$y = -\frac{1}{2}x + 5$ Slope-Intercept Form

Know which of these forms you are being asked to answer in. If it is not specified, you can choose.

Both describe the same line.

* Help ω #195-198

neg. reciprocal slope.

195. Write an equation for the line through C(1,2) that is perpendicular to the line through A(2,4) and B(5,5).

Explain your reasoning

① Find slope of line

② neg. reciprocal

③ use slope formula + cross multiply to get in point slope form.

④ write equation for the line. (general or slope intercept)

① $m = \frac{5-4}{5-2} = \frac{1}{3}$ ② $\therefore m = -\frac{3}{1}$

③ $\frac{-3}{1} = \frac{y-2}{x-1}$

$-3(x-1) = y-2$

$-3x+3 = y-2$

④ $\boxed{y = -3x+5}$

or

$3x + y - 5 = 0$ (General Form).

196. Write an equation for the line through Q(1,2) that is perpendicular to the line through R(-2,0) and S(3,5).

⊥ = neg. reciprocal.

$m = \frac{5-0}{3-(-2)} = \frac{5}{5} = 1$ $\therefore m = -1$

$\frac{-1}{1} = \frac{y-2}{x-1}$

$-1(x-1) = y-2$

$-x+1 = y-2$

$\boxed{\therefore y = -x+3}$

or $x + y - 3 = 0$

* recall: $Ax + By + C = 0$ (general form)
$m = -\frac{A}{B}$

Determine the equation of the following lines. Answer in general form.

197. The line parallel to $2x - 3y + 1 = 0$ and passing through the point (1, 2).

$m = \frac{-A}{B} = \frac{-2}{(-3)} = \frac{2}{3}$

$\frac{2}{3} = \frac{y-2}{x-1}$

$2(x-1) = 3(y-2)$

$2x - 2 = 3y - 6$

$\therefore 2x - 3y + 4 = 0$

198. The line perpendicular to $x - 5y + 2 = 0$ and passing through the point (-2, 5).

$m = \frac{-A}{B} = \frac{-1}{(-5)}$ $\left(\frac{1}{5}\right)$ ⊥ $= -\frac{5}{1} = -5$

$\frac{-5}{1} = \frac{y-5}{x-(-2)}$

$-5(x+2) = y - 5$

$-5x - 10 = y - 5$

$0 = 5x - y + 5$

199. The line perpendicular to $3x - 12y + 16 = 0$ and having the same y-intercept as $14x - 13y - 52 = 0$.

* when an equation is in general form:

$m = -\frac{A}{B}$ and y-int $= \frac{-C}{B}$

① ⊥ to $3x - 12y + 16 = 0$

$m = \frac{-3}{-12} = \frac{3}{12}$ ∴ ⊥ $m = \frac{-12}{3} = \frac{-4}{1}$

② same y-int as $14x - 13y - 52 = 0$

y-int $= \frac{-C}{B} = \frac{-(-52)}{-13} = \frac{52}{-13} = -4$

So... $m = \frac{-4}{1}$ and y-int $(0, -4)$

③ write equation for the line

$\frac{-4}{1} = \frac{y-(-4)}{x-0}$

$-4(x-0) = y + 4$

$-4x = y + 4$

$+4x \quad +4x$

$\therefore 4x + y + 4 = 0$

200. Two perpendicular lines intersect on the x-axis. An equation of one line is $y = 3x + 9$ Find the equation of the other line.

∴ x-int is the same

x-intercept:

$(0) = 3x + 9$

$-9 \quad -9$

$\frac{-9}{3} = \frac{3x}{3}$ $\therefore x = -3$ $(-3, 0)$

⊥ slope: $m = \frac{3}{1}$ $\therefore$ ⊥ $= -\frac{1}{3}$

$\frac{-1}{3} = \frac{y-0}{x-(-3)}$ * use the point.

$-1(x+3) = 3(y-0)$

$-x - 3 = 3y$

$0 = x + 3y + 3$

or

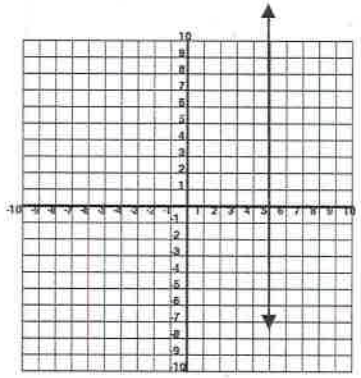
$\frac{-x-3}{3} = \frac{3y}{3}$

$y = -\frac{1}{3}x - 1$

Horizontal & Vertical Lines:

The equation of a horizontal line that is 3 units above the x-axis will be $y = 3$ or $y - 3 = 0$.
The equation of a horizontal line that is 12 units below the x-axis will be $y = -12$ or $y + 12 = 0$.

The equation of a vertical line 7 units to the right of the y-axis will be $x = 7$ or $x - 7 = 0$.
The equation of a vertical line 2 units to the left of the y-axis will be $x = -2$ or $x + 2 = 0$.

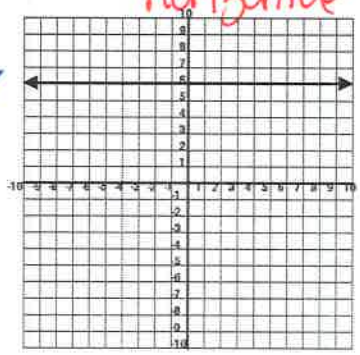


The equation of this line is $x = 5$.

Think to yourself...
Every point on this line has an x-coordinate of 5. It makes sense for the equation to be x=5.

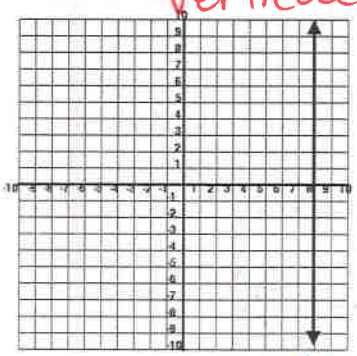
Write the equation of the following lines.

201. Horizontal



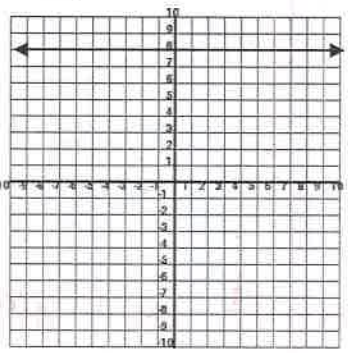
every point on this line has a y coordinate of $y=6$.
The x-coordinates will be different
$\therefore y=6$
or $y-6=0$.

202. Vertical



all $x=8$
y points will be different.
$\therefore x=8$
or $x-8=0$

203.



$y=8$
$y-8=0$