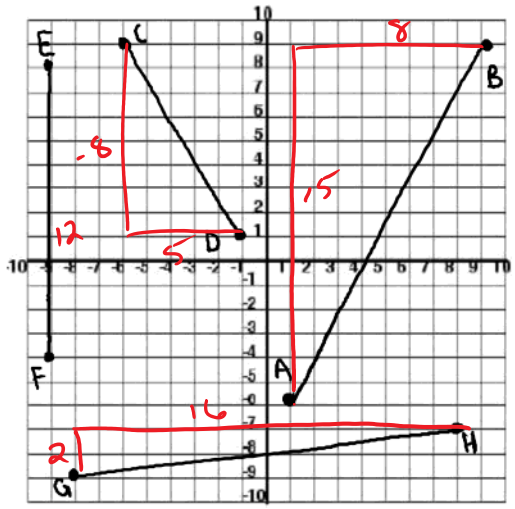
7 - Slope & Points on Lines

December 3, 2018 10:49 AM

7) slope & points on lines

Warm-Up:

1. Determine the slopes of the following lines:



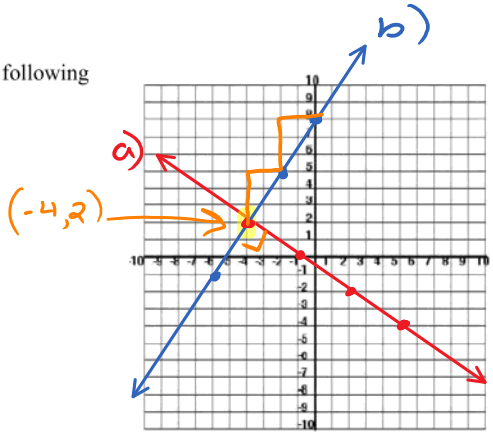
Line Segment	Slope
AB	$\frac{15}{8}$
CD	$\frac{-8}{5}$
EF	$\frac{12}{0}$ = undefined.
GH	$\frac{2}{16} = \frac{1}{8}$ (always simplify)

2. On the grid, draw a line through the point (-4, 2) with the following slope:

   a) Parallel to $-\frac{2}{3}$ $= \frac{-3}{3}$

   b) Perpendicular to $-\frac{2}{3}$ $= +\frac{3}{2}$

   negative reciprocal



3. Compute the slopes of the following line segments, using the coordinates provided:

   a) AB

   b) CD

A (1, 8)	C (2, 5)
B (7, 11)	D (6, 2)

A(1, 8) B(7, 11)
x_1, y_1 x_2, y_2

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 8}{7 - 1} = \frac{3}{6}$$

$= \frac{1}{2}$ ÷3 to simplify

$$m = \frac{2 - (5)}{6 - (2)} = \frac{-3}{4}$$

4. A line has a slope of $\frac{9}{7}$. It passes through (-1, -4) and (x, 5). Find the value of x.

solve for "x_2"

x_1, y_1 x_2, y_2

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{9}{7} = \frac{5-(-4)}{x_2-(-1)}$

$\frac{9}{7} = \frac{9}{x_2+1}$

$9(x_2+1) = 7 \times 9$

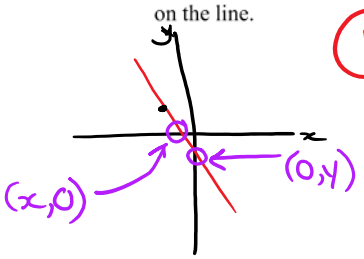
$9x_2 + 9 = 63$

$-9 \quad -9$

$\frac{9x}{9} = \frac{54}{9} \Rightarrow x_2 = 6$

5. The slope of a line is $-\frac{7}{5}$. The line passes through the point (-1, 3). Find the coordinates of another point on the line.

x_1, y_1 y-int: $(0, y)$ x_2, y_2



(1) $\frac{-7}{5} = \frac{y-3}{0-(-1)}$

$\frac{-7}{5} = \frac{y-3}{1}$

$-7 \cdot 1 = 5(y-3)$

$-7 = 5y - 15$

$+15 \quad +15$

$\frac{8}{5} = \frac{5y}{5}$

$y = \frac{8}{5}$ $(0, \frac{8}{5})$

$\frac{-7}{+5} = \frac{\Delta y}{\Delta x}$

$(-1+5, 3-7)$

$(4, -4)$

Part 1: Linear Relations

Linear Relations have a constant rate of change/slope.

Example #1: Determine whether the following relations are linear.

a)

x	y
4	10
6	16
8	22
10	28

+2, +2, +2 / +6, +6, +6

constant rate of change

Linear: (YES) or NO

Slope: $\frac{\Delta y}{\Delta x} = \frac{+6}{+2} = 3$

b)

x	y
5	8
10	7
15	6

+5, +5 / -1, -1

Linear: YES or NO

Slope: $\frac{\Delta y}{\Delta x} = \frac{-1}{5}$

c)

x	y
1	-9
2	-6
3	-1
4	6

+1 / +3, +5, +7

Δ by different amounts NOT constant.

Linear: YES or (NO)

Slope: n/a

slope: $m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Part 2: Finding the Slope from an Equation

To find the slope from an equation, always change the equation to be in the form $y = mx + b$.

Example #2: Find the slope of the following lines (without graphing).

a) $y = -4x + 7$

$m = -4 = \frac{-4}{1} = \frac{\text{rise}}{\text{run}}$

b) $3x - 2y = -18$

① convert to $y = mx + b$ (coeff. of $y = 1$)

$-3x \quad -3x$

$\frac{-2y}{-2} = \frac{-18 - 3x}{-2}$

$y = 9 + \frac{3}{2}x$

$y = \frac{3}{2}x + 9$

$m = \frac{3}{2}$

c) $y - 2 = \frac{1}{4}(x + 3)$

$\times 4 \quad \times 4$

$4y - 8 = 1(x + 3)$

$4y - 8 = x + 3$

$+8 \quad +8$

$\frac{4y}{4} = \frac{x + 11}{4}$

$y = \frac{1}{4}x + \frac{11}{4}$

$m = \frac{1}{4}$

Part 3: Determining if Points are on a Line

If a point falls on a line, it must 'satisfy' the equation of the line (ie: it must fit into the equation and remain equal)

Example #3: Do the following points fall on the line $2x + 5y = 20$?

(x, y) — if it still = 20 "true statement" (x, y) ARE on the line

a) (5, 2) — x y

$2x + 5y = 20$

$2(5) + 5(2) = 20$

$10 + 10 = 20$

$20 = 20$

✓True

b) (-5, 6)

$2(-5) + 5(6) = 20$

$-10 + 30 = 20$

$20 = 20$

✓True.

c) (4, 0)

$2(4) + 5(0) = 20$

$8 + 0 = 20$

$8 \neq 20$ not equal

✗ not on the line

d) (0, 4)

$2(0) + 5(4) = 20$

$0 + 20 = 20$

✓$20 = 20$

Example #4: Do the following points fall on the line $y = -x^2$? ← NOT Linear (x^2)

a) (2, -4)

$y = -(2)^2$

$-4 = -(4)$

$-4 = -4$ ✓

yes, on the line.

b) (-3, 9) — x y

$y = -x^2$

$9 = -(-3)^2$

$9 = -(9)$

$9 \neq -9$ NOT true

(-3, 9) is NOT on the line.

c) (-9, -81)

$y = -(-9)^2$

$-81 = -(81)$

$-81 = -81$ ✓

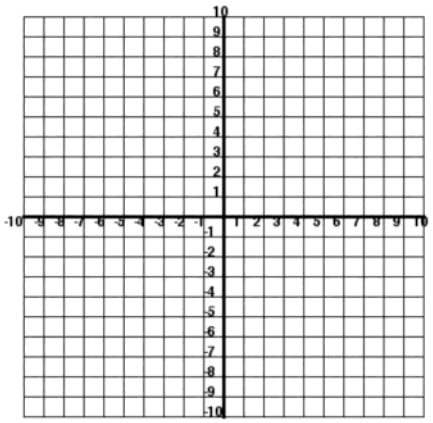
Homework

assignment # 7
pages #18-24-questions #64-97

Slope of a Line

Challenge #4:

60. Plot the following points:
$(-1, -5), (2, -4), (5, -3), (8, -2)$



61. Draw a line through the points you plotted.

62. Choose three sections of the line you just plotted and find their slopes.

Slope of section 1:

Slope of section 2:

Slope of section 3:

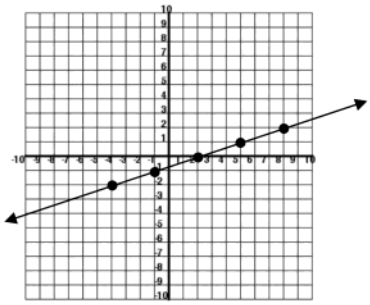
63. What do you notice?

Some notes here…

Slope of a Line

Recall from our discussion of line segments that slope can be calculated using: $m = \frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{rise}{run}$

For a straight line, the slopes of all segments on the line are equal. That is, if you find the slope of any two parts of the line, they will be equal.



Pick any three segments of the line and calculate the slope.

Slope will always be $\frac{1}{3}$.

The equations discussed earlier in this booklet result in lines that continue in two directions. Working with slope allows us to extend the line if we need to.

Remember:

- Parallel lines have equal slopes.
- Perpendicular lines have slopes that are negative reciprocals.

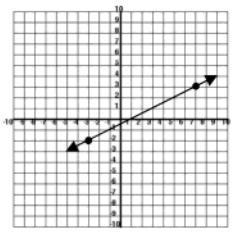
64. Find the slope of the line represented by the equation $y = 3x - 5$.

65. Find the slope of the line represented by the equation $2x + 5y = 20$.

66. Find the slope of the line represented by the equation $y - 4 = 3(x - 5)$.

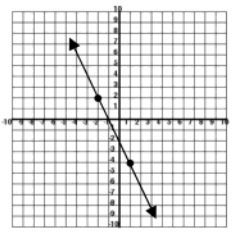
67. Find the slope of the line represented by the equation $\frac{1}{3}(x + 2) = y - 1$.

68. Find the slope of the line below.



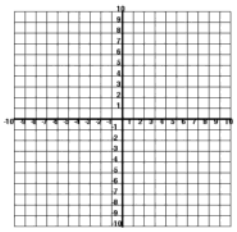
Slope is _____

69. Find the slope of the line below.

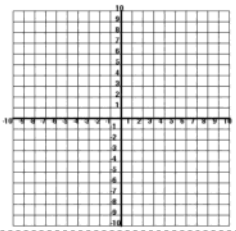


Slope is _____

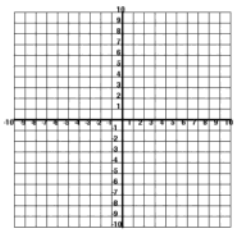
70. Draw a line through T(5,7) with slope $\frac{2}{5}$.



71. Draw a line through U(2, -2) parallel to the line in the previous question.



72. Draw a line through U(2, -2) perpendicular to the line in the question above.



73. If you were given a triangle with its vertices drawn as coordinates on an x-y coordinate plane, how could you determine if the triangle was a right triangle?

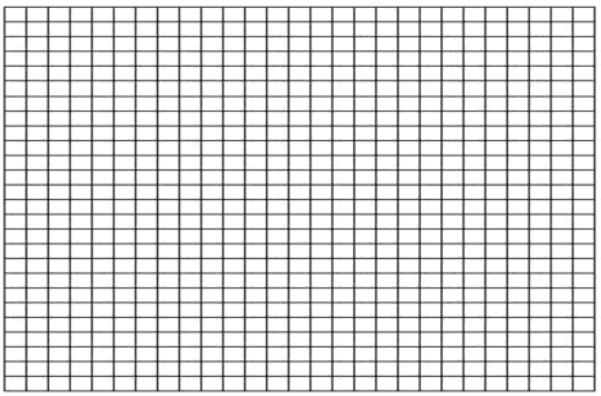
Do you know another way?

74. The slope of a line is $\frac{3}{2}$. If the line passes through point $B(5,2)$, find the coordinates of another point.

75. The slope of a line is -2.5. If the line passes through point $C(-1,2)$, find the coordinates of another point.

76. Julanya's internet provider charges a flat fee for the first 8 hr of access per month, plus an hourly rate for additional access. One month, 15 hr of usage cost her $25.88. The next month, 27 hr of access cost her $49.76.
a) Graph the data.

cost



Hours of usage

77. Find the hourly **rate** for access above 8 hr/month.

78. What word is synonymous with rate in this unit?

79. Find the flat fee for the first 8 hours. (Where will you find this value on the graph?)

Find the slope of the line passing through the points:

80. $(2,1)$ and $(6,6)$

81. $(-5,2)$ and $(4,2)$

82. $(-3,0)$ and $(3,-4)$

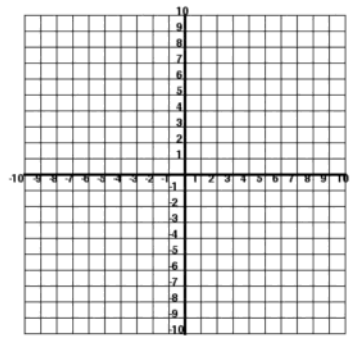
83. The slope of a line is -2. The line passes through (0,0) and (-3,y). Find the value of y.

84. A line has a slope of 1.5. It passes through (-2,1) and (x,7). Find the value of x.

85. **Challenge#5:** Show that (7, -1) is on the line $y = 2x - 15$

Algebraically:

Graphically:



The Equation of a Line

As you have seen, equations such as $2x + 3y = 12$ or $3y = x + 9$ or $y = \frac{5}{6}x - 4$ produce straight lines when graphed. They are **linear equations**.

Linear Equations may be written in several forms:

Slope-Intercept Form: $y = mx + b$	$y = 3x + 2$
Point-Slope Form: $y_2 - y_1 = m(x_2 - x_1)$	$(y - 2) = 3(x - 0)$
General Form: $Ax + By + C = 0$	$3x - y + 2 = 0$

Recall the ***Equation of a Line Property:***

The coordinates of every point on the line will satisfy the equation of the line.

Eg.1. Show that (7, -1) is on the line $y = 2x - 15$

$y = 2x - 15$	If (7, -1) is on the line, it will satisfy the equation.
$(-1) = 2(7) - 15$	Substitue the ordered pair into the equation.
$-1 = 14 - 15$	Does the left side = right side?
$-1 = -1$	Yes. The point IS on the line.

Do you recall the "text box" like this on page 10?

Determine if the following points lie on the line $y = 2x + 4$ (HINT: substitution!)

86. (-10, 24)

87. (5,14)

88. (-7,-10)

Determine if the following points lie on the line $3x - 2y + 6 = 0$

89. (10, 18)

90. (0,-3)

91. (-6,-6)

92. Determine if the point $(2,-3)$ is on the line $y = 3x - 9$.

$-3 = 3(2) - 9$

$-3 = 6 - 9$

$-3 = -3$

Explain why or why not:

Yes, it is on the line because when the coordinates 2, -3 are substituted into the equation, left side and right side are equal.

93. Determine if the point $(-1,-4)$ is on the line $3x - 2y - 11 = 0$.

Explain why or why not:

94. Determine if the point $(2,-3)$ is on the line $y + 1 = \frac{3x}{2}$.

Explain why or why not:

95. Determine if the set of ordered pairs represents a linear relation.

$(2,3), (3,4), (4,5), (5,6)$

Explain why or why not:

96. Determine if the set of ordered pairs represents a linear relation.

$(1,1), (1,2), (1,3), (1,4)$

Explain why or why not:

97. Determine if the set of ordered pairs represents a linear relation.

$(2,1), (3,0), (4,-1), (5,-2)$

Explain why or why not: