

9 - Equations of Lines in THREE FORMS

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9) equations of lines in three forms

The Three Forms of Writing Equations of Lines

1. Point-Slope Form:
 $y - y_1 = m(x - x_1)$
 ↑
 a point (coordinate) on the line (x_1, y_1) ← slope

2. Slope-Intercept Form:
 $y = mx + b$
 ↑ slope ↑ y-int

3. General Form
 $Ax + By + C = 0$ ← always $= \emptyset$
 a = ⊕
 must be Integers: • no fractions
 *order IS important:
 $-x + -y + * = 0$

Part I: Writing the Equation of a Line in General Form

Example #1: Write the following equations in general form.

a) $y = 4x - 10$
 $0 = 4x - 10 - y$
 $4x - y - 10 = 0$

b) $\frac{3}{4}y - 4 = 5x$
 $\frac{12}{4}y - 16 = 20x$
 $+3y - 16 = 20x$
 $-3y + 16 = -20x$
 $0 = 20x - 3y + 16$
 $20x - 3y + 16 = 0$

c) $1 = -\frac{2}{5}x + \frac{1}{2}y$
 LCM of 5 and 2 = 10
 $10 = -\frac{20}{5}x + \frac{10}{2}y$
 $+10 = -4x + 5y$
 $+4x - 5y + 10 = 0$
 $4x - 5y + 10 = 0$

Part 2: Writing the Equation of a Line in Three Forms

Example #2: Use the following slope and point on the line to write the equation of the line in all three forms.

When you have a slope and a point, ALWAYS come up with your equations in this order:

1. Slope-Point Form / Point-Slope Form
2. Slope-Intercept Form
3. General Form
4. Check: if you plug your point back into all three equations, does it work?

a) slope $m = 2$ Point $(4, 7)$
 x, y

① Point-Slope Form:

$$y - y_1 = m(x - x_1)$$

*substitute in values

$$y - (7) = (2)(x - (4))$$

$$y - 7 = 2(x - 4)$$

② Slope-Intercept Form:
 $(y = mx + b)$

solve for 'y' ... expand brackets + rearrange

$$y - 7 = 2(x - 4)$$

$$y - 7 = 2x - 8$$

$$y = 2x - 1$$

now we have y-int $(0, -1)$

③ General Form:
 $(Ax \pm By \pm C = 0)$

remove fraction + rearrange

$$y = 2x - 1$$

$$-y = -2x + 1$$

$$0 = 2x - y - 1$$

b) $m = -\frac{3}{4}$ $(12, 4)$

① Point-Slope Form

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{3}{4}(x - 12)$$

② Slope-Intercept Form:
 $(y = mx + b)$

*multiply to get rid of fraction

$$y - 4 = -\frac{3}{4}(x - 12)$$

*expand brackets

$$4y - 16 = -3(x - 12)$$

*solve for 'y'

$$4y - 16 = -3x + 36$$

$$4y = -3x + 52$$

$$y = -\frac{3}{4}x + 13$$

check your work is the slope correct? yes! $m = -\frac{3}{4}$

③ General Form:
 $Ax \pm By \pm C = 0$

*multiply out the fraction (LCM)

$$y = -\frac{3}{4}x + 13$$

$$+4y = -3x + 52$$

$$+3x - 52 = 0$$

rearrange

$$3x + 4y - 52 = 0$$

c) $m = \frac{1}{3}$ $(-6, 2)$

① Point-Slope Form

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{1}{3}(x - (-6))$$

$$y - 2 = \frac{1}{3}(x + 6)$$

② Slope-Intercept Form: $(y = mx + b)$

→ solve for "y"

$$y - 2 = \frac{1}{3}(x + 6)$$

$$y - 2 = \frac{1}{3}x + \left(-\frac{6}{3}\right)$$

$$y - 2 = \frac{1}{3}x - 2$$

$$y = \frac{1}{3}x + 0 \quad \leftarrow y\text{-int} = 0$$

$$y = \frac{1}{3}x$$

③ General Form:

$$y = \frac{1}{3}x$$

rearrange

$$3y = x$$

$$x - 3y = 0$$

d) $m = -\frac{2}{5}$ $(4, -3)$

① Point-Slope Form:

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = -\frac{2}{5}(x - 4)$$

$$y + 3 = -\frac{2}{5}(x - 4)$$

② Slope-Intercept Form: $(y = mx + b)$

$$y + 3 = -\frac{2}{5}(x - 4)$$

$$\times 5 \quad (y + 3 = -\frac{2}{5}x + \frac{8}{5}) \times 5$$

$$5y + 15 = -2x + 8$$

$$5y = -2x - 7$$

$$y = -\frac{2}{5}x - \frac{7}{5}$$

← y-int = $-\frac{7}{5}$

check with $m = -\frac{2}{5}$ ✓


③ General Form:

$$y = -\frac{2}{5}x - \frac{7}{5}$$

$$5y = -2x - 7$$

rearrange

$$2x + 5y + 7 = 0$$



Homework

assignment # 9
pages #30-35 questions #138-156

Quiz Thursday
Lessons #5-9

The Equation of a Line

The three forms

Slope-Intercept Form	Point-Slope Form	General Form
$y = mx + b$	$y - y_1 = m(x - x_1)$	$Ax + By + C = 0$
m is the slope b is the y -intercept	Derived from $m = \frac{y_2 - y_1}{x_2 - x_1}$ Cross multiply to get point-	A must be positive. A, B, C are integers.

The Equation of a Line

The three forms

Slope-Intercept Form	Point-Slope Form	General Form
$y = mx + b$	$y - y_1 = m(x - x_1)$	$Ax + By + C = 0$
m is the slope b is the y -intercept	Derived from $m = \frac{y_2 - y_1}{x_2 - x_1}$ Cross multiply to get point-slope form. Need one point and slope	A must be positive. A, B, C are integers.

Write in general form.

138. $y = 3x - 5$	139. $y - 5 = x + 7$	140. $5 - 2x = -4y + 2$
141. $-\frac{1}{3}x - 4y = 2$	142. $y - 5 = \frac{2}{3}x + 7$	143. $5 = \frac{2}{3}y + \frac{3}{4}x$

144. Challenge #6

Write the equation of the line that passes through $A(2,5)$ and has slope 3. Express your answer in general form and in slope intercept form.

The Equation of a Line

IMPORTANT!!! There is only one line that passes through a given point with a given slope.

Given the slope and a point:

Eg.1. A line passes through A(2,5) and has slope 3. Write the equation of the line.

Use the slope formula :

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Cross-Multiply. This creates the Point-Slope form of an equation.}$$

$$m(x_2 - x_1) = y_2 - y_1 \quad \text{Fill in what you know. } m = 3. \text{ Substitute the given point in for } x_1 \text{ and } y_1.$$

$$3(x - 2) = (y - 5) \quad \text{This is our equation in **point-slope form**.}$$

We no longer need the subscripts on x and y

$$3x - 6 = y - 5 \quad \text{Expanded.}$$

$$3x - y - 1 = 0 \quad \text{Collecting the terms to the left side is called writing the equation in **general form**.}$$

Or

$$y = 3x - 1 \quad \text{Isolate for 'y' to get the equation in **slope-intercept form**.}$$

Write the equation of the line that passes through the given point and has the given slope.
Express the equation in a) point-slope form b) general form c) slope-intercept form.

145. $(-2, 3), -2$

$$y - 3 = -2(x - (-2))$$

146. $(-5, 2), 2$

$$y - 2 = 2(x + 5)$$

point-slope

$$y - 2 = 2x + 10$$

$$y = 2x + 12$$

slope-intercept

$$2x - y + 12 = 0$$

general

$$a) y - 3 = -2(x + 2)$$

a)

a)

$$2(x+2)$$

$$\text{b) } 2x+y+1=0$$

$$\text{c) } y = -2x - 1$$

$$148. (-3, 4), -\frac{1}{3}$$

$$149. (2, 4), \frac{1}{2}$$

$$150. (0, 7), -1$$

a)

a)

a)

b)

b)

b)

c)

c)

c)

Write the equation of the line that passes through the given point and has the given slope.
Express the equation in a) point-slope form b) slope-intercept form c) general form.

151. $(3, -6), m = -3$

*Start with Point-**Slope formula:*

$$y_2 - y_1 = m(x_2 - x_1)$$

$$y - -6 = -3(x -$$

$$3)$$

$$y + 6 = -3(x -$$

$$3)$$

$$y + 6 = -3x + 9$$

$$y = -3x + 3$$

$$3x + y - 3 = 0$$

a) $y + 6 = -3(x - 3)$

b) $y = -3x + 3$

c) $3x + y - 3 =$

152. $(4, 6), m = 5$

153. $(-2, -1), m = \frac{1}{2}$

a)

b)

c)

a)

b)

c)

O

154. $(5, -6), m = -\frac{3}{4}$

155. $(\frac{1}{2}, 6), m = \frac{4}{3}$

156. $(-2, 1), m = 1.5$

a)

a)

a)

b)

b)

b)

c)

c)

c)