9) equations of lines in three forms

The Three Forms of Writing Equations of Lines

1. Point-Slope Form:
   \[ y - y_1 = m(x - x_1) \]
   - a point (coordinate) on the line \((x_1, y_1)\)

2. Slope-Intercept Form:
   \[ y = mx + b \]
   - \(m\) = slope
   - \(b\) = \(y\)-intercept

3. General Form
   \[ Ax + By + C = 0 \]
   - \(A\), \(B\), \(C\) must be integers; no fractions

*order is important:
- \(-x + y + \frac{1}{2} = 0\)

Part 1: Writing the Equation of a Line in General Form

Example #1: Write the following equations in general form.

a) \(y = 4x - 10\)

   \[ 4x - y - 10 = 0 \]

b) \(\frac{12}{y} - 4 = 5x\)

   \[ 12 - 4y = 5x\]
   \[ 12 - 4y - 5x = 0 \]

   \[ \text{L.C.M.} \quad \frac{20}{5} + \frac{5}{1} = 10 \]

   \[ 20x - 3y + 16 = 0 \]

Example #2:
   \[ 4x - 5y + 10 = 0 \]
Part 2: Writing the Equation of a Line in Three Forms

Example #2: Use the following slope and point on the line to write the equation of the line in all three forms.

When you have a slope and a point, ALWAYS come up with your equations in this order:
1. Slope-Point Form
2. Slope-Intercept Form
3. General Form
4. Check: if you plug your point back into all three equations, does it work?

a) \( m = 2 \), \( P(4, 7) \)

1. Point-Slope Form:
   \[ y - y_1 = m (x - x_1) \]
   \[ y - 7 = 2(x - 4) \]

2. Slope-Intercept Form:
   \( y = mx + b \)
   \[ y - 7 = 2(x - 4) \]
   \[ y - 7 = 2x - 8 \]
   \[ y = 2x - 1 \]

3. General Form:
   \( Ax + By + C = 0 \)
   \[ y = 2x - 1 \]
   \[ -2x + y = -1 \]

b) \( m = -\frac{3}{4} \), \( P(2, 4) \)

1. Point-Slope Form
   \[ y - y_1 = m (x - x_1) \]
   \[ y - 4 = -\frac{3}{4} (x - 2) \]

2. Slope-Intercept Form
   \( y = mx + b \)
   \[ 4 - 4 = -\frac{3}{4} (x - 2) \]
   \[ 0 = -\frac{3}{4} x + \frac{3}{2} \]
   \[ 0 = -\frac{3}{4} x + 1.5 \]
   \[ \frac{3}{4} y = \frac{3}{4} x + 1.5 \]
   \[ y = \frac{3}{4} x + 1.5 \]

3. General Form
   \( Ax + By + C = 0 \)
   \[ 3x + 4y - 5.2 = 0 \]

Check: if you plug your point back into all three equations, does it work?
### Point-Slope Form

\[ y - y_1 = m(x - x_1) \]

- **c)** \( m = \frac{1}{3} \) \( x_1, y_1 \) (6, 3)
  
  \[ y - 2 = -\frac{1}{3}(x + 6) \]

### Slope-Intercept Form

- **a)** \( y = -\frac{2}{3}x + 3 \)

### General Form

- **b)** \( 2x + 5y + 7 = 0 \)

### Assignment #9

- Pages #30-35 questions #138-156

### Quiz Thursday

- Lessons #5-9

---

**The Equation of a Line**

<table>
<thead>
<tr>
<th>Form</th>
<th>( y = mx + b )</th>
<th>( y - y_1 = m(x - x_1) )</th>
<th>( Ax + By + C = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Slope-Intercept Form</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Point-Slope Form</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>General Form</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **m** is the slope
- **b** is the y-intercept
- Derived from \( m = \frac{y_2 - y_1}{x_2 - x_1} \)
- Cross multiply to get point
- A must be positive.
- \( A, B, C \) are integers.
### The Equation of a Line

The three forms

<table>
<thead>
<tr>
<th>Slope Intercept Form</th>
<th>Point-Slope Form</th>
<th>General Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = mx + b )</td>
<td>( y - y_1 = m(x - x_1) )</td>
<td>( Ax + By + C = 0 )</td>
</tr>
</tbody>
</table>

- \( m \) is the slope
- \( b \) is the \( y \)-intercept
- Derived from \( m = \frac{y_2-y_1}{x_2-x_1} \)
- Cross multiply to get point-slope form.
- Need one point and slope

#### Write in general form.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>138. ( y = 3x - 5 )</td>
<td>139. ( y - 5 = x + 7 )</td>
<td>140. ( 5 - 2x = -9y + 2 )</td>
</tr>
<tr>
<td>141. ( -\frac{2}{3}x - 4y = 2 )</td>
<td>142. ( y - 5 = \frac{3}{7}x + 7 )</td>
<td>143. ( 5 = \frac{2}{7}y + \frac{2}{3}x )</td>
</tr>
</tbody>
</table>

#### Challenge #6

Write the equation of the line that passes through \( A(2, 5) \) and has slope 3. Express your answer in general form and in slope intercept form.
The Equation of a Line

IMPORTANT!!! There is only one line that passes through a given point with a given slope.

**Given the slope and a point:**

Eq. 1. A line passes through A(2,5) and has slope 3. Write the equation of the line.

Use the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$  
Cross-Multiply. This creates the **Point-Slope form** of an equation.

$$m(x_2 - x_1) = y_2 - y_1$$  
Fill in what you know. $m = 3$. Substitute the given point in for $x_1$ and $y_1$.

$$3(x - 2) = (y - 5)$$  
This is our equation in **point-slope form**.

We no longer need the subscripts on $x$ and $y$

$$3x - 6 = y - 5$$  
Expanded.

$$3x - y = 1$$  
Collecting the terms to the left side is called writing the equation in **general form**.

**Or**

$$y = 3x - 1$$  
Isolate for $y$ to get the equation in **slope-intercept form**.
Write the equation of the line that passes through the given point and has the given slope. Express the equation in a) point-slope form b) general form c) slope-intercept form.

<table>
<thead>
<tr>
<th>145. (2,3), -2</th>
<th>146. (-5,2),2</th>
<th>147. (-5,-1), -2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y - 3 = -2(x - 2) )</td>
<td>( y - 3 = 2(x + 2) ) <strong>point-slope</strong></td>
<td>( y - 3 = -2x - 4 ) <strong>slope-intercept</strong></td>
</tr>
<tr>
<td>( y = -2x + 1 ) <strong>slope-intercept</strong></td>
<td>( 2x + y + 4 = 0 ) <strong>general</strong></td>
<td>( y - 3 = -2x - 4 ) <strong>general</strong></td>
</tr>
</tbody>
</table>
Write the equation of the line that passes through the given point and has the given slope.
Express the equation in a) point-slope form b) slope-intercept form c) general form.
151. \((3, -6), m = -3\)

Start with Point-Slope formula:

\[ y_2 - y_1 = m(x_2 - x_1) \]

\[ y - (-6) = -3(x - 3) \]

\[ y + 6 = -3(x - 3) \]

\[ y + 6 = -3x + 9 \]

\[ y = -3x + 3 \]

\[ 3x + y - 3 = 0 \]

\[ a) \ y + 6 = -3(x - 3) \]

\[ a) \]

\[ b) \ y = -3x + 3 \]

\[ b) \]

\[ c) \ 3x + y - 3 = 0 \]

\[ c) \]

152. \((3, 6), m = 5\)

153. \((-2, -1), m = \frac{1}{2}\)
### Slope + Linear Relations

154. $(5, -6), m = -\frac{3}{4}$

155. $(\frac{1}{2}, 4), m = \frac{1}{3}$

156. $(-2, 1), m = 1.5$

---

<table>
<thead>
<tr>
<th>a)</th>
<th>a)</th>
<th>a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>b)</td>
<td>b)</td>
<td>b)</td>
</tr>
<tr>
<td>c)</td>
<td>c)</td>
<td>c)</td>
</tr>
</tbody>
</table>