Exponents Lesson 1

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Exponents: Integral & Rational

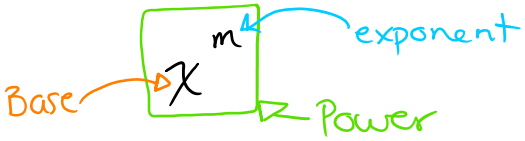
Term	Definition	Example
Power	$2^1, 2^2, 2^3, 2^4, \ldots$ are powers of 2. A power is made up of a base and an exponent.	
Exponent	The smaller number written to the upper right of the base that tells you how many times to multiply the base by itself.	$2^4 = 2 \times 2 \times 2 \times 2$ 4 is the exponent.
Base	The "larger" number that the exponent is applied to. (The bottom number in a power)	$2^4 = 2 \times 2 \times 2 \times 2$ 2 is the base.
Rational number	Numbers that can be written as fractions.	
Rational Exponent	The exponent on a power is a rational number (fraction). $x^{\frac{2}{3}} = \left(\sqrt[3]{x}\right)^2$	$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = (3)^2 = 9$
Integral number	An integer {...-3,-2,-1,0,1,2,3,...}.	
Integral Exponent	The exponent on a power is an integer.	Such as x^2, x^{-3}.
Coefficient	The numbers in front of the letters in mathematical expressions.	In $3x^2$, 3 is the coefficient.
Variable	The letters in mathematical expressions.	In $3x^2$,'x' is the variable.
Undefined	If there is no good way to describe something, we say it is undefined.	$\frac{3}{0}$ is undefined because we cannot divide by zero.
Radical form	$\left(\sqrt[3]{8}\right)^2$ is in radical form.	
Exponential Form	$8^{\frac{2}{3}}$ is in exponential form.	
Zero Exponent	Any expression to the power of 0 will equal 1.	$(2xyz)^0 = 1$
Negative Exponent	Reciprocate the base and perform repeated multiplication OR use repeated division.	$5^{-3} = \left(\frac{1}{5}\right)^3 = \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = \frac{1}{125}$
Multiply Powers with the Same base	Add the exponents.	$m^5 \times m^2 = m^7$
Dividing Powers with the same base.	Subtract the exponents.	$q^6 \div q^4 = q^2$
Power of a Power	Multiply the exponents.	$(x^2)^4 = x^8$
Power of a Product	Apply the exponent to all factors.	$(3x^2)^3 = 27x^6$
Power of a Quotient	Apply the exponent to both numerator AND denominator	$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$

Math 10

Unit 2: Exponents

Lesson 1: pages 1-9

Vocabulary:



Exponent Laws:

From Math 9, you should have learned how to simplify the following monomial expressions using the following exponent laws:

Note: DO NOT use exponent laws when bases aren't equal ← bases must be the same to use

simplify = to write as a power

evaluate = "solve" (2dp: is standard)

Exponent Laws	Examples (simplify & evaluate where possible)
Product of Powers $a^m \times a^n = a^{m+n}$ (same base)	a) $0.8^2 \times 0.8^7 = 0.8^{2+7} = 0.8^9$ (show work) $= 0.13$ (simplified, evaluate) b) $3^4 \times 3^1 = 3^{4+1} = 3^5 = 243$ c) $10^{10} \times 10^{-6} = 10^{10+(-6)} = 10^{10-6} = 10^4 = 10\,000$
Quotient of Powers $a^m \div a^n = a^{m-n}$ (base "a")	a) $5^5 \div 5^3 = 5^{5-3} = 5^2 = 25$ (⊖⊖=⊕) b) $\left(-\frac{4}{5}\right)^{-6} \div \left(-\frac{4}{5}\right)^{-20} = \left(-\frac{4}{5}\right)^{-6-(-20)} = \left(-\frac{4}{5}\right)^{14} = 0.04$ (exponent even ⊕) c) $40m^8 \div 5m^1 = 40 \div 5 = 8$, $m^{8-1} = m^7$ (4 coefficient) $= 8m^7$ simplified (cannot evaluate w/ variable)
Negative Exponent $a^{-m} = \frac{1}{a^m}$ (Flip, ⊕)	a) $25^{-3} = \frac{1}{25^3} = \frac{1}{15625}$ (top, evaluated) b) $6^3 \div 6^5 = 6^{3-5} = 6^{-2} = \frac{1}{6^2} = \frac{1}{36}$ (simplified)
Zero Exponent $a^0 = 1$	a) $(-7x^5y^{-6})^0 = (-7)^0 \cdot x^{5\cdot0} \cdot y^{-6\cdot0} = 1 \cdot 1 \cdot 1 = 1$ b) $\left(\frac{5}{2}\right)^4 \div \left(\frac{5}{2}\right)^4 = \left(\frac{5}{2}\right)^{4-4} = \left(\frac{5}{2}\right)^0 = 1$

$(-\frac{4}{5})(-\frac{4}{5}) = ⊖⊖ = ⊕$ EVEN

$(-\frac{4}{5})(-\frac{4}{5})(-\frac{4}{5}) = ⊕⊖⊖ =$ ODD

$3 = \frac{3}{1}$

$(a)^0 = 1$ 0 applies to everything in the brackets.

Watch for:

$a^0 = 1$ $-(a)^0 = -1 \iff -(a^0) = -1$ ($-a$ exponent applies to base only.)

$(-a)^0 = 1$ carry ⊖ through

Example: Evaluate or simplify the following expressions.

1. $3^2 =$ $3 \cdot 3 = 9$

2. $(-3)^2 =$ $(-3)\cdot(-3) = 9$ (recall $\ominus \times \ominus = \oplus$)

3. $-3^2 =$ applies to base only. $-[(3)\cdot(3)] = -[9] = -9$

4. $-5^0 =$ $\ominus \cdot 5^\circ = -1$ carry through. (NOTE: $-5^\circ \neq (-5)^\circ$; $=-1$ $=1$)

5. $\left(\frac{6^{-2}}{1}\right) =$ Flip $= \frac{1}{6^2} = \frac{1}{36}$

6. $-2^{-4} =$ applies to 2 as the base $\ominus \cdot \frac{1}{2^4} = -\frac{1}{2^4} = -\frac{1}{16}$ (simplify, evaluate)

7. $(-2)^{-4} =$ Base is -2 $\frac{1}{(-2)^4} = \frac{1}{(-2)\cdot(-2)\cdot(-2)\cdot(-2)} = \frac{1}{16}$ *even exponent. means $\oplus$ answer.

8. $x^3 \cdot x^4 =$ $x^{3+1} = x^7$

9. $x^3 \cdot x^{\frac{1}{4}} =$ multiply $x^{3+\frac{1}{4}} = x^{\frac{13}{4}}$ simplified or $x^{3\frac{1}{4}}$ $3 + \frac{1}{4} = \frac{12}{4} + \frac{1}{4} = \frac{13}{4}$ ($\times 4$, $\times 4$)

10. $6m^4 \cdot 2m^1 \div 3m^{-2} =$

$6 \cdot 2 \div 3$

$12 \div 3 = 4$ coefficient

$m^4 \cdot m^1 \div m^{-2}$

$m^5 \div m^{-2}$

$m^{5-(-2)} = m^{5+2} = m^7$ variable

$= 4m^7$ simplified

(HW p1-9 all)

Introduction to Exponents

Challenge #1: Solve each riddle using any strategy that works.

1. Evaluate. $3^2 \times 3^2$	2. Evaluate. $2^2 \times 2^2 \div 2^3$	3. Evaluate. $x^3 \times x^5$	4. Evaluate. $8x^4 \div 4x^3$
Rate the riddle: Easy, Medium, Hard	Rate the riddle: Easy, Medium, Hard	Rate the riddle: Easy, Medium, Hard	Rate the riddle: Easy, Medium, Hard

Rate the riddle: Easy, Medium, Hard	Rate the riddle: Easy, Medium, Hard	Rate the riddle: Easy, Medium, Hard	Rate the riddle: Easy, Medium, Hard

5. Find a strategy that is different from the one you used in Question 1 and solve the question again.

6. Find a strategy that is different from the one you used in Question 4 and solve the question again.

What is an Exponent?

Exponents are symbols that indicate an operation to be performed on the base.

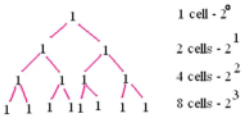
positive exponents → Repeated Multiplication
negative exponents → Repeated Division

b^e b is the base, and e is the exponent. Together, we call them a *power*.

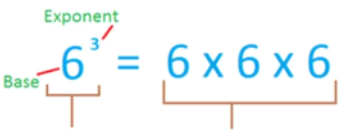
Some examples…

$2^1, 2^2, 2^3, 2^4, 2^5$ are the first five ***powers of 2***. x^1, x^2, x^3, x^4, x^5 are the first five ***powers of x.***

All organisms begin as one cell and then through a process called mitosis the single cell splits into two, then each of those split into two, etc. Eventually, these cells together form a multi-celled organism with trillions of cells.



** Guess the next few numbers _____, _____, _____



When numbers are written in a form such as 2^3 it is called a _____, the "2" is the _____ and the "3" is the _____. The exponent represents the number of times the base is multiplied by itself.

a^x	a is the base, x is the exponent and a^x is the power.
5^2	Is read 5 to the exponent 2 and equals 5×5 as a repeated multiplication and evaluates to 25.
2^5	Is read 2 to the exponent 5 and equals $2 \times 2 \times 2 \times 2 \times 2$ as a repeated multiplication and evaluates to 32.

Positive Integral Exponent (multiplication)	Zero Exponent	Negative Integral Exponent (repeated division)
$a^n = 1 \times a \times a \times a \times \ldots \times a$ (n factors)	$a^0 = 1, \quad (a \neq 0)$	$a^{-n} = 1 \div a^n$ $= \frac{1}{a^n}$
Eg. $3^4 = 1 \times 3 \times 3 \times 3 \times 3 = 81$	Eg. $5^0 = 1, \quad \left(\frac{3}{2}\right)^0 = 1$	Eg. $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

Challenge #2

7. Evaluate each of the following and examine the pattern:

$2^4 =$

$2^3 =$

$2^2 =$

$2^1 =$

$2^0 =$

$2^{-1} =$

$2^{-2} =$

$2^{-3} =$

$2^{-4} =$

8. What patterns do you notice in the list you created to the left?

9. Does the value of 2^0 make sense when put into this list?

10. Do negative exponents make sense in this list?

11. Why might people say negative exponents mean "repeated division?"

12. Identify the base in the following equation.

$4^3 = 64$

13. Identify the power in the following equation.

$2^5 = 32$

14. Identify the exponent in the following equation.

$-3^2 = -9$

15. Which of the following is equivalent to -16?

-4^2

$(-4)^2$

4^{-2}

-4^{-2}

16. Which of the following is equivalent to -81?

-9^2

$(-3)^4$

9^{-2}

-3^{-4}

17. Which of the following are equivalent to 1.

-3^0 $\frac{2x^3}{2x^3}$ $(5x)^0$

18. Which of the following is equivalent to 9?

-3^2

$(-3)^2$

3^{-2}

$(-3)^{-2}$

19. Evaluate.

-2^6

$= -1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= -64$

20. Evaluate.

$(-3)^3$

21. -4^2

22. $(-4)^{-2}$

23. -4^{-2}

24. 3^{-4}

$= \frac{1}{3^4}$

$= \frac{1}{81}$

$= 1 \div 3 \div 3 \div 3 \div 3$

$= 1 \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$

$= \frac{1}{81}$

25. $(-3)^{-4}$

26. -3^{-4}

27. 4^2

28. $(-4)^2$

29. $-(4)^2$

30. 5^0	31. -5^0	32. $\left(\frac{34a^2}{2x}\right)^0$

The Exponent Laws:

Challenge #3

33. Multiply. $a^3 \times a^6$	Explain your steps.

Challenge #4

34. Divide. $g^7 \div g^3$	Explain your steps.

Challenge #5

35. Multiply. $5m^4 \times 3m^2$	Explain your steps.

Simplify the following, write your answers using exponents.

36. $a^3 \times a^6$
$= a^{3+6}$
$= a^9$

37. $a^2 \times a^{-4}$

38. $f^2 \times f^x$

39. $x^{\frac{1}{4}} \times x^{\frac{6}{8}}$

40. $2^3 \times 2^{-5}$

41. $g^7 \div g^3$
$= g^{7-3}$
$= g^4$

42. $m^4 \div m^0$

43. $t^0 \div t^{-5}$

44. $\frac{x^{13}}{x^3}$

45. $5m^4 \times 3m^2$
$= 5 \times 3 \times m^{4+2}$
$= 15m^6$

46. $-10x^4 \div -2x^{-2}$

47. $\frac{4a^4}{-8a^2}$

48. $\frac{2}{3}x^3 \times \frac{6}{5}x^4$

49. $\frac{2}{a^3} \div \frac{6}{a^6}$

50. Evaluate.
$\left(\frac{2}{3}\right)^3 \left(\frac{-6}{4}\right)^2$

Multiplying Powers with the same Base:
Add the exponents.

Eg.
$x^5 \times x^2 = x^{5+2} = x^7$

$a^{\frac{2}{5}} \times a^{\frac{1}{5}} = a^{\frac{3}{5}} = a^1 = a$

$3x^2 \times 2x^5 = 3 \times 2 \times x^2 \times x^5 = 6x^7$

Dividing Powers with the same Base:
Subtract the exponents.

Eg.
$d^4 \div d^3 = d^{4-3} = d^1 = d$

$\frac{y^6}{y^{-2}} = y^{6-(-2)} = y^8$