Exponents Lesson 3

October 23, 2018 11:11 AM

① Journal ② lesson #3 Warm-up.

Math 10

Unit 2: Exponents

Lesson 3: pages 13-16

Warm-Up: Simplify or evaluate as far as possible. Express answers with positive exponents.

1. 7^{-3} $= \frac{1}{7^3} = \frac{1}{343}$

2. $2^6 \times 2^4$ $= 2^{6+4} = 2^{10} = 1024$

3. $x^9 \div x^3$ $= x^{9-3} = x^6$

4. $7m^4 \times 2m$

$7 \cdot 2 = 14$

$m^4 \cdot m^1 = m^{4+1} = m^5$ } $14m^5$

5. $(-8xy^5)^2$ $= (-8)^2 \cdot (x)^2 \cdot (y^5)^2$

$= 64x^2y^{5 \cdot 2}$

$= 64x^2y^{10}$

6. $50p^9 \div 10p^{-2}$

$50 \div 10 = 5$

$p^9 \div p^{-2} = p^{9-(-2)} = p^{11}$ } $5p^{11}$

7. $(3m^0)(9m)^0$

$(3 \cdot 1) \cdot (9^0 \cdot m^0)$

$3 \cdot 1 \cdot 1 = 3$

8. $(5m)^{-2}$ $= \frac{1}{(5m)^2} = \frac{1}{5^2 \cdot m^2} = \frac{1}{25m^2}$

9. $(2^{-3})^{-2}$ $= 2^{-3 \cdot -2}$ $\in \cdot \in = \in$

$= 2^6$

10. $(10y^{-3})(6y^4)^2$ $= 10y^{-3} \cdot 6^2 \cdot (y^4)^2$ ← 36, ← $4 \cdot 2 = 8$

$10 \cdot 36 \cdot y^{-3} \cdot y^8$ y^{-3+8}

$= 360y^5$

11. $(4x^2y^3)^{-3}$ $= 4^{-3} \cdot (x^2)^{-3} \cdot (y^3)^{-3}$ ($2 \cdot -3$, $3 \cdot -3$)

move ⊖ exp to bottom

$\frac{4^{-3} \cdot x^{-6} \cdot y^{-9}}{1} = \frac{1}{64x^6y^9}$

12. $\frac{6m^8y^2z^{-4}}{12my^5z^{-8}}$ $= \frac{6m^8y^2z^8}{12my^5z^4} = \frac{1}{2}m^7y^{-3}z^4 = \frac{m^7z^4}{2y^3}$

$6 \div 12 = \frac{1}{2}$

$m^8 \div m^1 = m^{8-1} = m^7$

$y^2 \div y^5 = y^{2-5} = y^{-3}$

$z^{-8} \div z^4 = z^{-8-4} = z^{-4}$

13. $\frac{-10ab^{-1}c^{-4}}{4a^{-2}c^7}$ $= \frac{-10a^1 \cdot a^2}{4c^7b^1c^4} = \frac{-10a^3}{4bc^{11}} = \frac{-5a^3}{2bc^{11}}$

$\frac{-10}{4} \xrightarrow{\text{simplify} \div 2} \frac{-5}{2}$

Lowest terms

14. $x^{-3} \cdot x^{-\frac{4}{3}} \cdot x^{\frac{1}{3}}$ add exp. $\frac{-3}{1} + \left(-\frac{4}{3}\right) + \frac{1}{3}$

$\frac{-9}{3} + \left(-\frac{4}{3}\right) + \frac{1}{3} = \frac{-12}{3} = -4$ ← exp

$x^{-4} = \frac{1}{x^4}$

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Exponent Laws:

From Math 9, you should have learned how to simplify the following monomial expressions using the following exponent laws:

Exponent Laws	Examples (simplify & evaluate where possible)
Power of a Quotient $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{y^{-3}}{x^{\frac{3}{5}}}\right)^5 = \frac{y^{-3\cdot5}}{x^{\frac{3}{5}\cdot5}} = \frac{y^{-15}}{x^3} = \frac{1}{x^3 y^{15}}$
* **Power of a Quotient** $\left(\frac{a}{b}\right)^{-m} = \frac{b^m}{a^m}$	$\left(\frac{y^{-3}}{x^{\frac{3}{5}}}\right)^{-5} = \left(\frac{x^{\frac{3}{5}}}{y^{-3}}\right)^5 = \frac{x^{\frac{3}{5}\cdot5}}{y^{-3\cdot5}} = \frac{x^3}{y^{-15}} = x^3 y^{15}$

FLIP Fraction $\left(\frac{b}{a}\right)^m$ ← exponent becomes ① dont change signs inside ( )

(More Complicated) Examples ☺ : Evaluate or simplify the following expressions.

1. $\left(\frac{x^4y^4m}{x^7y^2m^5}\right)^{-6} = \left(\frac{x^4y^4}{x^7y^2m^4}\right)^6 = \left(x^{4-7}\cdot y^{4-2}\cdot m^{-4}\right)^6 = \left(x^{-3}\cdot y^2\cdot m^{-4}\right)^6$

① Flip Fraction ② Cancel + simplify brackets ③ Apply exp. ④ exp.

$= x^{-3\cdot6}\cdot y^{2\cdot6}\cdot m^{-4\cdot6} = x^{-18}\cdot y^{12}\cdot m^{-24} = \frac{y^{12}}{x^{18}m^{24}}$

2. $\frac{(5m^{-1}y^3)^2}{my} = \frac{5^2\cdot m^{-1\cdot2}\cdot y^{3\cdot2}}{my} = \frac{5^2\cdot m^{-2}\cdot y^6}{my} = 25\cdot m^{-2-1}\cdot y^{6-1} = 25m^{-3}y^5 = \frac{25y^5}{m^3}$

3. $\left(\frac{7x^{-1}y^6}{x^{-4}y^4}\right)^{-2} = \left(\frac{x^{-4}y^4}{7x^{-1}y^6}\right)^2 = \frac{x^{-4\cdot2}\cdot y^{4\cdot2}}{7^2\cdot x^{-1\cdot2}\cdot y^{6\cdot2}} = \frac{x^{-8}\cdot y^8}{49x^{-2}y^{12}} = \frac{x^{-8-(-2)}\cdot y^{8-12}}{49} = \frac{x^{-6}y^{-4}}{49} = \frac{1}{49x^6y^4}$

*Flip Fraction ④ exp.

4. $\frac{\left(\frac{1}{2}a^4b^5\right)^{-4}}{4} = \frac{\left(\frac{1}{2}\right)^{-4}\cdot(a^4)^{-4}\cdot(b^5)^{-4}}{4} = \frac{\left(\frac{2}{1}\right)^4\cdot a^{4\cdot(-4)}\cdot b^{5(-4)}}{4} = \frac{16\cdot a^{-16}\cdot b^{-20}}{4} = \frac{16}{4a^{16}b^{20}} = \frac{4}{a^{16}b^{20}}$

16 ÷ 4 = 4

5. $\left(\frac{8xb^{-7}}{-12x^2b^{-9}}\right)^{-3} = \left(\frac{-12x^2b^{-9}}{8xb^{-7}}\right)^3 = \frac{(-12)^3\cdot(x^2)^3\cdot(b^{-9})^3}{8^3\cdot x^3\cdot(b^{-7})^3} = \frac{-1728\cdot x^6\cdot b^{-27}}{512\cdot x^3\cdot b^{-21}}$

$= \frac{-1728}{512}x^{6-3}b^{-27-(-21)} = \frac{-1728x^3b^{-6}}{512} = \frac{-27x^3}{8b^6}$

HW : p 13-16

Power of a Quotient:
Apply the exponent to numerator AND denominator.

THE RULE:

$\left(\frac{a}{b}\right)^m \quad \frac{a^m}{}$

HW : p 13-16

Power of a Quotient:
Apply the exponent to numerator AND denominator.

Eg. $\left(\frac{2}{5}\right)^3 = \left(\frac{2}{5}\right) \times \left(\frac{2}{5}\right) \times \left(\frac{2}{5}\right)$

$= \frac{2\times2\times2}{5\times5\times5}$

$= \frac{2^3}{5^3}$ If asked to write using exponents

$= \frac{8}{125}$ If asked to simplify.

$\left(\frac{2}{5}\right)^{-3}$ The negative exponent means "flip the base".

$= \frac{5\times5\times5}{2\times2\times2}$

$= \frac{5^3}{2^3}$

$= \frac{125}{8}$

THE RULE:

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$\left(\frac{a}{b}\right)^{-m} = \frac{b^m}{a^m}$$

Simplify the following.

67. $\left(\frac{x}{2}\right)^3$

$= \frac{x^3}{2^3}$

$= \frac{x^3}{8}$

68. $\left(\frac{a}{b}\right)^4$

69. $\left(\frac{x^2}{y^3}\right)^5$

70. $\left(\frac{-2a^2}{3y^3}\right)^3$

71. $\left(\frac{a^{-3}}{b^{-2}}\right)^{-2}$

72. $\left(\frac{4x}{3y}\right)^2$

73. $\left(\frac{6x^5y^3}{8y^4}\right)^{-2}$

$= \frac{(8)^2(y^4)^2}{(6)^2(x^5)^2(y^3)^2}$

$= \frac{64y^8}{36x^{10}y^6}$

$= \frac{16y^2}{9x^{10}}$

74. $\left(\frac{5ab^2c^3}{2a^{-2}c^{-3}}\right)^2$

75. $\left[\left(\frac{2m^2n^2}{mn^3}\right)^{-1}\right]^3$

Simplify the following.

76. $\left(\frac{6ab^3}{2ab}\right)^3$

77. $\left(\frac{4x^{-3}y^4}{8x^2y^{-2}}\right)^{-2}$

78. Show why $\frac{2a^2}{b^3}$ is the same as $2a^2 \times b^{-3}$.

79. Show why $\frac{12x^3}{y}$ is the same as $12x^3 \times y^{-1}$.

Challenge #13

80. Write the following without using any negative exponents.

$3a^2b^{-5}$

81. Write the following without using any negative exponents.

$\frac{3}{a^{-2}b^5}$

Challenge #14

82. Simplify using positive exponents.

$\left(\frac{2x^{-2}y^4}{x^{-3}y^3}\right)^{-3}$

Explain your steps

Writing Expressions with Positive Exponents. (Why? Because it is standard practice.)

An expression with powers is simplified if there are no brackets and no negative exponents.

Sometimes you will use the laws above and end up with an answer with negative exponents. The quick way to convert a negative exponent into a positive exponent is to ***move it across the division line***. The solution in question 83 shows why this works.

Simplify the following. (No brackets, no negative exponents)

83. $3a^2b^{-5}$

$= 3a^2 \times \frac{1}{b^5}$

$= \frac{3a^2}{b^5}$

84. a^2b^{-3}

85. $\frac{2xy^5}{x^{-4}}$

86. $3a^2b^{-3}c^{-5}$

87. $(x^4y^{-3}z^{-1})^{-2}$

88. $\frac{(3x^{-3}y^{-5})^2}{2xy}$

89. $\left(\frac{2x^{-2}y^4}{x^{-3}y^3}\right)^{-3}$

$= \left(\frac{x^{-3}y^3}{2x^{-2}y^4}\right)^3$

$= \frac{x^{-9}y^9}{8x^{-6}y^{12}}$

$= \frac{x^{-3}y^{-3}}{8}$

$= \frac{1}{8x^3y^3}$

90. $\frac{\left(\frac{2a^3b^2}{4a^{-2}b^{-1}}\right)^{-3}}{2}$

91. $\frac{(4m^2n^2)(7m^{-3}n^2)}{14mn^5}$

92. Why does moving a power across the division line in a fraction change the sign on the exponent?

Simplify the following. (No brackets, no negative exponents)

93. $\left(\frac{12x^3y^{-1}}{-8x^{-1}y^5}\right)^{-2}$

94. $\left(\frac{4a^3b^{-2}}{6a^2b^{-1}}\right)^{-3}$

95. $\left(\frac{8x^2y^{-3}}{4x^{-1}y^{-5}}\right)^{-3}$

96. $\left(\frac{12x^{-3}y^5}{16x^3y^{-2}}\right)^{-1}$