

Exponents Lesson 4

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Lesson 4 Warm-Up

Math 10

Unit 2: Exponents

Lesson 4: pages 17-19

Warm-Up #1: Simplify or evaluate as far as possible. Express answers with positive exponents.

1. $\frac{3x^2y^4}{4x^3y^3} = \frac{3x^{2-3}y^{4-3}}{4} = \frac{3x^{-1}y^1}{4} = \frac{3y}{4x}$

2. $\frac{2ab^3}{-2a^3b^2} = \frac{1a^{-3}b^1}{-1} = \frac{a^{-3}b^1}{-1} = \frac{b}{-a^3} = -\frac{b}{a^3}$

3. $\frac{-2x^2y^{-5}}{3x^{-4}y^3} = \frac{-2x^{2-(-4)}y^{-5-3}}{3} = \frac{-2x^6y^{-8}}{3} = \frac{-2x^6}{3y^8}$

4. $\frac{(n^2)(-n^0)^3}{-n^2} = \frac{n^2 \cdot (-n^0)^3}{-n^2} = \frac{n^2 \cdot (-1)^3}{-n^2} = \frac{-n^2}{-n^2} = n^0 = 1$

5. $\frac{2a^2bc^{-4}}{5a^{-3}b^3c^2} = \frac{2a^{2-(-3)}b^{1-3}c^{-4-2}}{5} = \frac{2a^5b^{-2}c^{-6}}{5} = \frac{2a^5}{5b^2c^6}$

6. $\left(\frac{x^2y}{mp^3}\right)^5 = \frac{x^{2 \cdot 5}y^5}{m^5p^{3 \cdot 5}} = \frac{x^{10}y^5}{m^5p^{15}}$

7. $\left(\frac{3c}{5d}\right)^2 = \frac{3^2c^2}{5^2d^2} = \frac{9c^2}{25d^2}$

8. $\left(\frac{15m^3y}{3my^{-5}}\right)^{-3} = \frac{15^{-3}m^{3 \cdot (-3)}y^{3 \cdot (-3)}}{3^{-3}m^{(-3) \cdot 1}y^{(-3) \cdot (-5)}} = \frac{1 \cdot m^{-9}y^{-9}}{27m^{-3}y^{15}} = \frac{m^{-9}y^{-9}}{27m^{-3}y^{15}} = \frac{m^{-9-(-3)}y^{-9-15}}{27} = \frac{m^{-6}y^{-24}}{27} = \frac{1}{27m^6y^{24}}$

9. $\left(\frac{27m^2n}{9mn}\right)^{-2} = \left(\frac{3m}{n}\right)^{-2} = \frac{3^{-2}m^{-2}n^2}{1} = \frac{1}{9m^2}n^2 = \frac{n^2}{9m^2}$

10. $\left(\frac{2a^2b^{-2}}{8a^{-3}b}\right)^4 = \frac{2^4a^{2 \cdot 4}b^{-2 \cdot 4}}{8^4a^{-3 \cdot 4}b^{1 \cdot 4}} = \frac{16a^8b^{-8}}{4096a^{-12}b^4} = \frac{16a^8b^{-8}}{4096a^{-12}b^4} = \frac{16a^{8-(-12)}b^{-8-4}}{4096} = \frac{16a^{20}b^{-12}}{4096} = \frac{16a^{20}}{4096b^{12}} = \frac{a^{20}}{256b^{12}}$

ON CALC. $4^{\frac{1}{2}} = 4$ $4^{(1 \div 2)} = 2$

depends on calc.

Warm-Up #2: Use your calculator to complete the following tables:

1.

x	$\frac{1}{x^2}$
1	1
4	2
9	3
16	4
25	5

$\sqrt{4} = 2$
 $\sqrt{25} = 5$

Explain the effect the exponent $\frac{1}{2}$ has on the value of x:
what is the pattern?
if exponent is $\frac{1}{2} = \sqrt{x}$

Write a rule to describe this relationship:
 $x^{\frac{1}{2}} = \sqrt{x}$ or \sqrt{x}

CALC:

9	3	$9^{1/2}$
16	4	$16^{1/2}$
25	5	$25^{1/2}$
36	6	$36^{1/2}$

$\sqrt{25} = 5$

Write a rule to describe this relationship:
 $x^{1/2} = \sqrt{x}$ or \sqrt{x}
 related.

CALC:
 $\sqrt{\square}$ or $\sqrt{\square}$

2.

y	$y^{1/3}$	
1	1	$1^{1/3}$
8	2	$8^{1/3}$
27	3	$27^{1/3}$
64	4	$64^{1/3}$
125	5	$125^{1/3}$
216	6	$216^{1/3}$

$\sqrt[3]{8} = 2$

$\sqrt[3]{125} = 5$

Explain what effect the exponent $\frac{1}{3}$ has on the value of y.
 if the exponent is $\frac{1}{3} = \sqrt[3]{y}$
 Write a rule to describe this relationship:
 $y^{1/3} = \sqrt[3]{y}$

CALC:
 $\sqrt[\square]{\square}$ or $\sqrt[\square]{\square}$

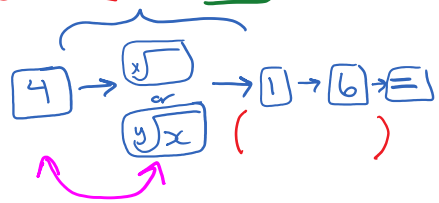
3. What do you think $x^{1/4}$ means? Test your prediction on your calculator (letting $x = 16$).

$x^{1/4} = \sqrt[4]{x}$ $16^{1/4} = \sqrt[4]{16} = 2$

4. What would $x^{1/n}$ mean (as a radical)?

$n = \text{any integer}$

$x^{1/n} = \sqrt[n]{x}$



- $\frac{1}{2} = \sqrt{\quad}$
- $\frac{1}{3} = \sqrt[3]{\quad}$
- $\frac{1}{4} = \sqrt[4]{\quad}$
- ∴ etc.

Exponent Law:

Exponent Laws	Example #1 (simplify & evaluate where possible)
	a) $100^{\frac{1}{2}} = \sqrt[2]{100} = 10$ <small>simplify evaluate</small>
	b) $(-8)^{\frac{1}{3}} = \sqrt[3]{-8} = -2$
	c) $1024^{\frac{1}{4}} = \sqrt[4]{1024} = 4$
	d) $(625m^4)^{\frac{1}{4}} = \sqrt[4]{625m^4} = \sqrt[4]{625} \cdot \sqrt[4]{m^4}$ <small>cancel</small> $= 5 \cdot m = 5m$
	e) $(81m)^{\frac{1}{4}} = \sqrt[4]{81m} = \sqrt[4]{81} \cdot \sqrt[4]{m}$ $= 3 \cdot \sqrt[4]{m}$
	f) $-343^{\frac{1}{3}} = \sqrt[3]{-343} = -7$
	g) $(-49)^{\frac{1}{2}} = \sqrt{-49} = \text{no solution}$ (cannot take $\sqrt{\ominus}$ \neq no solution)
	h) $16^{\frac{-1}{4}} \xrightarrow{\text{FLIP}} = \frac{1}{16^{\frac{1}{4}}} = \frac{1}{\sqrt[4]{16}} = \frac{1}{2}$
	i) $1000^{\frac{-1}{3}} \xrightarrow{\text{FLIP}} = \frac{1}{1000^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{1000}} = \frac{1}{10}$

write as a radical
 solve

$\sqrt{-x} \neq$ no answer
 $\sqrt[3]{-x} \checkmark$ ok!
 $= 0$
 $\sqrt{-x} \neq$ NO
 $\sqrt[3]{-x} \checkmark$ ok
 $= 0$
 leave in radical form when can't simplify.
 cannot take $\sqrt{\ominus} \neq$ no solution

Rational Exponents
 (with numerator = 1)
 $x^{\frac{1}{n}} = \sqrt[n]{x}$
 $n=1$

EXPONENTIAL

Example #2: Simplify the following in radical form

- $\sqrt[2]{121} = \text{square root} = \sqrt[2]{121} = 121^{\frac{1}{2}}$
given
- $\sqrt[5]{-32} = (-32)^{\frac{1}{5}}$
opp # ok
 $-\sqrt[5]{32} \neq \sqrt[5]{-32}$
- $\frac{1}{\sqrt[3]{125}} = \frac{1}{125^{\frac{1}{3}}} \xrightarrow{\text{FLIP}} = 125^{-\frac{1}{3}}$
- $10\sqrt[2]{3xy} = 10 \cdot \sqrt[2]{3xy} = 10 \cdot (3xy)^{\frac{1}{2}} = 10(3xy)^{\frac{1}{2}}$

HW: p. 17 - 19

Mon Oct 29 - Review + Practice Test
 Tue's Oct 30 - Unit Test.

97. **Challenge #15**

If $\sqrt{9} \times \sqrt{9} = 9$,

and $9^a \times 9^a = 9$

Then what is the value of 'a'?

Explain:

.....

.....

.....

.....

98. **Challenge #16**

If $\sqrt[3]{2} \times \sqrt[3]{2} \times \sqrt[3]{2} = 2$,

and $2^a \times 2^a \times 2^a = 2$

Then what is the value of 'a'?

Explain:

.....

.....

.....

.....

99. Write a "rule" that relates a **rational** (fraction) **exponent** to an equivalent **radical expression**.

Rational Exponents in the form: $x^{\frac{1}{n}}$

Remember, *rational* often refers to fractions.

What does a rational exponent mean?

Recall: $\sqrt{9} \times \sqrt{9} = 9$.

If $\sqrt[3]{2} \times \sqrt[3]{2} \times \sqrt[3]{2} = 2$

But $9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 9$

But $2^{\frac{1}{3}} \times 2^{\frac{1}{3}} \times 2^{\frac{1}{3}} = 2$

And $3 \times 3 = 9$

So, $\sqrt[3]{2} = 2^{\frac{1}{3}}$

So, $\sqrt{9} = 9^{\frac{1}{2}} = 3$

100. Write another statement like the one to the left.

.....

.....

.....

.....

The Rule...

$$a^{\frac{1}{n}} = \sqrt[n]{a} \quad \text{and} \quad a^{-\frac{1}{n}} = \frac{1}{\sqrt[n]{a}}$$

Evaluate or simplify the following.

101. $49^{\frac{1}{2}}$	102. $-16^{\frac{1}{2}}$	103. $(-16)^{\frac{1}{2}}$
104. $64^{\frac{1}{3}}$	105. $27^{-\frac{1}{3}}$	106. $32^{-\frac{1}{5}}$
107. $10000^{\frac{1}{4}}$	108. $(4x^2)^{\frac{1}{2}}$	109. $(27x^6)^{-\frac{1}{3}}$

Write in radical form.

110. $7^{\frac{1}{2}}$	111. $(3x)^{\frac{1}{3}}$	112. $4^{\frac{1}{5}}$
113. $4^{-\frac{1}{5}}$	114. $-64^{\frac{1}{3}}$	115. $64^{-\frac{1}{3}}$

Write in exponential form.

116. $\sqrt{13}$	117. $-3\sqrt{x}$	118. $\sqrt{2y}$
119. $\sqrt[4]{4}$	120. $\sqrt[4]{4}$	121. $\frac{1}{\sqrt[5]{3x}}$

Consider the following...

Step 1: $32^{\frac{3}{5}} = \left(32^{\frac{1}{5}}\right)^3$

Step 2: $32^{\frac{3}{5}} = \left(\sqrt[5]{32}\right)^3$

Step 3: $32^{\frac{3}{5}} = (2)^3$

Step 4: $32^{\frac{3}{5}} = 8$

122. Challenge #17. Complete the following as shown above.

Step 1: $27^{\frac{2}{3}} =$ _____ Explain: _____

Step 2: $27^{\frac{2}{3}} =$ _____

Step 3: $27^{\frac{2}{3}} =$ _____

Step 4: $27^{\frac{2}{3}} =$ _____