Lesson 1

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Lesson #1 – Intro to Polynomials & Addition/Subtraction

Term: A number and or variable connected by addition, subtraction or multiplication, division (also called a monomial) ← constant; operations

ex. $5x^2 + y^3(5 + x) - 7$

coefficient (5); exponents (2, 3); variables: letters to represent numbers (y, x); constant: term with NO variable (−7), sign ⊖/⊕ belongs to the constant

**Coefficients must be real numbers and exponents must be whole numbers (0, 1, 2, 3, ... etc.)

ie. $3y^{-2}$, $2x^{1/2}$, 3^x ⇐ NOT polynomials.

$2x^{-1} = \frac{2}{x}$ NOT polynomial

	# of Terms	Example
Monomial	1	$4y$
Binomial	2	$3x + 2$
Trinomial	3	$8x^2 + 9x + 2$
Polynomial*	more than 1+	$3x^3 + 6xy + 2x + 10$

"Polynomials" many.

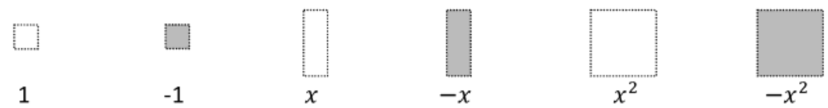
* is a general name for an expression with 1 or more terms.

Degree of a...

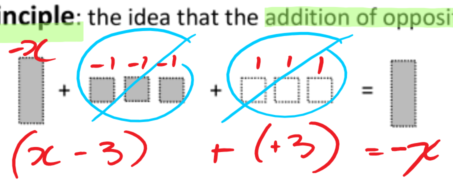
Term: add all the exponents within the 1 term. ie. $4x^2 + 4xy^2 + 6$ (degree = 2; degree: 1+2=3)

Polynomial: the highest degree of all terms ie. $x^4 + 4x^2 + 4xy^2 + 6$ — degree of polynomial = 4

Algebra Tile Legend: white ⊕, shaded ⊖



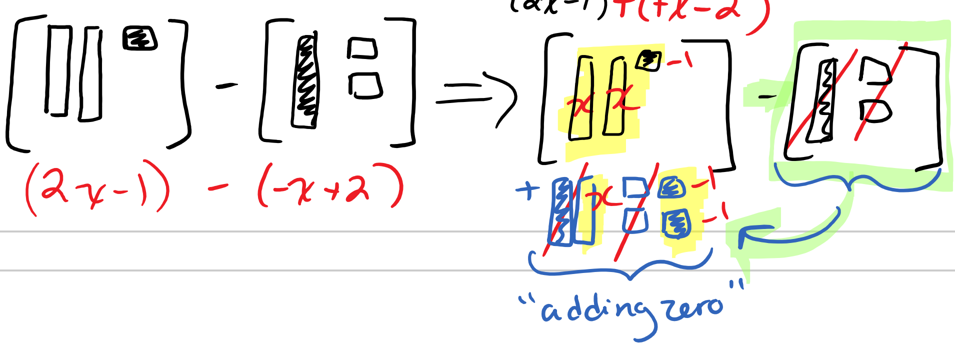
Zero Principle: the idea that the addition of opposites cancel each other out and the result is zero



"zero pair" – set of opposites

example: Subtract the following using algebra tiles $(2x - 1) - (-x + 2)$

$(2x-1) + (+x-2)$



$= 3x - 3$

I. Simplify the following

- you can simplify expressions by collecting _______________ terms (terms with ___________________ variables and exponents)

1. $7x + 3y + 5x - 2y$

2. $3x^2 + 4xy - 6xy + 8x^2 - 3yx$

II. Add/Subtract the following

3. $(x^2 + 4x - 2) + (2x^2 - 6x + 9)$

4. $(4x^2 - 2x + 3) - (3x^2 + 5x - 2)$

ASSIGNMENT # 1
pages 4- 11 Questions #1-53

What is a Polynomial?

What is a Term?

A **term** is a number and/or variable connected by multiplication or division. One term is also called a **monomial**.

Numerical coefficient: 5

$5x$

Variable: x

Constant: 3

$-2a + 3$

The following are terms: 5, x, $3x$, $5x^2$, $\frac{3x}{4}$, $-2xy^2z^3$

Each term may have a **coefficient, variable(s) and exponents**. One term is also called a **monomial**.

If there is no variable present…we call the term a **constant.**

Answer the questions below.

1. What is/are the coefficients below? $5xy^2 - 7x + 3$	2. What is/are the constant(s) below? $12x^2 - 5x + 13$	3. What is/are the variable(s) below? $5xy^2 + 3$

A **polynomial** is an expression made up of **one or more terms** connected to the next by addition or subtraction.

We say a polynomial is any expression where the **coefficients are real** numbers and all **exponents are whole** numbers. That is, no variables under radicals (rational exponents), no variables in denominators (negative exponents).

The following are polynomials:

x, $2x - 5$, $5 + 3x^2 - 12y^3$, $\frac{x^2+3x+2}{2}$, $\sqrt{3}x^2 + 5y - z$

The following are **NOT** polynomials:

x^{-2}, $3\sqrt{x}$, $4xy + 3xy^{-3}$, $12xz + 3^x$

Which of the following are not polynomials? Indicate why.

4. $3xyz - \frac{2}{x}$	5. $\frac{1}{-5}x^3 - 5y$	6. $2x - 4y^{-2}$
7. $(3x+2)^{\frac{1}{3}}$	8. $\sqrt{3} + x^2 - 5$	9. $\frac{5}{3}x - 2^x$

Classifying polynomials:

By Number of Terms:

- **Monomial**: one term. Eg. $7x$, 5, $-3xy^3$
- **Binomial**: two terms Eg. $x+2$, $5x-3y$, $y^3 + \frac{5x}{3}$
- **Trinomial**: three terms Eg. x^2+3x+1, $5xy - 3x + y^2$
- **Polynomial**: four terms Eg. $7x + y - z + 5$, $x^4 - 3x^3 + x^2 - 7x$

By Degree:
To find the degree of a *term*, add the exponents within that term.

Eg. $-3xy^3$ is a 4th degree term because the sum of the exponents is 4.
$5z^4y^2x^3$ is a 9th degree term because the sum of the exponents is 9.

To find the degree of a polynomial first calculate the degree of each term. The highest degree amongst the terms is the degree of the polynomial.

Eg. $x^4 - 3x^3 + x^2 - 7x$ is a 4th degree polynomial. The highest degree term is x^4.
$3xyz^4 - 2x^2y^3$ is a 6th degree binomial. The highest degree term is $3xyz^4$ (6th degree)

Classify each of the following by degree and by number of terms.

10. $2x+3$ Degree: _1__ Name: _Binomial__	11. $x^3 - 2x^2 + 7$ Degree: _____ Name: __________	12. $2a^3b^4 + a^2b^4 - 27c^5 + 3$ Degree: _____ Name: __________
13. 7 Degree: _____ Name: __________	14. Write a polynomial with one term that is degree 3.	15. Write a polynomial with three terms that is degree 5.

Algebra Tiles

The following will be used as a legend for algebra tiles in this guidebook.

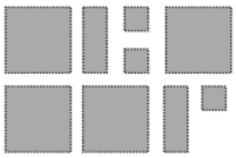
Write an expression that can be represented by each of the following diagrams.

16.

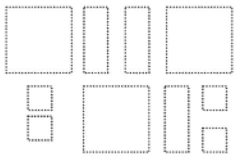
17.

18.

19.



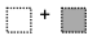
20.



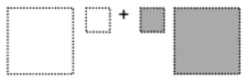
21. Draw a diagram to represent the following polynomial.
$$3x^2 - 5x + 6$$

22. Draw a diagram to represent the following polynomial.
$$-3x^2 + x - 2$$

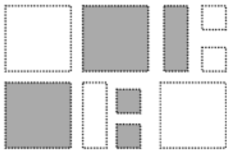
23. What happens when you add the following?



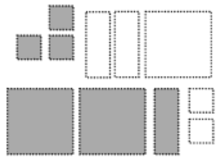
24. What happens when you add the following?



25. Simplify by cancelling out tiles that add to zero. Write the remaining expression.



26. Simplify by cancelling out tiles that add to zero. Write the remaining expression.



27. Represent the following addition using algebra tiles. Do not add.
$x + (x - 1)$

28. Represent the following addition using algebra tiles. Do not add.

$(5x + 3) + (2x + 1)$

29. Use Algebra tiles to add the following polynomials. (collect like-terms)

$(2x - 1) + (-5x + 5)$

30. Use Algebra tiles to add the following polynomials. (collect like-terms)

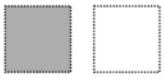
$(2x^2 + 5x - 3) + (-3x^2 + 5)$

<u>The Zero Principle</u>:

The idea that opposites cancel each other out and the result is zero.

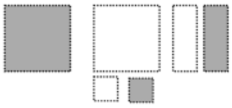
Eg. $x + 3 + (-3) = x$ The addition of opposites did not change the initial expression.

31. What is the sum of the following tiles?



Sum__________

32. If you add the following to an expression, what have you increased the expression by?

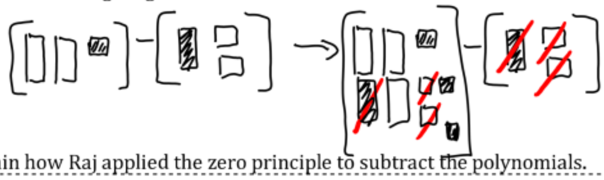


33. Represent the following subtraction using algebra tiles.

$(2x-1)-(-x+2)$

34. Why can you not simply "collect like-terms" when subtracting the two binomials in the previous question?

35. When asked to subtract $(2x-1)-(-x+2)$, Raj drew the following diagram:



Explain how Raj applied the zero principle to subtract the polynomials.

36. Use Algebra tiles to subtract the following polynomials.

$(2x-1)-(-5x+5)$

37. Use Algebra tiles to subtract the following polynomials.

$(2x^2+5x-3)-(-3x^2+5)$

38. Use Algebra tiles to subtract the following polynomials.

$(-2x^2-4x-3)-(-3x^2+5)$

Like Terms

39. When considering algebra tiles, what makes two tiles "alike"?

40. What do you think makes two algebraic terms alike? (Remember, tiles are used to represent the parts of an expression.)

Collecting Like Terms without tiles:

Exactly the same variable & exponents.

You have previously been taught to combine like terms in algebraic expressions.

Terms that have the same variable factors, such as $7x$ and $5x$, are called ***like terms***.

Simplify any expression containing like terms by adding their coefficients.

Eg.1. Simplify:

$7x + 3y + 5x - 2y$
$7x + 5x + 3y - 2y$
$= 12x + y$

Eg.2. Simplify

$3x^2 + 4xy - 6xy + 8x^2 - 3yx$
$3x^2 + 8x^2 + 4xy - 6xy - 3xy$
$= 11x^2 - 5xy$

Remember … $3yx$ is the same

Simplify by collecting like terms. Then evaluate each expression for $x = 3, y = -2$.

41. $3x + 7y - 12x + 2y$

42. $2x^2 + 3x^3 - 7x^2 - 6$

43. $5x^2y^3 - 5 + 6x^2y^3$

Adding & Subtracting Polynomials without TILES.

ADDITION
To add polynomials, collect like terms.

Eg.1. $(x^2 + 4x - 2) + (2x^2 - 6x + 9)$

Horizontal Method:

$=x^2 + 4x - 2 + 2x^2 - 6x + 9$
$=x^2 + 2x^2 + 4x - 6x - 2 + 9$

$=3x^2 - 2x + 7$

Vertical Method:

$x^2 + 4x - 2$
$\underline{2x^2 - 6x + 9}$

$= 3x^2 - 2x + 7$

SUBTRACTION
It is important to remember that the subtraction refers to all terms in the bracket immediately after it.

To subtract a polynomial, determine the opposite and add.

Eg.2. $(4x^2 - 2x + 3) - (3x^2 + 5x - 2)$

Multiplying each term by -1 will remove the brackets from the **second** polynomial.

This question means the same as:

$(4x^2 - 2x + 3) - \mathbf{1}(3x^2 + 5x - 2)$
$= 4x^2 - 2x + 3 - 3x^2 - 5x + 2$
$= 4x^2 - 3x^2 - 2x - 5x + 3 + 2$

$= x^2 - 7x + 5$

We could have used vertical addition once the opposite was determined if we chose.

Add or subtract the following polynomials as indicated.

44. $(4x + 8) + (2x + 9)$

45. $(3a + 7b) + (9a - 3b)$

46. $(7x + 9) - (3x + 5)$

47. Add.
$(4a - 2b)$
$\underline{+(3a + 2b)}$

48. Subtract.
$(7x - 3y)$
$\underline{-(-5x + 2y)}$

49. Subtract.
$(12a - 5b)$
$\underline{-(-7a - 2b)}$

Add or subtract the following polynomials as indicated.

50. $(5x^2 - 4x - 2) + (8x^2 + 3x - 3)$

51. $(3m^2n + mn - 7n) - (5m^2n + 3mn - 8n)$

52. $(8y^2 + 5y - 7) - (9y^2 + 3y - 3)$

53. $(2x^2 - 6xy + 9) + (8x^2 + 3x - 3)$

Your notes here…