Lesson 3 Domain & Range

November 14, 2018 3:47 PM

3) RELATIONS & FUNCTIONS: DOMAIN AND RANGE

relationship between 2 quantities (variables)

Warm-Up #1: List all the values of x in each relation.

(x, y)

a) (1, 3), (2, 5), (9, 4) ⟹ a) $x = 1, 2, 9$

b)

x	y
−3	4
−1	7
0	1

b) $x = -3, -1, 0$

c) $y = 2x - 3$ ⟹ c) x = any value

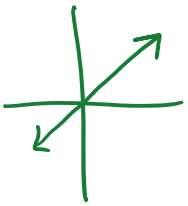
Warm- Up #2: List all the values of y in each relation.

a) (1, 3), (2, 5), (9, 4) a) $y = 3, 5, 4$

b)

x	y
−3	4
−1	7
0	1

⟹ b) $y = 4, 7, 1$



< less than

> greater than

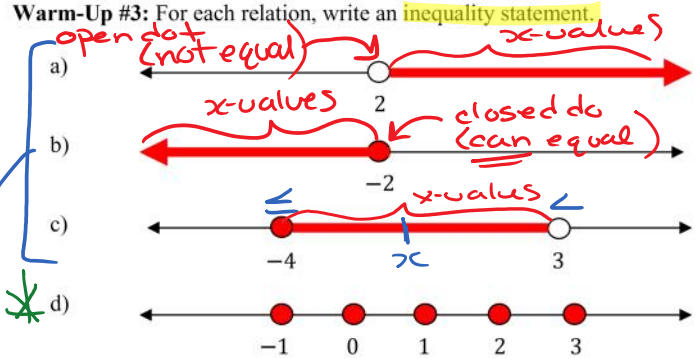
≤ less than or equal to

≥ greater than or equal to

c) $y = 2x - 3$ ⟶ c) y = any value

Warm-Up #3: For each relation, write an inequality statement.

x-values



a) $x > 2$

b) $x \leq -2$

c) $-4 \leq x < 3$

d) $x = -1, 0, 1, 2, 3$

What is different about part d in the warm up?

a)-c) represent continuous data (decimals, fractions, etc) It includes values between data points eg. age, height

d) Discrete data - includes only the values listed, no points between. eg. # of people

Warm-Up #4: Fill in the table below, with the correct symbol for each number set. Then, identify the number set represented in the examples from ~~warm-up #1~~. warm-up 3

Number Set	Symbol
Real	R
Rational	Q
Irrational	$\bar{Q}$
Integer	Z
Whole	W
Natural	N

Review your answer in the warm up. Identify the appropriate number set for each inequality.

a) R, Q, $\bar{Q}$, Z, W, N $x > 2$

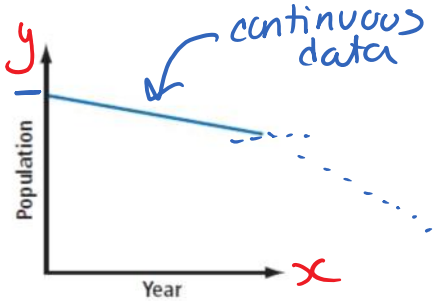
b) R, Q, $\bar{Q}$, Z $x \geq -2$ ← does not include (W, N)

c) R, Q, $\bar{Q}$, Z, W, N $-4 \leq x < 3$

d) R, Q, Z $x = -1, 0, 1, 2, 3$

When interpreting information to solve a problem, it is important to make sense of the possible values of each quantity being compared.

Example #1: Determine the possible values for each quantity in the given relation.



continuous data

y = population → $y \geq 0$
- no negative
- all real numbers

x = year (Time) → $x \geq 0$
- no negatives
- all real numbers.

When comparing two quantities, the words **DOMAIN** and **RANGE** are used to describe the values that are appropriate.

The Domain is the set of all possible values for the **independent** variable in a relation. (x-values)

The Range is the set of all possible values for the **dependent** variable in a relation. (y-values)

relationship between values.

There are a variety of ways to express the domain and range of a relation.

1. **Words** – A description of the value that are allowed.

   Example: the range is the set of all whole numbers less than twenty

2. **Number Line** - A picture of the values that are allowed. $y \leq 20$

   Example: 

   y = range

3. **A List** – used for discrete data (coordinates) ⟹ discrete data
   - no lines
   - points are not connected.

   Example: For the relation $\{(3, 1), (2, -3), (7, 0.4)\}$

   x the domain is $\{3, 2, 7\}$

   y the range is $\{1, -3, 0.4\}$

4. **Set Notation** – a formal way to give the values of the domain and range.

   Example: $\{x | x \geq -1, x \in \mathbb{Z}\}$

   Domain: x (Range: y)

   "such that"

   x is an integer (-neg., ∅, decimals, fract.)

   inequality statement

   always use {curly brackets}

   What does it all mean???

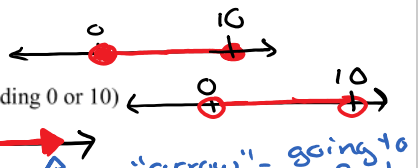
   { } = type of brackets used for a set
   | means "such that"
   ∈ means "is an element of" (or "belongs to")
   This statement is read as: "x is such that, x is greater than or equal to -1, and x is an integer"

5. **Interval Notation** – uses different brackets to indicate an interval

   Example: $[0, 10]$ means all numbers between zero to ten inclusive.

   Example: $(0, 10)$ means all numbers between zero and ten (not including 0 or 10)

   Example: $(10, \infty)$ means all numbers greater than 10

   ∞ infinity

   "arrow" = going to infinity

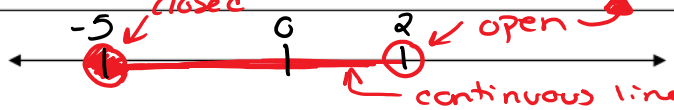
[square brackets] : equal to : ●——● : closed dots (includes the points)

(curved brackets) : can't equal : ○——○ : open dots (doesn't include points)

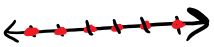
Example #2: Consider the Relation: *all real numbers between -5 and 2 including -5 but not including 2.*

Domain (x) R

$-5 \leq x < 2$ inequality statement

Number Line:	
Set Notation:	$\{x \mid -5 \leq x < 2, x \in R\}$ "x is all real #'s"
Interval Notation:	includes → $[-5, 2)$ ← not included.
Could I use a list? Explain.	Yes... but not practical. Because Lists are only good for discrete data. We have a continuous LINE

Example #3: Complete the table. (Range : y)

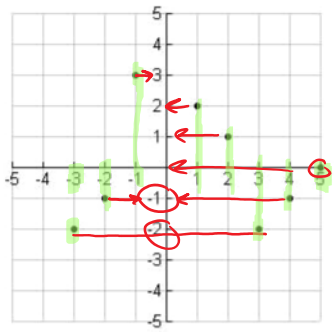
Words: your age from grade 1 until now	6 yrs old.
Number Line:	
List:	is this data discrete? or continuous? 
Set Notation:	$\{y \mid 6 \leq y \leq 16, y \in R\}$ we know you can be 6.5, 12.2
Interval Notation:	$[6, 15]$ or ~~$(6, 15)$~~ or ~~$[6, 15)$~~ included

Let's see how this applies to graphs!

x-values ↔ y-values

Example #4: Write the domain and range for each relation, as specified.

a)

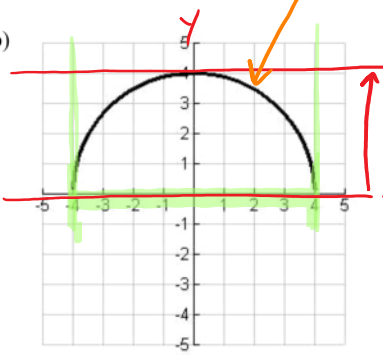


Discrete data

Domain as a list: $\{-3,-2,-1,1,2,3,4,5\}$

Range in set notation: $\{y \mid y=-2,-1,0,1,2,3, y \in \mathbb{Z}\}$ (integers)

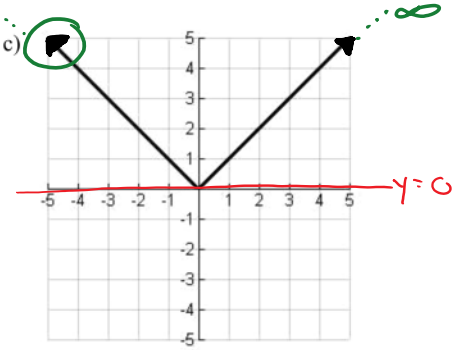
b)



Domain in words: x is between -4 and 4

Range in interval notation: $[0,4]$ (included)

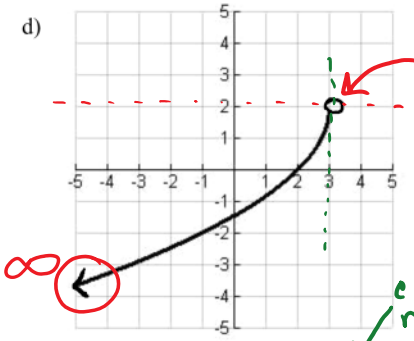
c)



Domain in set notation: $\{x \mid -\infty < x < \infty, x \in \mathbb{R}\}$

Range in set notation: $\{y \mid 0 \le y < \infty, y \in \mathbb{R}\}$

d)



Domain in interval notation: $(-\infty, 3)$ (can't ever reach infinity; NOT included)

Range in interval notation: $(-\infty, 2)$

Discrete data

e) $\{(4,-1),(-1,4),(-1,3),(4,-3),(-3,0)\}$

Domain in set notation: $\{x \mid x=-3,-1,4, x \in \mathbb{Z}\}$

Range in set notation: $\{y \mid y=-3,-1,0,3,4, y \in \mathbb{Z}\}$

Homework

ASSIGNMENT # 3

Pages 13-19 Questions #36-66

Domain & Range (continued)

Recall, (2,5) and (-3,7) are called ***ordered pairs*** because the order of the two ***elements*** is important.

- The first set of elements in the ordered pair is called the ***domain*** of the relation.
- The second set of elements in the ordered pair is called the ***range*** of the relation.

36. Challenge Question:
List the domain and range for the relation (1,1), (2, 4), (3,9), (4,16)

Answer:
Domain: {1,2,3,4} Range: {1,4,9,16}

37. Which of the following is/are true?

a. The domain is the set of permissible values of x.
b. The domain is the set of permissible values of y.
c. The range is the set of permissible values of x.
d. The range is the set of permissible values of y.

Your notes here…

Domain & Range of Discrete Data (points):

[Definition on page 25]

Remember, domain is all "first elements" and range is all "second elements".

Since we are often working with graphs that have an x-axis and a y-axis.

Domain is often described as all permissible values of *x*.

Range is often described as all permissible values of *y*.

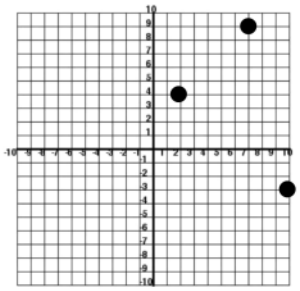
Find the domain and range:

Example:
Find the domain and range of the relation:
(2,3), (3,4), (4,5), (5, 6)

Solution:
Simply list the first elements, then second:

domain: {2,3,4,5} range:{3,4,5,6}

Example:
Find the domain and range of the following relation.



Solution:
First, find the coordinates of the BIG points:
(2,4), (7,9), (10,-3)

Domain: {2,7,10}

Range: {4,9,-3}

**It's OK that there is no apparent pattern...this is still a relation.

Find each of the following.

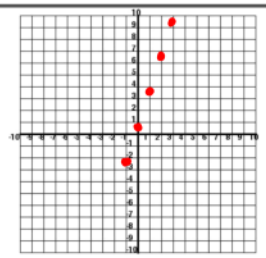
38. Find the domain for the following relation.

(−2,4), (3,5), (5,7), (8,11)

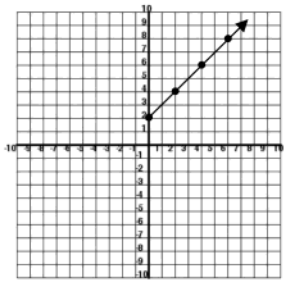
39. Find the range for the relation below.

(2,3), (4,3), (6,3), (8,3)

40. Find the domain for the graphed relation.



41. **Challenge Question:** Find the domain of the following graph.



42. How many items are there in the domain of the relation above?

43. What is the smallest item in the domain?

44. What is the biggest value in the domain?

45. How many items are there in the range?

46. What is the smallest item in the range?

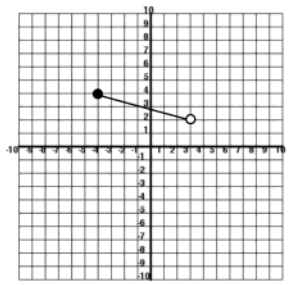
47. What is the biggest item in the range?

Domain & Range of Continuous Data (Lines and Curves): [Definition on page 25]

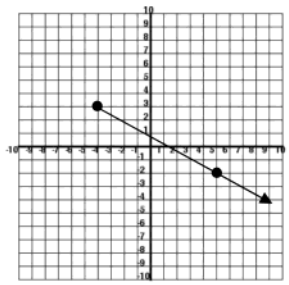
When the graph of a relation is a line or curve, the domain and range cannot be expressed as a list of numbers as in the earlier questions. Why is this so?

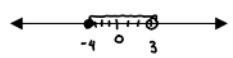
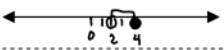
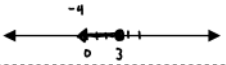
Consider Example A and B.

Example A

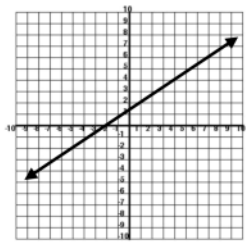


Example B



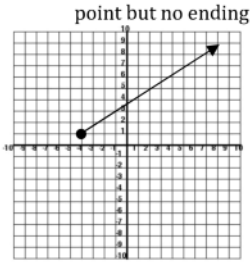
Use Inequalities	Use Interval Notation	Use a number line
Example A Domain: $-4 \leq x < 3$ Range: $2 < y \leq 4$	Example A Domain:$[-4,3)$ Range: $(2,4]$	Example A Domain:  Range: 
Example B Domain: $x \geq -4$ Range: $y \leq 3$	Example B Domain: $[-4, \infty)$ Range: $(\infty, 3]$	Example B Domain:  Range: 
The inequality symbols: $<, >, \leq, \geq, \neq$ **Set Notation:** $\boldsymbol{x \epsilon R}$: The domain is the set of real numbers. $\{\boldsymbol{y} \mid \boldsymbol{y \leq 0, y \epsilon R}\}$: The range is the set of real numbers less than or equal to zero.	Brackets are used to show the interval. **[** if the number is included **(** if the number is not included ∞ is used if the set does not end. $(-\infty, \infty)$: No upper or lower limit, or, "all real numbers". $(3, \infty)$: All real numbers greater than 3.	Solid circles indicate the number is included. Hollow circles indicate the number is not included.

48. If a relation continues in both directions:



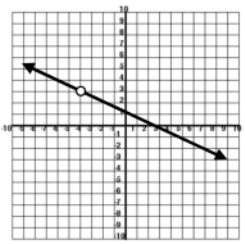
Use Interval Notation:
Domain:
Range:

49. The relation has a starting point but no ending point:



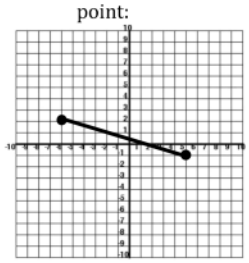
Use Inequalities:
Domain:
Range:

50. The relation has a **non-permissible** value:



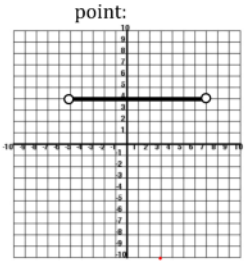
Use Number Lines:
Domain:
Range:

51. The relation has a starting point and a finishing point:



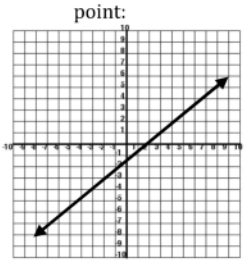
Use words:
Domain:
Range:

52. The relation has a starting point and a finishing point:



Use Inequalities:
Domain:
Range:

53. The relation has no starting point or finishing point:



Use Interval Notation:
Domain:
Range:

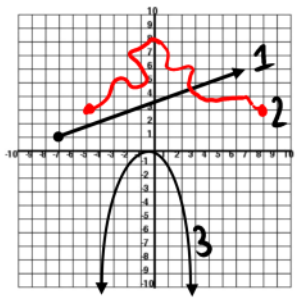
Write a set of instructions for finding the domain of a function in:

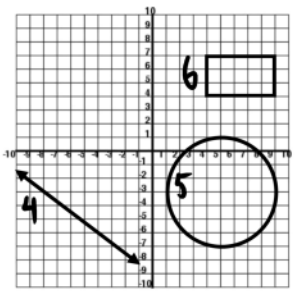
54. Interval Notation:

55. Using a Number Line:

56. Using Inequalities:

57. Try to match each of the following graphs with domain and range below. (There are three on each graph)





A. $x \epsilon R, \quad y \epsilon R$

B. $[1,9]\ and\ [-7,1]$

C. $\{x|x \epsilon R\},\ \{y|y \leq 0, y \epsilon R\}$

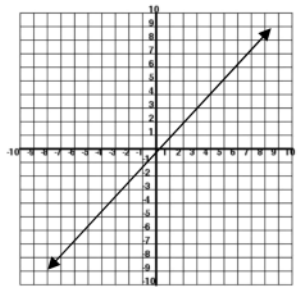
D. $domain[4,9], \quad range[4,7]$

E. $\{x|x \geq -7, x \epsilon R\}, \quad \{y|y \geq 1, y \epsilon R\}$

F. Domain is all real numbers from -5 to 8. Range is all real numbers from 3 to 8.

Find the domain and range for each of the following graphs.

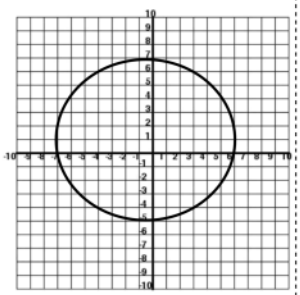
58. Use set notation:



domain_____

range_____

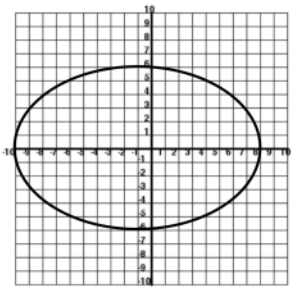
59. Use interval notation:



domain_____

range_____

60. Use number lines:

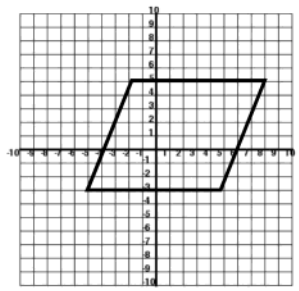


domain_____

range_____

Find the domain and range for each of the following graphs.

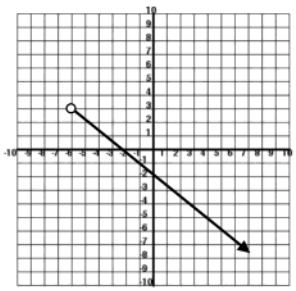
61. Use set notation:



domain_____

range_____

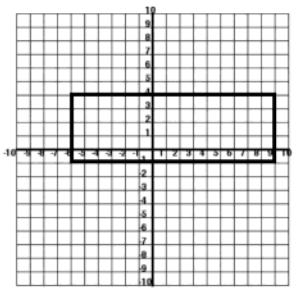
62. Use interval notation:



domain_____

range_____

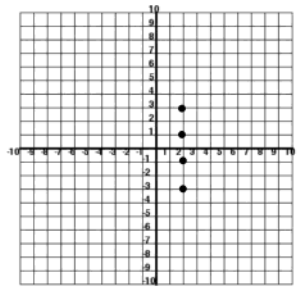
63. Use number lines:



domain_____

range_____

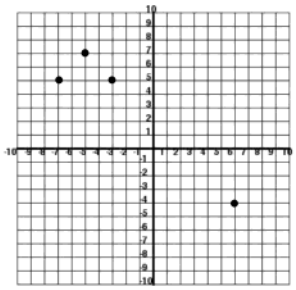
64. Use a list (discrete):



domain_____

range_____

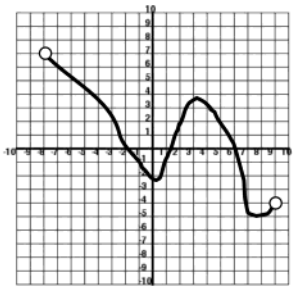
65. Use a list (discrete):



domain_____

range_____

66. Use interval notation:



domain_____

range_____