Lesson #5 - (Greatest Common Factor) Factoring

I. Factoring

Factoring is the reverse of multiplying.

Multiply → Expanding

$$5(x+2) = 5x + 10$$

Factor → Dividing

$$144: 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

II. Factoring a Monomial Common Factor (with the GCF)

1. $$10x - 10 = \frac{10(x-1)}{10}$$ Factorized

2. $$9x^3y^5 - 30x^4y = \frac{3 \cdot 3 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y}{3 \cdot 10 \cdot x \cdot x}$$ GCF: $$3x^2y$$

3. $$8x^3 + 12x^2y - 20x = \frac{4x}{4} (2x^2 + 3xy - 5)$$

4. $$3x + 11 = \text{NO GCF} \Rightarrow \text{no change} = 3x + 11$$

III. Factoring a Binomial Common Factor (with the GCF)

1. $$3x(x+2) + 7(x+2) = (x+2)(3x+7)$$

2. $$6a(a-5) - 11(a-5) = (a-5)(6a-11)$$

3. $$2x(x-3) + 9(x-3) = 2x(x-3) + 9(-x+3)$$

$$= (x-3)(2x-9)$$

Idea: Dividing almost common binomials... but not quite. To make them common x (1) changes + → -
**IV. Factoring by Grouping**

1. \( \frac{1}{x} + 2 \cdot \frac{1}{x} + 3 \cdot \frac{1}{x} + 6 \cdot \frac{1}{x} \)
   \[ \text{GCF: } \frac{1}{x} \]
   \[ \frac{1}{x} \left( x + 2 \right) + 3 \left( \frac{1}{x} + 2 \right) \]
   \[ \frac{1}{x} \left( x + 2 \right) + 3 \left( x + 2 \right) \]
   \[ \frac{1}{x} \left( x + 2 \right) \left( x + 3 \right) \]
   \[ \text{GCF: } (x+2) \]

2. \( 3a + 3b - a - b \)
   \[ \text{GCF: } 1 \]
   \[ 3(a+b) + x(-a-b) \]
   \[ 3(a+b) - x(a+b) \]
   \[ (a+b)(3-x) \]

3. \( 4m^2 - 12m + 15m - 5mt = \)
   \[ \text{GCF: } 4m \]
   \[ \text{GCF: } 5t \]
   \[ 4m(m-3) + 5t(3-m) \]
   \[ x(-1) \text{ to change the signs} \]
   \[ 4m(m-3) - 5t(m-3) \]
   \[ 4m(m-3) - 5t(m-3) \]
   \[ (m-3)(4m-5t) \]

4. \( xy + 10x + 2y + 5x = \)
   \[ \text{GCF: } y \]
   \[ \text{GCF: } 5 \]
   \[ y(x+2) + 5(x+2) \]
   \[ = (x+2)(y+5) \]

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**Homework Assignment #5**

Pages 28-34 Questions #154-186
Factoring:

When a number is written as a product of two other numbers, we say it is factored.

“Factor Fully” means to write as a product of **prime factors**.

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**Eg.1.** Write 15 as a product of its prime factors.

\[ 15 = 5 \times 3 \]

5 and 3 are the prime factors.

**Eg.2.** Write 48 as a product of its prime factors.

\[ 48 = 8 \times 6 \]
\[ = 2^4 \times 3 \]

\[ 48 = 2 \times 2 \times 2 \times 3 \times 2 \]

**Eg.3.** Write 120 as a product of its prime factors.

\[ 120 = 10 \times 12 \]
\[ = 2 \times 5 \times 2 \times 2 \times 3 \]
\[ = 2^3 \times 3 \times 5 \]

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154. Write 18 as a product of its prime factors.

155. Write 144 as a product of its prime factors.

156. Write 64 as a product of its prime factors.

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157. Find the greatest common factor (GCF) of 48 and 120.

Look at each factored form.

\[ 48 = 2^4 \times 3 \]
\[ 120 = 2^3 \times 3 \times 5 \]

Both contain \( 2 \times 2 \times 2 \times 3 \), therefore this is the GCF.

GCF is 24.
We can also write algebraic expressions in factored from.

Eg. 4. Write $36x^2y^3$ as a product of its factors.

$$36x^2y^3 = 9 \times 4 \times x \times x \times y \times y \times y$$
$$36x^2y^3 = 3^2 \times 2^2 \times x^2 \times y^3$$

160. Write $10ax^3b$ as a product of its factors.

161. Write $18ab^4c^3$ as a product of its factors.

162. Write $12b^5c^2$ as a product of its factors.

163. Find the greatest common factor (GCF) of $10a^2b$ and $18ab^2c^3$.

164. Find the greatest common factor (GCF) of $12b^5c^2$ and $18ab^4c^3$.

165. Find the greatest common factor (GCF) of $10a^2b$, $18ab^2c^3$, and $12b^5c^2$. 
Factoring Polynomials:

The process of factoring "undoes" the process of expanding, and vice versa. They are opposites. You must be able to interchange a polynomial between these two forms.

Factoring means *"write as a product of factors."*

The method you use depends on the type of polynomial you are factoring.

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**Challenge Question:**
Write a multiplication that would be equal to $5x + 10$.

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**Challenge Question:**
Write a multiplication that would be equal to $3x^2 + 6x^2 - 12x$.

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The answers to the above questions are called the "FACTORED FORM".
Factoring: Look for a Greatest Common Factor

Hint: Always look for a GCF first.

Ask yourself: "Do all terms have a common integral or variable factor?"

Eg.1. Factor the expression.

\[ 5x + 10 \]

Think...what factor do 5x and 10 have in common?
Both are divisible by 5...that is the GCF.

\[ = 5(x + 2) \]

Write the GCF outside the brackets, remaining factors inside.

Eg.2. Factor the expression

\[ 3ax^3 + 6ax^2 - 12ax \]

GCF = 3ax

\[ = 3ax(x^2 + 2x - 4) \]

You should check your answer by expanding. This will get you back to the original polynomial.

Eg.3. Factor the expression \( 4x + 4 \) using algebra tiles.

I draw the expression as a rectangle using algebra tiles. Find length and width.

\[ 4(x + 1) = 4x + 4 \]
## Factor the following polynomials.

<table>
<thead>
<tr>
<th>Expression</th>
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<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>166. $5x + 25$</td>
<td>167. $4x + 13$</td>
<td>168. $8x + 8$</td>
</tr>
</tbody>
</table>

## Model the expression above using algebra tiles.

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<tr>
<td>169. $4x + 8xy - 6ax$</td>
<td>170. $24w^3 - 6w^3$</td>
<td>171. $3w^2y + 12wxy^2 - 3wy$</td>
</tr>
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<td>172. $27a^2b^2 + 9a^2b^2 - 18a^2b$</td>
<td>173. $6m^2n^2 + 18m^2n^2 - 12mn^2 + 24mn^2$</td>
<td>174. $3w^2y + 12wxy^2 - 3wy$</td>
</tr>
</tbody>
</table>
Factoring a Binomial Common Factor:

The common factor is the term in the brackets!

Eg. 1. Factor. \(4x(w + 1) + 5y(w + 1)\)

\[
4x(w + 1) + 5y(w + 1) = (w + 1)(4x + 5y)
\]

Eg. 2. Factor. \(3x(a + 7) - (a + 7)\)

\[
3x(a + 7) - (a + 7) = (a + 7)(3x - 1)
\]

Sometimes it is easier to understand if we substitute a letter, such as \(d\) where the common binomial is.

Consider Eg. 1.

\[
4x(w + 1) + 5y(w + 1) = 4xd + 5yd
\]

\[
d(4x + 5y)\]

\[
= (w + 1)(4x + 5y)
\]

Substitute \(d\) for \((w + 1)\).

Now replace \((w + 1)\).

Factor the following, if possible.

177. \(5x(a + b) + 3(a + b)\)

178. \(3m(x - 1) + 5(x - 1)\)

179. \(3t(x - y) + (x + y)\)

180. \(4t(m + 7) + (m + 7)\)

181. \(3t(y - x) + (y - x)\)

182. \(4y(p + q) - x(p + q)\)

Challenge Question:

Factor the expression \(ac + bd + ad + bc\).
Factoring: Factor by Grouping.

Sometimes a polynomial with 4 terms but no common factor can be arranged so that grouping the terms into two pairs allows you to factor.

You will use the concept covered above...common binomial factor.

Eq. 1. Factor \( ac + bd + ad + bc \)

\[
\begin{align*}
ac + bd & + ad + bc \\
& = (a + b)(c + d)
\end{align*}
\]

Group terms that have a common factor.

Notice the newly created binomial factor, \((a + b)\).

Factor out the binomial factor.

Eq. 2. Factor \(5m^2t - 10m^2 + t^2 - 2t\)

\[
\begin{align*}
5m^2t - 10m^2 & - t^2 + 2t \\
& = (t - 2)(5m^2 - t)
\end{align*}
\]

Group.

Notice that I factored out a \(-t\) in the second group.

This made the binomials into common factors, \((t - 2)\).

183. \( wx + wy + xz + yz \)

184. \( x^2 + x - xy - y \)

185. \( xy + 12 + 4x + 3y \)

186. \( 2x^2 + 6y + 4x + 3xy \)

187. \( m^2 - 4n + 4m - mn \)

188. \( 3a^2 + 6ab - 9a - 2ab^2 \)