# Lesson 5

November 1, 2018 6:44 PM

Name: ____________________

## Lesson #5 – (Greatest Common Factor) Factoring

### I. Factoring

Factoring is the reverse of multiplying.

GCF Review:

Multiply × → Expanding

5(x+2) = 5x + 10

Factor ÷ ← Dividing.

144 → 12, 12 → 2, 6, 3, 4 → 2, 3, 2, 2

64 → 8, 8 → 2, 4, 2, 4 → 2, 2, 2, 2

GCF: $2^4$

144: $\boxed{2\cdot 2\cdot 2\cdot 2}\cdot 3\cdot 3$

64: $\boxed{2\cdot 2\cdot 2\cdot 2}\cdot 2\cdot 2$

### II. Factoring a Monomial Common Factor (with the GCF)

GCF: what can you divide by?

1. $10x - 10 =$ $10(x-1)$ Factored

$\frac{10x}{10}$ $\frac{10}{10}$

*Quick check by expanding (mentally)

2. $9x^2y^5 - 30x^4y =$ $\boxed{\begin{matrix} 3\cdot 3\cdot x\cdot x\cdot y\cdot y\cdot y\cdot y\cdot y \\ 2\cdot 3\cdot 5\cdot x\cdot x\cdot x\cdot x\cdot y \end{matrix}}$ GCF: $3x^2y(3y^4 - 10x^2)$

3 3 3 10 2 5

*check $9x^2y^5 - 3x^4y$

3. $8x^3 + 12x^2y - 20x =$ $4x(2x^2 + 3xy - 5)$

4x 4x 4x

GCF: 4x

4. $3x + 11 =$ NO GCF ∴ no change $\boxed{= 3x + 11}$

### III. Factoring a Binomial Common Factor (with the GCF)

Binomial is the GCF!

Factor out (x+2)

1. $3x(x+2) + 7(x+2) =$ $(x+2)(3x+7)$

2. $6a(a-5) - 11(a-5) =$ $(a-5)(6a-11)$

3. $2x(x-3) + 9(3-x) =$

almost common binomials... but not quite.

$2x(x-3) + 9(-x+3)$ ×(-1)

To make them common × (-1)

changes + → −, − → +

GCF = (x-3)

$2x(x-3) - 9(x-3)$ $\boxed{= (x-3)(2x-9)}$

Quiz on lessons 1-5.5

## IV. Factoring by Grouping

• 4 terms, with no overall common factor
• rearrange to group terms into pairs that DO have a common factor

⇑ just factor binomials

1. $mx + 2m + 3x + 6 =$

GCF: m  GCF: 3

$m(x+2) + 3(x+2)$  GCF = $(x+2)$

$\boxed{= (x+2)(m+3)}$

2. $3a + 3b - ax - bx =$

GCF = 3  GCF = x

$3(a+b) + x(-a-b)$ ×(-1)

multiply 2nd binomial by (-1) to change the signs to make a common binomial pair

$3(a+b) - x(+a+b)$ ← now factor out $(a+b)$

$\boxed{(a+b)(3-x)}$

3. $4m^2 - 12m + 15t - 5mt =$

GCF: 4m  GCF: 5t

* factor out the GCF from both terms.

$4m(m-3) + 5t(3-m)$  ×(-1) to change ⊕/⊖ signs

$(-m+3)$

$4m(m-3) - 5t(m-3)$

* Factor out the common binomial

$\boxed{= (m-3)(4m-5t)}$

4. $xy + 10 + 2y + 5x =$

$xy + 2y + 10 + 5x$

* Factor out 5 … ÷ 5

GCF: y  GCF: 5

$y(x+2) + 5(2+x)$

$(x+2)$

* factor out the common binomial

$\boxed{= (x+2)(y+5)}$

Homework  ASSIGNMENT # 5
pages 28-34 Questions #154-188

## Factoring:

When a number is written as a product of two other numbers, we say it is factored.

"Factor Fully" means to write as a product of **prime factors**.

Eg.1.
Write 15 as a product of its prime factors.

$$15 = 5 \times 3$$

5 and 3 are the prime factors.

Eg.2.
Write 48 as a product of its prime factors.

$$48 = 8 \times 6$$
$$48 = 2 \times 2 \times 2 \times 3 \times 2$$
$$48 = 2^4 \times 3$$

Eg.3.
Write 120 as a product of its prime factors.

$$120 = 10 \times 12$$
$$120 = 2 \times 5 \times 2 \times 2 \times 3$$
$$120 = 2^3 \times 3 \times 5$$

154. Write 18 as a product of its prime factors.

155. Write 144 as a product of its prime factors.

156. Write 64 as a product of its prime factors.

157. Find the greatest common factor (GCF) of 48 and 120.

Look at each factored form.

$$48 = 2^4 \times 3$$
$$120 = 2^3 \times 3 \times 5$$

Both contain $2 \times 2 \times 2 \times 3$, therefore this is the GCF,

GCF is 24.

158. Find the greatest common factor (GCF) of 144 and 64.

159. Find the greatest common factor (GCF) of 36 and 270.

We can also write algebraic expressions in factored from.

Eg.4. Write $36x^2y^3$ as a product of its factors.

$$36x^2y^3 = 9 \times 4 \times x \times x \times y \times y \times y$$
$$36x^2y^3 = 3^2 \times 2^2 \times x^2 \times y^3$$

160. Write $10a^2b$ as a product of its factors.

161. Write $18ab^2c^3$ as a product of its factors.

162. Write $12b^3c^2$ as a product of its factors.

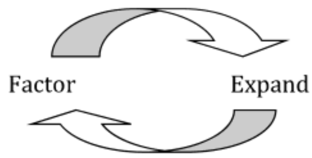
163. Find the greatest common factor (GCF) of $10a^2b$ and $18ab^2c^3$.

164. Find the greatest common factor (GCF) of $12b^3c^2$ and $18ab^2c^3$.

165. Find the greatest common factor (GCF) of $10a^2b$, $18ab^2c^3$, and $12b^3c^2$.

## Factoring Polynomials:




The process of factoring "undoes" the process of expanding, and vice versa.

They are opposites.

You must be able to interchange a polynomial between these two forms.

Factoring means "*write as a product of factors.*"

The method you use depends on the type of polynomial you are factoring.

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Challenge Question:

Write a multiplication that would be equal to $5x + 10$.

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Challenge Question:

Write a multiplication that would be equal to $3x^3 + 6x^2 - 12x$.

The answers to the above questions are called the "FACTORED FORM".

## Factoring: Look for a Greatest Common Factor

Hint: Always look for a GCF first.

Ask yourself: "Do all terms have a common integral or variable factor?"

Eg.1. Factor the expression.

$5x + 10$

Think...what factor do 5x and 10 have in common?
Both are divisible by 5...that is the GCF.

$= 5(x) + 5(2)$ Write each term as a product using the GCF.

$= \mathbf{5(x + 2)}$ Write the GCF outside the brackets, remaining factors inside.

You should check your answer by expanding. This will get you back to the original polynomial.

Eg.2.

Factor the expression

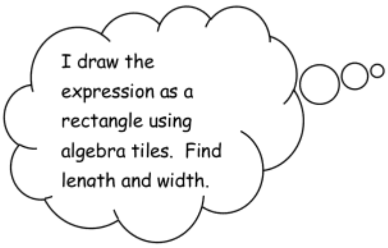
$3ax^3+6ax^2-12ax$

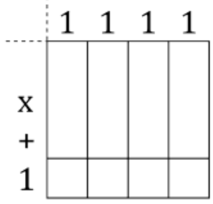
GCF = 3ax

$=3ax(x^2)+3ax(2x)+3ax(-4)$

$=3ax(x^2+2x-4)$

Eg.3. Factor the expression $4x + 4$ using algebra tiles.




$4(x + 1) = 4x + 4$

Factor the following polynomials.

166. $5x + 25$

167. $4x + 13$

168. $8x + 8$

169. Model the expression above using algebra tiles.

170. Model the expression above using algebra tiles.

171. Model the expression above using algebra tiles.

172. $4ax + 8ay - 6az$

173. $24w^5 - 6w^3$

174. $3w^3xy + 12wxy^2 - wxy$

175. $27a^2b^3 + 9a^2b^2 - 18a^3b^2$

176. $6m^3n^2 + 18m^2n^3 - 12mn^2 + 24mn^3$

**Factoring a Binomial Common Factor**: Hint: There are brackets with identical terms.

The common factor **IS** the term in the brackets!

Eg.1. Factor. $4x(w+1)+5y(w+1)$

$$\begin{aligned}&4x(w+1)+5y(w+1)\\&=(w+1)(4x)+(w+1)(5y)\\&=(w+1)(4x+5y)\end{aligned}$$

Eg.2. Factor. $3x(a+7)-(a+7)$

$$\begin{aligned}&3x(a+7)-(a+7)\\&=(a+7)(3x)-(a+7)(1)\\&=(a+7)(3x-1)\end{aligned}$$

Sometimes it is easier to understand if we substitute a letter, such as $d$ where the common binomial is.

Consider Eg.1.
$4x(w+1)+5y(w+1)$ Substitute $d$ for $(w+1)$.
$4xd+5yd$
$d(4x+5y)$ Now replace $(w+1)$.
$=(w+1)(4x+5y)$

Factor the following, if possible.

| | | |
|---|---|---|
| 177. $5x(a+b)+3(a+b)$ | 178. $3m(x-1)+5(x-1)$ | 179. $3t(x-y)+(x+y)$ |
| 180. $4t(m+7)+(m+7)$ | 181. $3t(x-y)+(y-x)$ | 182. $4y(p+q)-x(p+q)$ |

Challenge Question:
Factor the expression $ac+bd+ad+bc$.

**Factoring: *Factor by Grouping*.** Hint: 4 terms!

Sometimes a polynomial with 4 terms but no common factor can be arranged so that grouping the terms into two pairs allows you to factor.

You will use the concept covered above…common binomial factor.

Eg.1. Factor $ac + bd + ad + bc$

| | |
|---|---|
| $ac + bc \quad + ad + bd$ | Group terms that have a common factor. |
| $c(a + b) + d(a + b)$ | Notice the newly created binomial factor, $(a + b)$. |
| $= \boldsymbol{(a + b)(c + d)}$ | Factor out the binomial factor. |

Eg.2. Factor $5m^2t - 10m^2 + t^2 - 2t$

| | |
|---|---|
| $5m^2t - 10m^2 \quad - t^2 + 2t$ | Group. |
| $5m^2(t - 2) - t(t - 2)$ | *Notice that I factored out a $-t$ in the second group. This made the binomials into common factors, $(t - 2)$. |
| $= \boldsymbol{(t - 2)(5m^2 - t)}$ | |

| | | |
|---|---|---|
| 183. $wx + wy + xz + yz$ | 184. $x^2 + x - xy - y$ | 185. $xy + 12 + 4x + 3y$ |
| 186. $2x^2 + 6y + 4x + 3xy$ | 187. $m^2 - 4n + 4m - mn$ | 188. $3a^2 + 6b^2 - 9a - 2ab^2$ |