

Lesson 7

November 4, 2018 8:05 PM

Name: _____

dividing by the GCF "pulling out the GCF"

NOT 1

Lesson #7 - Factoring Trinomials ($ax^2 + bx + c$), where $a \neq 1$

Lesson Focus:

- To use an algebraic method to factor a trinomial of the form $ax^2 + bx + c$, using one of two strategies:
 - Strategy #1: The Decomposition Method / *The box method*
 - Strategy #2: The X-Method (or The Trial & Error Method)

Lesson 6

Review Example: Factor $3x^2 - 9x - 12$ completely.

GCF: 3

$$\begin{aligned}
 &= 3(x^2 - 3x - 4) \\
 &= 3(x+1)(x-4) \\
 &= 3(x+1)(x-4)
 \end{aligned}$$

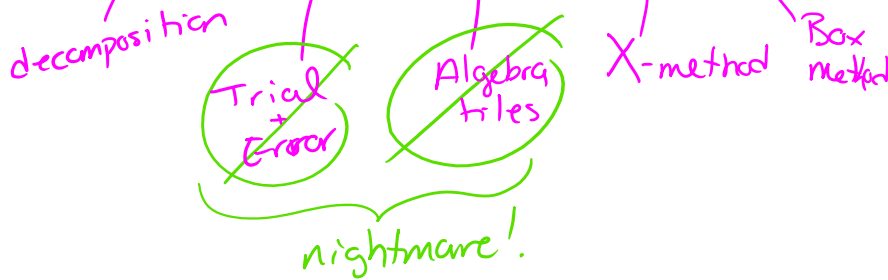
Trinomial where a=1
 $x + 0 = -4$ $1 \cdot -4 = -4$
 $+ + 0 = -3$ $1 + (-4) = -3$ ✓

Note: In this example, after we remove the GCF, the coefficient on the "a" term (the x^2) term is 1.

Best case scenario

What if $a \neq 1$, even after common factoring?? (or if there is no GCF)

(ONLY use these two strategies if $a \neq 1$. If $a = 1$, look back at Lesson #6)



$$ax^2 \pm bxc \pm c \quad \text{where } a \neq 1$$

\uparrow \uparrow
 add to = b want 2 numbers that multiply to = a.c

Strategy #1: The Decomposition Method

Steps	Example:
1. Mentally figure out two numbers that multiply to "ac" and add to "b". -72 $\begin{array}{r} 1 \cdot 72 \\ 2 \cdot 36 \\ -3 \cdot 24 \\ 4 \cdot 18 \\ 6 \cdot 12 \\ \dots \end{array}$ 2. Decompose the middle term (ie the "b" term) using the answer from step #1. (NOTE: The order that you list the decomposed middle terms doesn't matter) $9x^2 + 24x - 8$ $9x^2 + 24x - 3x - 8$ $9x^2 - 3x + 24x - 8$ $3x(3x-1) + 8(3x-1)$ $(3x-1)(3x+8)$ 3. Now you have four terms, so let's factor by grouping! or... the Box! method! answer \Rightarrow	$a \neq 1$ $a = 9$ $a.c = 9 \cdot (-8) = -72$ $b = +24$ \Rightarrow need 2 numbers that multiply to = -72 and add to = $+24$ $(-3) \cdot 24 = -72$ $(-3) + 24 = +24$ ✓ $9x^2 + 24x - 8$ $9x^2 + 24x - 3x - 8$ (decompose "b" term) $9x^2 - 3x + 24x - 8$ $3x(3x-1) + 8(3x-1)$ * Factoring a common binomial $(3x-1)(3x+8)$ $(3x-1)(3x+8) \neq E$ $9x^2 + 24x - 3x - 8$ $\sqrt{9x^2 + 21x - 8}$ original trinomial
4. Check your answer using FOIL CHECK: by expanding	$(3x-1)(3x+8)$ $\begin{array}{c c} 3x & 8 \\ \hline 3x & 9x^2 + 24x \\ -1 & -3x - 8 \end{array}$ grid is the area = answer ... solve for side lengths

1 of the numbers must be -

or... the Box! method!

answer \Rightarrow

CHECK: by expanding

$$9x^2 + 24x - 3x - 8$$

$$(3x-1)(3x+8)$$

	$3x$	8
$3x$	$9x^2$	$24x$
-1	$-3x$	-8

- 1st term top left.
- last term bottom right.
- Fill in x terms

grid is the area = answer ... solve for side lengths

Strategy #2: The X Method (or The Trial & Error Method)

Steps	Example: Factor $2x^2 + 3x - 2$
<p>1. Draw a large X under the trinomial, leaving one line of space in between.</p> <p>2. On the LHS of the X, write two numbers that multiply to "a" (ie. two factors of "a")</p> <p>3. On the RHS of the X, write two numbers that multiply to "c" (ie. two factors of "c")</p> <p>4. Cross multiply, and check to see if the two numbers can add to "b". Keep trying new combinations of numbers until you find the "winning" numbers. Put "+" or "-" signs on the RIGHT HAND SIDE ONLY. Put the variable on the LEFT HAND SIDE ONLY.</p> <p>5. Write the numbers in the X as factors. The top two numbers form one factor. The bottom two numbers form the other factor.</p> <p>6. Check your answer using FOIL</p>	<p><i>not a good method.</i></p>

$a \neq 1$

Example. Factor the following completely, using one of the two strategies

a) $6x^2 + 5x - 6$

* $6 \cdot (-6) = -36$

1	36
2	18
3	12
4	9

$-4 \cdot 9 = -36$
 $-4 + 9 = +5$

$6x^2 + 9x - 4x - 6$

$= (2x+3)(3x-2)$

$3x$	$6x^2$	$9x$
-2	$-4x$	-6

b) $2x^2 + 5x + 2$

$2 \cdot 2 = 4$

1	4
2	2

add to $= 5$

$1 \cdot 4 = 4$
 $1 + 4 = 5$

decompose

$2x^2 + 5x + 2$

$2x^2 + 1x + 4x + 2$

$= (x+2)(2x+1)$

$2x$	$2x^2$	$4x$
$+1$	$1x$	2

$2x^2 + 1x + 4x + 2$

$x(2x+1) + 2(2x+1)$

$(2x+1)(x+2)$

Final Thoughts on Trinomial Factoring:

- Only 2 methods have been outlined in this section. There are even more, but these are the ones I like! You may have learned an alternative method last year, in fact.
- Every teacher has their preferred method.
- Every student has their preferred method.
- YOU MAY CHOOSE WHICHEVER METHOD YOU WISH. YOU ONLY NEED TO KNOW ONE METHOD. PICK ONE AND MASTER IT!

Do not recycle the Polynomials notes! It is absolutely imperative that you remember how to factor next year and years to come. You will not be taught again, but you will be expected to know how to do it. *I wouldn't recycle any of Math 10, if I were you, but especially not Chapter 3.

Homework

ASSIGNMENT # 7
 pages 39-42 Questions #217-234

Factoring $ax^2 + bx + c$ where $a \neq 1$

When the trinomial has an x^2 term with a coefficient other than 1 on the x^2 term, you cannot use the same method as you did when the coefficient is 1.

We will discuss 3 other methods:

1. Trial & Error
2. Decomposition
3. Algebra Tiles

Trial & Error:

Eg.1. Factor $2x^2 + 5x + 3$.

$$2x^2 + 5x + 3 = (\quad)(\quad)$$

We know the first terms in the brackets have product of $2x^2$

$$2x^2 + 5x + 3 = (2x \quad)(x \quad)$$

$2x$ and x have a product of $2x^2$, place them at front of brackets.

The product of the second terms is 3. (1, 3 or -1, -3).
These will fill in the second part of the binomials.

List the possible combinations of factors.

$$\begin{aligned} &(2x + 1)(x + 3) \\ &(2x + 3)(x + 1) \\ &(2x - 1)(x - 3) \\ &(2x - 3)(x - 1) \end{aligned}$$

IF $2x^2 + 5x + 3$ is factorable, one of these must be the solution.

Expand each until you find the right one.

$$(2x + 3)(x + 1) = 2x^2 + 2x + 3x + 3 = 2x^2 + 5x + 3. \quad \text{This is the factored form.}$$

Decomposition:

Using this method, you will break apart the middle term in the trinomial, then factor by grouping.

To factor $ax^2 + bx + c$, look for two numbers with a product of ac and a sum of b .

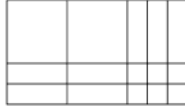
Eg.1. Factor. $3x^2 - 10x + 8$

1. We see that $ac = 3 \times 8 = 24$; and $b = -10$
We need two numbers with a product of 24, but add to -10...
-6 and -4.

$$\begin{aligned} &3x^2 - 6x - 4x + 8 && 2. \text{ Break apart the middle term.} \\ &3x(x - 2) - 4(x - 2) && 3. \text{ Factor by grouping.} \\ &= (x - 2)(3x - 4) \end{aligned}$$

Eg.2. Factor. $3a^2 - 22a + 7$ We need numbers that multiply to 21, but add to -22...
 $3a^2 - 21a - 1a + 7$ **-21 and -1**
 $3a(a - 7) - 1(a - 7)$ Decompose middle term.
 Factor by grouping.
 $= (a - 7)(3a - 1)$

Eg.3. Factor $2x^2 + 7x + 6$ using algebra tiles.



Arrange the tiles into a rectangle (notice the "ones" are again grouped together at the corner of the x^2 tiles)

Side lengths are $(2x + 3)$ and $(x + 2)$ $\therefore 2x^2 + 7x + 6 = (2x + 3)(x + 2)$

Your notes here...

Factor the following if possible.

217. $2a^2 + 11a + 12$

218. $5a^2 - 7a + 2$

219. $3x^2 - 11x + 6$

Factor the following if possible.

$$220. 2y^2 + 9y + 9$$

$$221. 5y^2 - 14y - 3$$

$$222. 10x^2 - 17x + 3$$

$$223. 2x^2 + 3x + 1$$

$$224. 6k^2 - 5k - 4$$

$$225. 6y^2 + 11y + 3$$

$$226. 3x^2 - 16x - 12$$

$$227. 3x^3 - 5x^2 - 2x$$

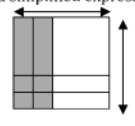
$$228. 9x^2 + 15x + 4$$

Factor the following if possible.

$229. 21x^2 + 37x + 12$	$230. 6x^3 - 15x - x^2$	$231. 4t + 10t^2 - 6$
$232. 3x^2 - 22xy + 7y^2$	$233. 4c^2 - 4cd + d^2$	$234. 2x^4 + 7x^2 + 6$

Challenge Question

Write a simplified expression for the following diagram of algebra tiles.



What two binomials are being multiplied in the diagram above?

Write an equation using the binomials above and the simplified product.