Lesson 8

November 4, 2018 8:06 PM

Lesson #8 – Factoring Special Polynomials

Lesson Focus:

- To learn the shortcut for expanding $(a \pm b)^2$
- To learn to identify and factor the following special polynomials: Perfect Square Trinomials and Difference of Squares Binomials

Expanding $(a \pm b)^2$

$(a+b)^2$	$(a-b)^2$
$=(a+b)(a+b)$ $a^2 + \underbrace{ab + ba} + b^2$ $= a^2 + 2ab + b^2$	$=(a-b)(a-b)$ $a^2 \underbrace{- ab - ba} + b^2$ $= a^2 - 2ab + b^2$

Note: A polynomial of the form $(a \pm b)^2$ is called a Perfect Square Trinomial.

• 1st term + last term are perfect squares.

Example: Expand the following polynomials.

a) $(5x - 2)^2$ (a = 5x, b = 2)

$(5x-2)(5x-2)$

$25x^2 \underbrace{-10x - 10x} + 4$

$\underset{ax^2}{25x^2} \underset{2 \cdot ab}{- 20x} \underset{b^2}{+4}$

b) $(6x + 7)^2$ (a = 6, b = 7)

$(6x+7)(6x+7)$

$36x^2 + \underbrace{42x + 42x} + 49$

$\underset{ax^2}{36x^2} + \underset{2 \cdot ab}{84x} + \underset{b^2}{49}$

$36x^2 + 84x + 49$ match our prediction

Factoring Perfect Square Trinomials

- All perfect square trinomials (PSTs) can be factored into: $(a \pm b)^2$
- Therefore, if you can identify one, you can skip ALL FACTORING STEPS and go straight to the final answer. NO WORK NEED BE SHOWN.

Algebraically, we can spot one by first noticing the following: $\overbrace{ax^2 + bx + c}^{\text{Trinomial}}$

(1) a and c are both perfect squares

(2) check if the middle term $= 2 \cdot \sqrt{a}\sqrt{c}$

Note: "a" and "c" MUST be positive for the polynomial to be a perfect square trinomial. **WHY?**

when you square any number = (+)

$5^2 = 25$

$(-5)^2 = 25$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

recognize this pattern to save alot of time!

Example. Factor the following completely. Check your answer by expanding (FOIL).

a) $x^2 - 6x + 9$

$= x$ $= 3$

$(x-3)^2$

b) $121d^2 + 66d + 9$

$= 11d$ $= 3$

$(11d + 3)^2$

Factoring Difference of Squares Binomials

subtract 2 terms

- 'a' and 'b' terms are both perfect squares
- always have a $\ominus$ sign.

A difference of squares binomial is a binomial in the form $a^2 - b^2$.
For example: $x^2 - 81$

Some other examples:

$(x^2 - 64)$, $(4y^2 - 16)$, $(w^6 - 1)$

Note: It must be a DIFFERENCE (-) NOT a sum (+).

$(10x - 10) = 10(x-1)$

The ONLY ways to factor a binomial are:
1. Common Factor (Remove the GCF)
2. Difference of Squares

Difference of Squares Rule

$$a^2 - b^2 = (a - b)(a + b)$$

$a^2 + ab - ab - b^2$

0

$a^2 - b^2$

Example: Factor $x^2 - 81$ completely.

$= x$ $= 9$

$(x+9)(x-9)$

Don't forget the Golden Rule of factoring! ***The first step of any factoring process is to ALWAYS…***

Find the GFC and factor it out!

$(2a^2 - 2b^2)$

$2(a^2 - b^2)$

Let's Play….. *SPOT THE DIFFERENCE OF SQUARES!!!!!*

Circle the numbers of questions that are differences of squares. If it isn't, circle the part of the binomial that is preventing it from being a difference of squares. Lastly, factor each question.

1. $x^2 - 4y^2$

   $(x+2y)(x-2y)$

2. $49x^3 - 16$

   not a perfect square

3. $25n^2 + 100$

4. $18k^2 - 98$ GCF: 2

   $2(9k^2 - 49)$ diff. of sq.

   $\sqrt{9k^2}$ $\sqrt{49}$

   $2(3k+7)(3k-7)$

5. $\frac{x^4}{9} - y^6$

   $\frac{x^2}{3} \cdot \frac{x^2}{3}$ $y^3 \cdot y^3 = y^6$

   exponent must be even

   $\left(\frac{x^2}{3} + y^3\right)\left(\frac{x^2}{3} - y^3\right)$

6. $x^4 - 16$

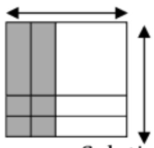
   $(x^2+4)(x^2-4)$ Factor Fully

   $(x^2+4)(x+2)(x-2)$

ASSIGNMENT # 8
pages 43-47 Questions #235-260

A Difference of Squares

235. Write a simplified expression for the following diagram.



Solution: $x^2 - 2x + 2x - 4$

What two binomials are being multiplied in the diagram above?

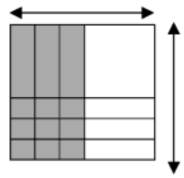
$(x-2)(x+2)$

Write an equation using the binomials above and the simplified product.

$x^2 - 4 = (x-2)(x+2)$

Factored Form

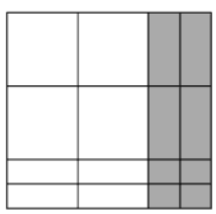
236. Write a simplified expression for the following diagram.



What two binomials are being multiplied above?

Write an equation using the binomials above and the simplified product.

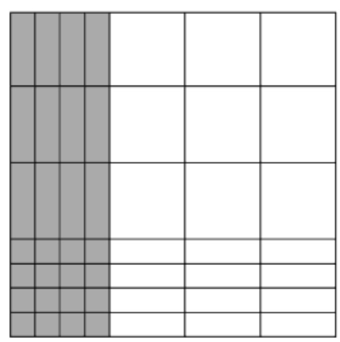
237. Write a simplified expression for the following diagram.



What two binomials are being multiplied above?

Factor the polynomial represented above by writing the binomials as a product (multiplication).

238. Write a simplified expression for the following diagram.



What two binomials are being multiplied above?

Factor the polynomial represented above by writing the binomials as a product (multiplication).

Factoring a Difference of Squares: $a^2 - b^2$

Conjugates: Sum of two terms and a difference of two terms.

Learn the pattern that exists for multiplying conjugates.

$(x+2)(x-2) = x^2 - 2x + 2x - 4 = x^2 - 4$ The two middle terms cancel each other out.

We can use this knowledge to quickly factor polynomials that look like $x^2 - 4$.

Eg.1. Factor $x^2 - 9$.

$= (x+3)(x-3)$ Square root each term, place them in 2 brackets with opposite signs (+ and -).

Eg.2. Factor $100a^2 - 81b^2$

$= (10a + 9b)(10a - 9b)$ Square root each term, place them in 2 brackets with opposite signs (+ and -).

Factor the following completely.

239. $a^2 - 25$	240. $x^2 - 144$	241. $1 - c^2$

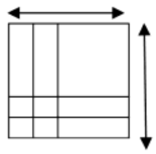
I recognize a polynomial is a difference of squares because____

Factor the following completely.

242. $4x^2 - 36$	243. $9x^2 - y^2$	244. $25a^4 - 36$
245. $49t^2 - 36u^2$	246. $7x^2 - 28y^2$	247. $-18a^2 + 2b^2$
248. $-9 + d^4$	249. $\frac{a^2}{9} - \frac{b^2}{16}$	250. $\frac{x^2y^2}{49} - 1$

Factoring a Perfect Square Trinomial

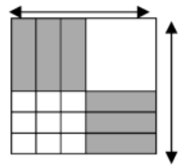
251. Write an expression for the following diagram (do not simplify):



What two binomials are being multiplied above?

Write an equation using the binomials above and the simplified product.

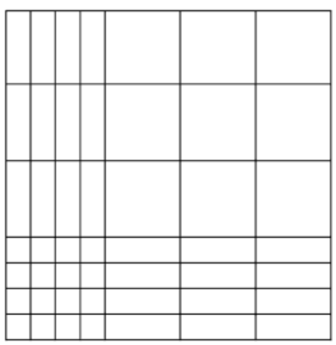
252. Write an expression for the following diagram (do not simplify):



What two binomials are being multiplied above?

Write an equation using the binomials above and the simplified product.

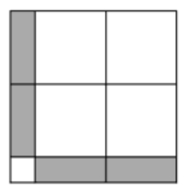
253. Write an expression for the following diagram (do not simplify):



What two binomials are being multiplied above?

Write an equation using the binomials above and the simplified product.

254. Write an expression for the following diagram (do not simplify):



What two binomials are being multiplied above?

Write an equation using the binomials above and the simplified product.

PERFECT SQUARE TRINOMIALS

You may use the methods for factoring trinomials to factor ***trinomial squares*** but recognizing them could make factoring them quicker and easier.

Eg.1. Factor.

$x^2 + 6x + 9$ Recognize that the first and last terms are both perfect squares.

$(x + 3)^2$ Guess by taking the square root of the first and last terms and put them in two sets of brackets.

Check: Does $2(x)(3) = 6x$
Yes! Trinomial Square!

In a trinomial square, the middle term will be double the product of the square root of first and last terms. Wow, that's a mouthful!

$(x + 3)^2$ Answer in simplest form.

Eg.2. Factor.

$121m^2 - 22m + 1$

$(11m - 1)^2$ Guess & Check. $2(11m \times -1) = -22m$.
Since the middle term is negative, binomial answer will be a subtraction.

Factor the following.

255. $x^2 + 14x + 49$	256. $4x^2 - 4x + 1$	257. $9b^2 - 24b + 16$
258. $64m^2 - 32m + 4$	259. $81n^2 + 90n + 25$	260. $81x^2 - 144xy + 64y^2$