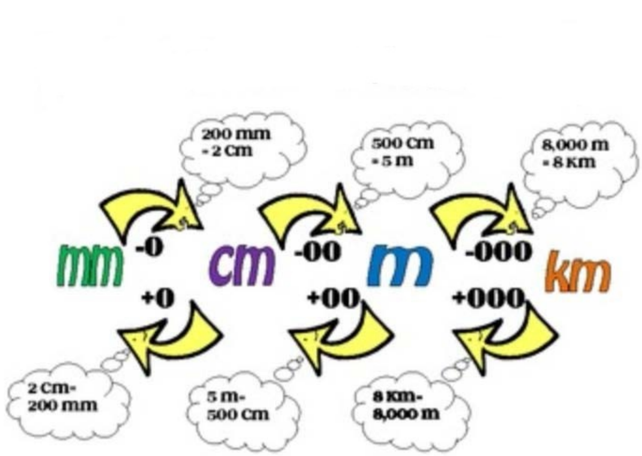
Chemistry 11

Measurement 2: Unit Conversions & Scientific Notation



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Unit Converstions

A ______________________________ is a *fraction* or factor written so that the denominator and numerator are *equivalent values* with *different units*.

One of the most useful conversion factors allows the user to convert from the __________________ to the ____________________________________ and vice versa.
Since 1 inch is exactly the same length as 2.54 cm, the factor may be expressed as:

____

These two lengths are identical so multiplication of a given length by the conversion factor will not change the length. It will simply express it in a different unit.

Now if you wish to determine how many centimetres are in a yard, you have two things to consider.

First, which of the two forms of the conversion factor will allow you to __________________ the imperial unit, converting it to a metric unit?

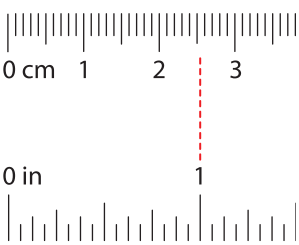


Figure 1.4.2 *A ruler with both imperial and metric scales shows that 1 inch = 2.54 cm.*

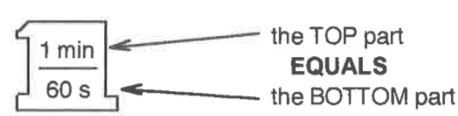
Second, what other conversion factors will you need to complete the task?
Assuming you know, or can access, these equivalencies:
1 yard = ____feet and 1 foot = _____ inches
...your approach would be as follows:

Notice that as with the multiplication of any fractions, it is possible to ____________ ____.
We've simply followed a numerator-to-denominator pattern to convert yards to feet to inches to cm.

The number of feet in a yard and inches in a foot are _____________________ *values*. They are not things we measured. Thus they _____________________ affect the number of significant figures in our answer.

This will be the case for any ____ in which the numerator and denominator are in the same system (both metric or both imperial).
As all three of the conversion factors we used are ____________________________, only the original value of 1.00 yards influences the significant figures in our answer. Hence we round the answer to three sig figs.

Example: How many minutes are there in 3480 seconds?



Both 60 s and 1 min are the same length of time. "Equal to" , this is the converstion factor.
Multiplying by the converstion facor did not change the VALUE fo the time.
However, the units are different after using the conversion factor:
we started with a <u>LARGE</u> number of ***small units*** and ended up with a ***small number*** of <u>LARGE</u> units.

The method of unit conversions uses conversion factors to change the units associated with an expression to a different set of units.

Every unit conversion problem has three major pieces of information which must be identified:

i) the unknown amount and its UNITS,
ii) the initial amount and its UNITS, and
iii) a conversion factor which relates or connects the initial UNITS to the UNITS of the unknown.

INCREDIBLY, VITALLY IMPORTANT NOTE!

In all the calculations which follow you must **ALWAYS** include the units, for they are the "major players" in the calculation. If you are tempted to omit or "forget about" the units, DON'T! The course you fail could be Chem 11!

Example: If a car can go 80 km in 1 h, how far can the car go in 8.5 h ?

CONVERSION STATEMENT   UNKNOWN AMOUNT   INITIAL AMOUNT

Example: If 0.200 mL of gold has a mass of 3.86 g, what is the mass of 5.00 mL of gold ?

CONVERSION STATEMENT   UNKNOWN AMOUNT   INITIAL AMOUNT

Example: If 0.200 mL of gold has a mass of 3.86 g, what is the volume occupied by 100.0 g of gold ?

CONVERSION STATEMENT   UNKNOWN AMOUNT   INITIAL AMOUNT

EXERCISE: *Show **FULL WORKING OUT** on **THIS PAGE** in the space provided below.*

2. Solve the following using the method of unit conversions.

   a) If there are 6.02×10^{23} atoms in 1 mol of atoms, how many atoms are there in 5.5 mol of atoms?

   b) If one mole of a gas has a volume of 22.4 L, how many moles are there in 25.0 L of gas?

   c) If one mole of nitrogen has a mass of 28 g, how many moles of nitrogen gas are in 7.0 g of nitrogen gas?

   d) How many seconds must an electrical current of 35 coulombs/s flow in order to deliver 200.0 coulombs?

   e) A quiet sound exerts a pressure of 4×10^{-8} kPa ("kPa" = kilopascals, an SI pressure unit). What is this pressure in atmospheres if 1 atmosphere is 101.3 kPa?

   f) A large nugget of naturally occurring silver metal has a mass of 3.20×10^{4} troy ounces. What is the mass in kilograms if 1 troy ounce is equivalent to 0.0311 kg?

   g) A reaction is essentially complete in 5.0×10^{-4} s. If one millisecond (1 ms) equals 10^{-3} s, how many milliseconds does the reaction take?

   h) If 1 mol of octane produces 5450 kJ of heat when burned, how many moles of octane must be burned to produce 15 100 kJ of heat?

   i) Our fingers can detect a movement of 0.05 micron. If 1 micron is 10^{-3} mm, what is this movement expressed in millimetres (mm)?

   j) If concentrated hydrochloric acid has a concentration of 11.7 mol/L, what volume of hydrochloric acid is required in order to have 0.0358 mol of hydrochloric acid?

Multiple Unit Converstions

What happens when there is **more than one** converstion factor involved in a problem?

REMEMBER: your conversion factor must include a fraction where the numerator (top) and denominator (bottom) are ***equivalent values*** with ***different units***.

Example: If eggs are $1.44/doz and if there are 12 eggs/doz, how many individual eggs can be bought for $4.32?

UNKNOWN AMOUNT: INITIAL AMOUNT: CONVERSION FACTORS:

OVERALL CONVERSTION REQUIRED:

Example: The gas tank of a Canadian tourist holds 39.4 L of gas. If 1 L is equal to 0.264 gal in the US, and gas is $1.26/gal, how much will it cost to fill up south of the border?

UNKNOWN AMOUNT: INITIAL AMOUNT: CONVERSTION FACTORS:

OVERALL CONVERSTION REQUIRED:

EXERCISES: *Show **FULL WORKING OUT** on **THIS PAGE** in the space provided below.*

3. An old barometer hanging on the wall of a mountain hut has a reading of 27.0 inches of mercury. If 1 inch of mercury equals 0.0334 atm ("atmospheres") and 1 atm = 101.3 kPa ("kilopascals"), what is the pressure reading of the barometer, in kilopascals?
4. It requires 334 kJ of heat to melt 1 kg of ice.
   (a) The largest known iceberg had a volume of about 3.1×10^{13} m^3. How much heat was required to melt the iceberg if 1 m^3 of ice has a mass of 917 kg?
   (b) The explosive "TNT" releases 1.51×10^4 kJ of energy for every kilogram of TNT which explodes. Provided that all the energy of an explosion went into melting the ice, how many kilograms of TNT would be needed to melt the iceberg in part (a) of this question?

*Show **FULL WORKING OUT** on **THIS PAGE** in the space provided below.*

5. Sugar costs \$0.980/kg. 1 t = 1000 kg. How many tonnes ("t") of sugar can you buy for \$350?
6. The Cullinan diamond, the largest diamond ever found, had an uncut volume of 177 mL. If 1 mL of diamond has a mass of 3.51 g and 1 carat = 0.200 g, how many carats was the Cullinan diamond?

7. How many kilometres ("km") will a car travelling at 120 km/h go in: (a) 0.25 h? (b) 12 min?
8. Solve the following, using the fact that beakers cost \$8.40 per dozen.
   (a) Harry drops 3 dozen beakers. How much will the Chemistry teacher charge Harry?
   (b) Harry drops another 5 dozen beakers (clumsy!). If Burger Bob's hamburgers cost \$1.50 each, how many hamburgers could clumsy Harry have bought for the same amount of money as he has to pay for the second batch of beakers?
   (c) Harry does not learn very quickly, and breaks a third batch of beakers. If he has to pay \$13.30, what is the number of beakers he breaks the third time? (Express your answer in actual numbers of beakers, rather than in "dozens of beakers".)

Converting Within the Metric System

Measures	Unit Name	Symbol
length	metre	m
mass	gram	g
volume	litre	L
time	second	s

The metric system is based on powers of___________.
The power of 10 is indicated by a simple _______________. Table 1.4.1 is a list of SI prefixes.
You will need to memorize from "nano" 10^{-9} to "giga" 10^{9}.

You should highlight these.

Metric conversions require either one or two steps. You will recognize a one-step metric conversion by the presence of a __________________________ in the question.

The common base units in the metric system include: m, g, L and s.

SOME IMPORTANT EQUIVALENCES

$1\ mL = 1\ cm^3$
$1\ m^3 = 10^3\ L$
$1\ t = 10^3\ kg$

Example: re-write 5 kilograms using PREFIX and UNIT SYMBOLS and the correct EXPONENTIAL EQUIVALENT

Table 1.4.1 *SI Prefixes*

Prefix	Symbol	10^n
yotta	Y	10^{24}
zetta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deca	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	µ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}

Example: re-write 2.7×10^{-2} m using WRITTEN PREFIX and UNIT and the correct PREFIX SYMBOL

EXERCISES:

11. Re–write the following using PREFIX and UNIT SYMBOLS, and EXPONENTIAL EQUIVALENTS.
(a) 2.5 centimetres (b) 1.3 kilograms (c) 25.2 millimoles (d) 5.1 decigrams (e) 0.25 megalitres (f) 6.38 micrograms

a)____________ c)____________ e)____________
b)____________ d)____________ f)____________

12. Re–write the following using WRITTEN PREFIXES and UNITS, and EXPONENTIAL EQUIVALENTS.
(a) 2.5 mm (b) 6.5 dL (c) 1.9 kmol (d) 4 Mt (e) 9.94 cg (f) 1.25 µs

a)____________ c)____________ e)____________
b)____________ d)____________ f)____________

13. Re–write the following using PREFIX SYMBOLS, and WRITTEN PREFIXES and UNITS.
(a) 4.5×10^{-3} mol (b) 1.6×10^{3} m (c) 0.50×10^{-6} L (d) 2.68×10^{-1} g (e) 8.85×10^{6} t (f) 7.25×10^{-2} m

a)____________ c)____________ e)____________
b)____________ d)____________ f)____________

One & Two-Step Converstions

One step metric conversions involve ____________________(metres, litres, grams, or seconds) being converted to a ____________________ or a prefixed unit being converted to a base unit.

Metric conversions involve using unit conversions between prefix symbols and exponential equivalents.

EXAMPLES: (a) Write a conversion statement between **cm** and **m.**

Since "c" stands for "10^{-2}" then $1\ \text{cm} = 10^{-2}\ \text{m}$.

(b) Write a conversion statement between **ms** and **s.**

Since "m" stands for "10^{-3}" then $1\ \text{ms} = 10^{-3}\ \text{s}$.

Sample Problems — One-Step Metric Conversions

1. Convert 9.4 nm into m.

What to Think about	How to Do It
1. In any metric conversion, you must decide whether you need one step or two. There is a base unit in the question and only one prefix. This problem requires only one step. Set the units up to convert nm into m. Let the units lead you through the problem. You are given 9.4 nm, so the conversion factor must have nm in the denominator so it will cancel.	
2. Now determine the value of nano and fill it in appropriately. **1** nm = **10^{–9}** m Give the answer with the appropriate number of significant figures and the correct unit. Because the conversion factor is a defined equality, only the given value affects the number of sig figs in the answer.	

EXERCISE: *Show **FULL WORKING OUT** on **THIS PAGE** in the space provided below.*

15. Write conversion statements between each of the following.

(a) kg and g	(d) dm and m	(g) kL and L	(j) cL and L
(b) Mm and m	(e) cs and s	(h) μs and s	(k) dmol and mol
(c) μL and L	(f) mmol and mol	(i) Mg and g	(l) mg and g

2STEP CONVERSIONS

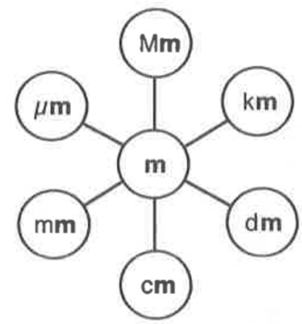
Two-step metric conversions require the use of

____.

Two factors will be required any time there are ___________________________________ in the question.

In a two-step metric conversion, you must ***always***____.

Example: How many micrometres are there in 5 cm ?



Sample Problems — Two-Step Metric Conversions

1. Convert 6.32 μm into km.

What to Think about

1. This problem presents with two prefixes so there must be two steps.
   The first step in such a problem is always to convert to the base unit. Set up the units to convert from μm to m and then to km.
2. Insert the values for 1 μm and 1 km.
   1 μm = 10^{-6} m
   1 km = 10^{3} m
3. Give the answer with the correct number of significant figures and the correct unit.

How to Do It

Practice Problems — One- and Two-Step Metric Conversions

1. Convert 16 s into ks.
2. Convert 75 000 mL into L.
3. Convert 457 ks into ms.
4. Convert 5.6×10^{-4} Mm into dm.

EXERCISES: *Show* ***FULL WORKING OUT*** *on* ***THIS PAGE*** *in the space provided below.*

16. (a) If 1 mg = 10^{-3} g and 1 Mg = 10^{6} g, how many milligrams are there in 0.25 Mg?
    (b) If 1 μs = 10^{-6} s and 1 cs = 10^{-2} s, how many centiseconds are there in 10 μs?
    (c) If 1 mm = 10^{-3} m and 1 cm = 10^{-2} m, how many millimetres are there in 15.8 cm?
    (d) If 1 kg = 10^{3} g and 1 mg = 10^{-3} g, how many kilograms are there in 250 mg?
    (e) If 1 dL = 10^{-1} L and 1 kL = 10^{3} L, how many decilitres are there in 0.5 kL?

17. Convert the following
    (a) 3 s into milliseconds
    (b) 50.0 mL into litres
    (c) 2 L into microlitres
    (d) 25 kg into grams
    (e) 3 Mm into metres
    (f) 2 L into decilitres
    (g) 7 μs into milliseconds
    (h) 51 kg into milligrams
    (i) 3125 μL into kilolitres
    (j) 1.7 μg into centigrams

Derived Unit Conversions

A **derived unit** is

Units like those used to express rate (km/h) or density (g/mL) are good examples of derived units.

EXAMPLE: The heat change occurring when the temperature of a water sample increases is given by

$$\Delta H = c \bullet m \bullet \Delta T$$

Therefore, **c**, is a ______________________________, having derived units, found by combinding three other quantities (____) and their units.

EXERCISE: *Show* ***FULL WORKING OUT*** *on* ***THIS PAGE*** *in the space provided below.*

29. Find the derived value and units for

(a) the molar concentration, $\boldsymbol{c}$, using the equation $\boldsymbol{c} = \dfrac{\boldsymbol{n}}{\boldsymbol{V}}$,

where: $\boldsymbol{n} = 0.250$ mol and $\boldsymbol{V} = 0.500$ L.

(b) the Universal Gas Constant, $\boldsymbol{R}$, using the equation $\boldsymbol{R} = \dfrac{\boldsymbol{P} \bullet \boldsymbol{V}}{\boldsymbol{n} \bullet \boldsymbol{T}}$,

i) where $\boldsymbol{P} = 1$ atm, $\boldsymbol{V} = 22.4$ L, $\boldsymbol{n} = 1$ mol and $\boldsymbol{T} = 273$ K (K is the temperature on the Kelvin scale.

ii) where $\boldsymbol{P} = 202.6$ kPa, $\boldsymbol{V} = 24.45$ L, $\boldsymbol{n} = 2$ mol and $\boldsymbol{T} = 298$ K.

(c) the entropy change for the boiling of water , $\Delta \boldsymbol{S}$, using the equation $\Delta \boldsymbol{H} = \boldsymbol{T} \bullet \Delta \boldsymbol{S}$,
where: $\Delta \boldsymbol{H} = 44.0$ kJ and $\boldsymbol{T} = 373$ K. (Hint: you will have to rearrange the equation first.)

(d) the kinetic energy of hydrogen gas at 0^{o}C, $\boldsymbol{KE}$, using the equation $\boldsymbol{KE} = \frac{1}{2}\, \boldsymbol{m} \bullet \boldsymbol{v}^2$,

where: $\boldsymbol{m} = 3.35 \times 10^{-27}$ kg and $\boldsymbol{v} = 1692\ \dfrac{\text{m}}{\text{s}}$.

Derived unit conversions require ____from numerator to denominator as usual AND from denominator to numerator).

Example: Express 5 Mg/mL in kolograms/litre

Sample Problem — Derived Unit Conversions

Convert 55.0 km/h into m/s

What to Think about

1. The numerator requires conversion of a prefixed metric unit to a base metric unit. This portion involves one step only and is similar to sample problem one above.

2. The denominator involves a time conversion from hours to minutes to seconds. The denominator conversion usually follows the numerator.
   Always begin by putting all conversion factors in place using *units only*. Now that this has been done, insert the appropriate numerical values for each conversion factor.

3. As always, state the answer with units and ro
   figures (in this case, three).

How to Do It

Practice Problems — Derived Unit Conversions

1. Convert 2.67 g/mL into kg/L. Why has the numerical value remained unchanged?

2. Convert the density of neon gas from 8.9994×10^{-4} mg/mL into kg/L.

3. Convert 35 mi/h (just over the speed limit in a U.S. city) into m/s. (Given: 5280 feet = 1 mile)

Use of a Derived Unit as a Conversion Factor

A quantity expressed with a derived unit may be used to convert a unit that measures one thing into a unit that measures something ______________________________.

The most common examples are the use of rate to convert between ____________ and ____________and the use of ______________ to convert between ___________and _________________.

The keys to this type of problem are determining which form of the conversion factor to use and where to start.

Example:

Suppose we wish to use the speed of sound (330 m/s) to determine the time (in hours) required for an explosion to be heard 5.0 km away.

It is always a good idea to begin any conversion problem by considering **what we are trying to find?**

Begin with the end in mind. This allows us to decide where to begin.

Do we start with 5.0 km or 330 m/s?

First, consider: are you attempting to convert a unit → unit, or a $\frac{\text{unit}}{\text{unit}} \rightarrow \frac{\text{unit}}{\text{unit}}$?

The answer is ________________ begin with the single unit: km.

The derived unit will serve as the conversion factor.

Second, which of the two possible forms of the conversion factor will allow conversion of a distance in km into a time in h?

Sample Problem — Use of Density as a Conversion Factor

What is the volume in L of a 15.0 kg piece of zinc metal? (Density of Zn = 7.13 g/mL)

What to Think about	How to Do It
1. Decide what form of the conversion factor to use: g/mL or the reciprocal, mL/g. Always begin by arranging the factors using *units* only. As the answer will contain one unit, begin with one unit, in this case, kg.	
2. Insert the appropriate numerical values for each conversion factor. In order to cancel a mass and convert to a volume, use the reciprocal of the density: $\frac{1\,\text{mL}}{7.13\text{ g}}$	
3. Calculate the answer with correct unit and number of significant digits.	

Practice Problems — Use of Rate and Density as Conversion Factors

1. The density of mercury metal is 13.6 g/mL. What is the mass of 2.5 L?

2. The density of lead is 11.2 g/cm^3. The volumes 1 cm^3 and 1 mL are exactly equivalent. What is the volume in L of a 16.5 kg piece of lead?

3. The speed of light is 3.0×10^{10} cm/s. Sunlight takes 8.29 min to travel from the photosphere (light-producing region) of the Sun to Earth. How many kilometres is Earth from the Sun?

Conversions Involving Units with Exponents (Another Kind of Derived Unit)

If a unit is _______________ or cubed, it may be cancelled in one of two ways.

It may be written more than once to convey that it is being multiplied by itself *or* it may be placed in brackets with the exponent applied to the *number* inside the brackets as well as to the *unit*.

Hence, the use of the equivalency 1 L = 1 dm^3 to convert 1 m^3 to L might appear in either of these formats:

$$1\ \text{m}^3 \times \frac{1\ \text{dm}}{10^{-1}\ \text{m}} \times \frac{1\ \text{dm}}{10^{-1}\ \text{m}} \times \frac{1\ \text{dm}}{10^{-1}\ \text{m}} \times \frac{1\ \text{L}}{1\ \text{dm}^3} \quad \textbf{OR}$$

Sample Problem – Use of Conversion Factors Containing Exponents

Convert 0.35 m^3 (cubic metres) into mL. (1 mL = 1 cm^3)

What to Think about	How to Do It
1. The unit cm must be cancelled three times. Do this by multiplying the conversion factor by itself three times or through the use of brackets.	
2. Once the units have been aligned correctly, insert the appropriate numerical values.	
3. Calculate the answer with the correct unit and number of significant figures.	

Practice Problems — Use of Conversion Factors Containing Exponents

1. Convert 4.3 dm^3 into cm^3.

2. Atmospheric pressure is 14.7 lb/in^2. Convert this to the metric unit, g/cm^2. (Given 454 g = 1.00 lb)

3. Convert a density of 8.2 kg/m^3 to lb/ft^3 using factors provided in this section.

Using Scientific Notation

-2.3445 x 10^3

Mantissa Exponent

Because it deals with atoms, and they are so incredibly small, the study of chemistry is notorious for using very large and very tiny numbers. For example, if you determine the total number of atoms in a sample of matter, the value will be very large. If, on the other hand, you determine an atom's diameter or the mass of an atom, the value will be extremely small. The method of reporting an ordinary, expanded number in scientific notation is very handy for both of these things.

________________________________ refers to the method of representing numbers in _______________________ *form.* Exponential numbers have two parts. Consider the following example:

24 500 becomes 2.45 10^4 in scientific notation

Convention states that the first portion of a value in scientific notation should always be expressed as a number __________________________________.

This portion is called the **mantissa** or the __________________________.
The second portion is the _______________________raised to some power.
It is called the **ordinate** or the _____________________________**portion**.

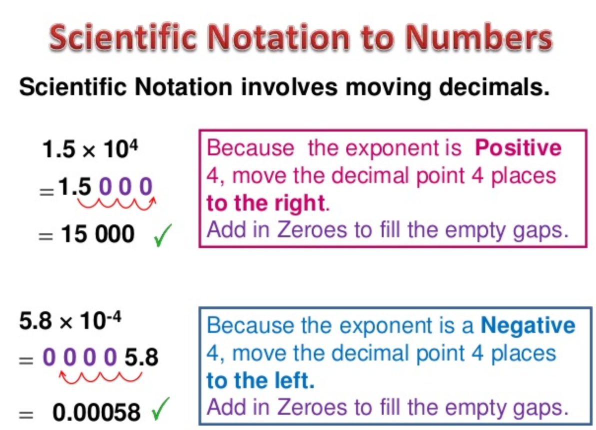
mantissa → $\underline{2.45} \times 10^4$ and $2.45 \times \underline{10^4}$ ← ordinate

A _______________________exponent in the ordinate indicates a __________________________ in scientific notation, while a _____________________ exponent indicates a _____________________

In fact the exponent indicates the number of 10s that must be multiplied together to arrive at the number represented by the scientific notation. If the exponents are negative, the exponent indicates the number of tenths that must be multiplied together to arrive at the number.

In other words, the exponent indicates the number of _____________________________________in the mantissa must be moved to correctly arrive at the _____________________**notation** (also called **standard notation**) version of the number.

A __________________*exponent* indicates the number of *places* the decimal must be moved to the __________, while a __________________*xponent* indicates the number of *places* the decimal must be moved to the ___________.



Quick Check

1. Change the following numbers from scientific notation to expanded notation.
   (a) 2.75×10^3 = ________________

   (b) 5.143×10^{-2} = __________________

2. Change the following numbers from expanded notation to scientific notation.
   (a) 69 547 = __________________

   (b) 0.001 68 = _________________

Multiplication and Division in Scientific Notation

To _________________two numbers in scientific notation, we *multiply the* ___________________ and state their product multiplied by 10, raised to a power that is the ____.

$$(A \times 10^a) \times (B \times 10^b) = (A \times B) \times 10^{(a+b)}$$

To *divide* two numbers in scientific notation, we divide one mantissa by the other and state their quotient multiplied by 10, raised to a power that is the *difference* between the exponents.

$$(A \times 10^a) \div (B \times 10^b) = (A \div B) \times 10^{(a-b)}$$

Sample Problems — Multiplication and Division Using Scientific Notation

Solve the following problems, expressing the answer in scientific notation.

1. (2.5×10^3) x $(3.2 \times 10^6) =$
2. $(9.4 \times 10^{-4}) \div (10^{-6}) =$

What to Think about	How to Do It
Question 1 1. Find the product of the mantissas. 2. Raise 10 to the sum of the exponents to determine the ordinate. 3. State the answer as the product of the new mantissa and ordinate. **Question 2** 1. Find the quotient of the mantissas. When no mantissa is shown, it is assumed that the mantissa is 1. 2. Raise 10 to the difference of the exponents to determine the ordinate. 3. State the answer as the product of the mantissa and ordinate.	

Practice Problems — Multiplication and Division Using Scientific Notation

Solve the following problems, expressing the answer in scientific notation, *without* using a calculator. Repeat the questions using a calculator and compare your answers. Compare your method of solving with a calculator with that of another student.

1. $(4 \times 10^3) \times (2 \times 10^4) =$ __________________
2. $(9.9 \times 10^5) \div (3.3 \times 10^3) =$ __________________
3. $[(3.1 \times 10^{-4}) \times (6.0 \times 10^7)] \div (2.0 \times 10^5) =$ __________________
4. $10^9 \div (5.0 \times 10^6) =$ __________________
5. $[(4.5 \times 10^{12}) \div (1.5 \times 10^4)] \times (2.5 \times 10^{-6}) =$ ________________

Addition and Subtraction in Scientific Notation

Remember that a number in proper scientific notation will always have a mantissa between ___ and ____.
Sometimes it becomes necessary to__________a decimal in order to express a number in ***proper scientific notation***.

The *number of places* shifted by the decimal is indicated by an *equivalent change* in the *value of the exponent*. If the decimal is shifted ___________, the *exponent* becomes______________; shifting the decimal to the _____________causes the *exponent* to become ____________________.

Another way to remember this is if the *mantissa becomes smaller* following a shift, the *exponent becomes larger*. Consequently, if the *exponent becomes larger*, the *mantissa becomes smaller*. Consider AB.C $\times 10^{x}$: if the decimal is shifted to change the value of the mantissa by 10^{n} times, the value of x changes $-n$ times.

For example,
A number such as $18\,235.0 \times 10^{2}$ (1 823 500 in standard notation) requires the decimal to be ____________________ places to the _______________ to give a mantissa between 1 and 10, that is 1.823 50.
A _____________shift_____places, means the exponent in the ordinate becomes _________________________ (from 10^{2} to 10^{6}).
The correct way to express $18\,235.0 \times 10^{2}$ in scientific notation is $1.823\,50 \times 10^{6}$.
Notice the new mantissa is 10^{4} *smaller*, so the exponent becomes 4 numbers larger.

Quick Check

Express each of the given values in proper scientific notation in the second column. Now write each of the given values from the *first* column in expanded form in the third column. Then write each of your answers from the *second* column in expanded form. How do the expanded answers compare?

	Given Value	Proper Notation	Expanded Form	Expanded Answer
1.	$6\,014.51 \times 10^{2}$			
2.	$0.001\,6 \times 10^{7}$			
3.	$38\,325.3 \times 10^{-6}$			
4.	0.4196×10^{-2}			

When adding or subtracting numbers in scientific notation, it is important to realize that we add or subtract only the mantissa. *Do not add or subtract the exponents!*

Shift the decimal to obtain the *same value for the exponent* in the ordinate of both numbers to be added or subtracted. Then simply *sum or take the difference of the mantissas***.** Convert back to proper scientific notation when finished.

Sample Problems — Addition and Subtraction in Scientific Notation

Solve the following problems, expressing the answer in proper scientific notation.

1. $(5.19 \times 10^3) - (3.14 \times 10^2) =$
2. $(2.17 \times 10^{-3}) + (6.40 \times 10^{-5}) =$

What to Think about	How to Do It
Question 1 1. Begin by shifting the decimal of one of the numbers and changing the exponent so that both numbers share the *same exponent*. For consistency, adjust one of the numbers so that *both* numbers have the *larger* of the two ordinates. The goal is for both mantissas to be multiplied by 10^3. This means the exponent in the second number should be increased by one. Increasing the exponent requires the decimal to shift to the left (so the mantissa becomes smaller). 2. Once both ordinates are the same, the mantissas are simply subtracted. **Question 1 — Another Approach** 1. It is interesting to note that we could have altered the first number instead. In that case, 5.19×10^3 would have become 51.9×10^2. 2. In this case, the difference results in a number that is not in proper scientific notation as the mantissa is greater than 10. 3. Consequently, a further step is needed to convert the answer back to proper scientific notation. Shifting the decimal one place to the left (mantissa becomes smaller) requires an increase of 1 to the exponent.	
Question 2 1. As with differences, begin by shifting the decimal of one of the numbers and changing the exponent so both numbers share the same ordinate. The *larger ordinate* in this case is 10^{-3}. 2. Increasing the exponent in the second number from –5 to –3 requires the decimal to be shifted two to the left (make the mantissa smaller). 3. Once the exponents agree, the mantissas are simply summed. 4. The alternative approach involves one extra step, but gives the same answer.	

Practice Problems — Addition and Subtraction in Scientific Notation

Solve the following problems, expressing the answer in scientific notation, *without* using a calculator. Repeat the questions using a calculator and compare your answers. Compare your use of the exponential function on the calculator with that of a partner.

1. $\begin{array}{r} 8.068 \times 10^{8} \\ \underline{-4.14 \times 10^{7}} \end{array}$

2. $\begin{array}{r} 6.228 \times 10^{-4} \\ \underline{+4.602 \times 10^{-3}} \end{array}$

3. $\begin{array}{r} 49.001 \times 10^{1} \\ \underline{+ \qquad\quad 10^{-1}} \end{array}$

Scientific Notation and Exponents

Occasionally a number in scientific notation will be raised to some power. When such a case arises, it's important to remember when one exponent is raised to the power of another, the ____________________________________ by one another.

Consider a problem like $(10^3)^2$. This is really just $(10 \times 10 \times 10)^2$ or $(10 \times 10 \times 10 \times 10 \times 10 \times 10)$. So we see this is the same as $10^{(3 \times 2)}$ or 10^6.

Topic Review:

Solve the following problems, expressing the answer in scientific notation, *without* the use of a calculator. Repeat the problems with a calculator and compare your answers.

1. $(10^3)^5$
2. $(2 \times 10^2)^3$
3. $(5 \times 10^4)^2$
4. $(3 \times 10^5)^2 \times (2 \times 10^4)^2$

5. Convert the following numbers from *scientific* notation to *expanded* notation and vice versa (be sure the scientific notation is expressed correctly).

Scientific Notation	Expanded Notation
3.08×10^4	
	960
4.75×10^{-3}	
	0.000 484
0.0062×10^5	

6. Give the product or quotient of each of the following problems (express all answers in proper form scientific notation). Do **not** use a calculator.

(a) (8.0×10^3) x (1.5×10^6) =

(b) $(1.5 \times 10^4) \div (2.0 \times 10^2)$ =

(c) $(3.5 \times 10^{-2}) \times (6.0 \times 10^5)$ =

(d) $(2.6 \times 10^7) \div (6.5 \times 10^{-4})$ =

7. Give the product or quotient of each of the following problems (express all answers in proper form scientific notation). Do **not** use a calculator.
   (a) $(3.5 \times 10^4) \times (3.0 \times 10^5) =$

   (b) $(7.0 \times 10^6) \div (1.75 \times 10^2) =$

   (c) $(2.5 \times 10^{-3}) \times (8.5 \times 10^{-5}) =$

   (d) $(2.6 \times 10^5) \div (6.5 \times 10^{-2}) =$

8. Solve the following problems, expressing the answer in scientific notation, *without* using a calculator. Repeat the questions using a calculator and compare your answers.

   (a) $$\begin{array}{r} 4.034 \times 10^5 \\ -2.12 \times 10^4 \\ \hline \end{array}$$

   (b) $$\begin{array}{r} 3.114 \times 10^{-6} \\ +2.301 \times 10^{-5} \\ \hline \end{array}$$

   (c) $$\begin{array}{r} 26.022 \times 10^2 \\ +7.04 \times 10^{-1} \\ \hline \end{array}$$

9. Solve the following problems, expressing the answer in scientific notation, *without* using a calculator. Repeat the questions using a calculator and compare your answers.

   (a) $$\begin{array}{r} 2.115 \times 10^8 \\ -1.11 \times 10^7 \\ \hline \end{array}$$

   (b) $$\begin{array}{r} 9.332 \times 10^{-3} \\ +6.903 \times 10^{-4} \\ \hline \end{array}$$

   (c) $$\begin{array}{r} 68.166 \times 10^2 \\ + \qquad \times 10^{-1} \\ \hline \end{array}$$

10. Solve each of the following problems *without* a calculator. Express your answer in correct form scientific notation. Repeat the questions using a calculator and compare.
    (a) $(10^{-4})^3$ (b) $(4 \times 10^5)^3$ (c) $(7 \times 10^9)^2$ d. $(10^2)^2 \times (2 \times 10)^3$

11. Solve each of the following problems *without* a calculator. Express your answer in correct form scientific notation. Repeat the questions using a calculator and compare.
    (a) $(6.4 \times 10^{-6} + 2.0 \times 10^{-7}) \div (2 \times 10^6 + 3.1 \times 10^7)$

    (b) $$\frac{3.4 \times 10^{-17} \times 1.5 \times 10^4}{1.5 \times 10^{-4}}$$

    (c) $(2 \times 10^3)^3 \times [(6.84 \times 10^3) \div (3.42 \times 10^3)]$

    (d) $$\frac{(3 \times 10^2)^3 + (4 \times 10^3)^2}{1 \times 10^4}$$