Unit # 4 : Polynomials

Submission Checklist: (make sure you have included all components for full marks)

- Cover page & Assignment Log
- Class Notes
- Homework (attached any extra pages to back)
- Quizzes (attached original quiz + corrections made on separate page)
- Practice Test/ Review Assignment

Assignment Rubric: Marking Criteria

<table>
<thead>
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<th>Good (4)</th>
<th>Satisfactory (3)</th>
<th>Needs Improvement (2)</th>
<th>Incomplete (1)</th>
<th>NHI (0)</th>
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# Homework Assignment Log

& Textbook Pages: ____________________________

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<thead>
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<th>Due Date</th>
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## Quizzes & Tests:

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<tr>
<th>What?</th>
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<tr>
<td>Quiz 1</td>
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<td>Quiz 2</td>
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<tr>
<td>Unit/ Chapter test</td>
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</table>
Polynomials

This booklet belongs to: _________________ Period ___

<table>
<thead>
<tr>
<th>LESSON #</th>
<th>DATE</th>
<th>QUESTIONS FROM NOTES</th>
<th>Questions that I find difficult</th>
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Your teacher has important instructions for you to write down below.

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### Polynomials: Key Terms

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
<th>Example</th>
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<tbody>
<tr>
<td>Term</td>
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<tr>
<td>Coefficient</td>
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<tr>
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<tr>
<td>Binomial</td>
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<tr>
<td>Trinomial</td>
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<tr>
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<td></td>
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<tr>
<td>Degree of a term</td>
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<tr>
<td>Degree of a Polynomial</td>
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<tr>
<td>Algebra Tiles</td>
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<tr>
<td>Combine like-terms</td>
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<td>Area Model</td>
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<td>Distribution (Expanding)</td>
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<td>Factoring by Grouping</td>
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<td>Factoring $ax^2 + bx + c$</td>
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<tr>
<td>when $a = 1$</td>
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<td>Factoring $ax^2 + bx + c$</td>
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<tr>
<td>when $a ≠ 1$</td>
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<td>Difference of Squares</td>
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<td>Perfect Square Trinomial</td>
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Lesson #1 – Intro to Polynomials & Addition/Subtraction

Term: A number and or variable connected by ______________ or _____________ (also called a monomial)

**Coefficients must be ______________ and exponents must be ______________________

ie.

<table>
<thead>
<tr>
<th># of Terms</th>
<th>Example</th>
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<tbody>
<tr>
<td>Monomial</td>
<td></td>
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<tr>
<td>Binomial</td>
<td></td>
</tr>
<tr>
<td>Trinomial</td>
<td></td>
</tr>
<tr>
<td>Polynomial*</td>
<td></td>
</tr>
</tbody>
</table>

* is a general name for an expression with 1 or more terms.

Degree of a...

Term: 

Polynomial:

Algebra Tile Legend:

Zero Principle: the idea that the addition of opposites cancel each other out and the result is zero

\[
\begin{array}{c}
\quad+
\quad-
\quad-x
\quad+x^2
\quad-x^2
\end{array}
\]

example: Subtract the following using algebra tiles \((2x - 1) - (-x + 2)\)
I. Simplify the following

- you can simplify expressions by collecting ____________ terms (terms with _______________ variables and exponents)

1. $7x + 3y + 5x - 2y$
2. $3x^2 + 4xy - 6xy + 8x^2 - 3yx$

II. Add/Subtract the following

3. $(x^2 + 4x - 2) + (2x^2 - 6x + 9)$
4. $(4x^2 - 2x + 3) - (3x^2 + 5x - 2)$
What is a Polynomial?

What is a Term?

A term is a number and/or variable connected by multiplication or division. One term is also called a monomial.

A polynomial is an expression made up of one or more terms connected to the next by addition or subtraction.

We say a polynomial is any expression where the coefficients are real numbers and all exponents are whole numbers. That is, no variables under radicals (rational exponents), no variables in denominators (negative exponents).

The following are polynomials:

\[ x, \quad 2x - 5, \quad 5 + 3x^2 - 12y^3, \quad \frac{x^2 + 3x + 2}{2}, \quad \sqrt{3}x^2 + 5y - z \]

The following are NOT polynomials:

\[ x^{-2}, \quad 3\sqrt{x}, \quad 4xy + 3xy^{-3}, \quad 12xz + 3^x \]

The following are terms:

5, x, 3x, 5x^2, \frac{3x}{4}, -2xy^2z^3

Each term may have a coefficient, variable(s) and exponents. One term is also called a monomial.

If there is no variable present...we call the term a constant.

Answer the questions below.

1. What is/are the coefficients below?
   \[ 5xy^2 - 7x + 3 \]

2. What is/are the constant(s) below?
   \[ 12x^2 - 5x + 13 \]

3. What is/are the variable(s) below?
   \[ 5xy^2 + 3 \]
Which of the following are not polynomials? Indicate why.

<p>| | | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>4.</td>
<td>$3xyz - \frac{2}{x}$</td>
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<tr>
<td>5.</td>
<td>$\frac{1}{y}x^3 - 5y$</td>
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<tr>
<td>6.</td>
<td>$2x - 4y^{-2}$</td>
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<tr>
<td>7.</td>
<td>$(3x + 2)^3$</td>
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<tr>
<td>8.</td>
<td>$\sqrt{3} + x^2 - 5$</td>
<td></td>
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<tr>
<td>9.</td>
<td>$\frac{5}{x} - z^2$</td>
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</tbody>
</table>

Classifying polynomials:

By Number of Terms:
- **Monomial**: one term.  
  Eg. $7x, 5, -3xy^3$
- **Binomial**: two terms  
  Eg. $x + 2, 5x - 3y, y^3 + \frac{5x}{3}$
- **Trinomial**: three terms  
  Eg. $x^2 + 3x + 1, 5xy - 3x + y^2$
- **Polynomial**: four terms  
  Eg. $7x + y - z + 5, x^4 - 3x^3 + x^2 - 7x$

By Degree:
To find the degree of a term, add the exponents within that term.

Eg. $-3xy^3$ is a $4^{th}$ degree term because the sum of the exponents is 4.
$5z^7y^2x^3$ is a $9^{th}$ degree term because the sum of the exponents is 9.

To find the degree of a polynomial first calculate the degree of each term. The highest degree amongst the terms is the degree of the polynomial.

Eg. $x^4 - 3x^3 + x^2 - 7x$ is a $4^{th}$ degree polynomial. The highest degree term is $x^4$.
$3xyz^4 - 2x^2y^3$ is a $6^{th}$ degree binomial. The highest degree term is $3xyz^4$ ($6^{th}$ degree)

Classify each of the following by degree and by number of terms.

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<tr>
<td>10.</td>
<td>$2x + 3$</td>
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<tr>
<td>11.</td>
<td>$x^3 - 2x^2 + 7$</td>
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<tr>
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<tr>
<td>Name:</td>
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<tr>
<td>12.</td>
<td>$2a^3b^4 + a^2b^4 - 27c^5 + 3$</td>
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<td>Name:</td>
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<td>13.</td>
<td>7</td>
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<td>Name:</td>
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<tr>
<td>14.</td>
<td>Write a polynomial with one term that is degree 3.</td>
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<td>15.</td>
<td>Write a polynomial with three terms that is degree 5.</td>
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</table>
Algebra Tiles

The following will be used as a legend for algebra tiles in this guidebook.

\[
\begin{array}{cccc}
1 & -1 & x & -x \\
\square & \square & \text{ } & \text{ } \\
\end{array}
\]

\[
\begin{array}{cccc}
x^2 & -x^2 \\
\text{ } & \text{ } \\
\end{array}
\]

Write an expression that can be represented by each of the following diagrams.

16.  

17.  

18.  

19.  

20.  

21. Draw a diagram to represent the following polynomial.  
\[3x^2 - 5x + 6\]

22. Draw a diagram to represent the following polynomial.  
\[-3x^2 + x - 2\]

23. What happens when you add the following?  

24. What happens when you add the following?  

25. Simplify by cancelling out tiles that add to zero. Write the remaining expression.

26. Simplify by cancelling out tiles that add to zero. Write the remaining expression.

27. Represent the following addition using algebra tiles. Do not add.
   \[ x + (x - 1) \]

28. Represent the following addition using algebra tiles. Do not add.
   \[ (5x + 3) + (2x + 1) \]

29. Use Algebra tiles to add the following polynomials. (collect like-terms)
   \[ (2x - 1) + (-5x + 5) \]

30. Use Algebra tiles to add the following polynomials. (collect like-terms)
   \[ (2x^2 + 5x - 3) + (-3x^2 + 5) \]

The Zero Principle:

The idea that opposites cancel each other out and the result is zero.

Eg. \[ x + 3 + (-3) = x \] The addition of opposites did not change the initial expression.

\[ \boxed{\quad} + \boxed{\quad} + \boxed{\quad} = \boxed{\quad} \]
31. What is the sum of the following tiles?

Sum

32. If you add the following to an expression, what have you increased the expression by?

33. Represent the following subtraction using algebra tiles.

\[(2x - 1) - (-x + 2)\]

34. Why can you not simply “collect like-terms” when subtracting the two binomials in the previous question?

35. When asked to subtract \((2x - 1) - (-x + 2)\), Raj drew the following diagram:

Explain how Raj applied the zero principle to subtract the polynomials.

36. Use Algebra tiles to subtract the following polynomials.

\[(2x - 1) - (-5x + 5)\]

37. Use Algebra tiles to subtract the following polynomials.

\[(2x^2 + 5x - 3) - (-3x^2 + 5)\]

38. Use Algebra tiles to subtract the following polynomials.

\[(-2x^2 - 4x - 3) - (-3x^2 + 5)\]
Like Terms

39. When considering algebra tiles, what makes two tiles “alike”?

40. What do you think makes two algebraic terms alike? (Remember, tiles are used to represent the parts of an expression.)

Collecting Like Terms without tiles:

You have previously been taught to combine like terms in algebraic expressions. Terms that have the same variable factors, such as $7x$ and $5x$, are called like terms.

Simplify any expression containing like terms by adding their coefficients.

Eg.1. Simplify:

$7x + 3y + 5x - 2y$
$7x + 5x + 3y - 2y$

$= 12x + y$

Eg.2. Simplify

$3x^2 + 4xy - 6xy + 8x^2 - 3yx$
$3x^2 + 8x^2 + 4xy - 6xy - 3xy$

$= 11x^2 - 5xy$

41. $3x + 7y - 12x + 2y$

42. $2x^2 + 3x^3 - 7x^2 - 6$

43. $5x^2y^3 - 5 + 6x^2y^3$

Simplify by collecting like terms. Then evaluate each expression for $x = 3, y = -2$. 

Exactly the same variable & exponents.

Remember $3yx$ is the same as $3xy$. 

Exactly the same variable & exponents.
Adding & Subtracting Polynomials without TILES.

ADDITION
To add polynomials, collect like terms.

Eg.1. \((x^2 + 4x - 2) + (2x^2 - 6x + 9)\)

**Horizontal Method:**
\[
= x^2 + 4x - 2 + 2x^2 - 6x + 9 \\
= x^2 + 2x^2 + 4x - 6x - 2 + 9 \\
= 3x^2 - 2x + 7
\]

**Vertical Method:**
\[
\begin{align*}
x^2 + 4x - 2 \\
2x^2 - 6x + 9
\end{align*}
\]

SUBTRACTION
It is important to remember that the subtraction refers to all terms in the bracket immediately after it.

To subtract a polynomial, determine the opposite and add.

Eg.2. \((4x^2 - 2x + 3) - (3x^2 + 5x - 2)\)

This question means the same as:
\[
(4x^2 - 2x + 3) - 1(3x^2 + 5x - 2)
\]
\[
= 4x^2 - 2x + 3 - 3x^2 - 5x + 2
\]
\[
= 4x^2 - 3x^2 - 2x - 5x + 3 + 2
\]
\[
= x^2 - 7x + 5
\]

We could have used vertical addition once the opposite was determined if we chose.

<table>
<thead>
<tr>
<th>Add or subtract the following polynomials as indicated.</th>
</tr>
</thead>
<tbody>
<tr>
<td>44. ((4x + 8) + (2x + 9))</td>
</tr>
<tr>
<td>45. ((3a + 7b) + (9a - 3b))</td>
</tr>
<tr>
<td>46. ((7x + 9) - (3x + 5))</td>
</tr>
<tr>
<td>47. Add. ((4a - 2b) + (3a + 2b))</td>
</tr>
<tr>
<td>48. Subtract. ((7x - 3y) - (-5x + 2y))</td>
</tr>
<tr>
<td>49. Subtract. ((12a - 5b) - (-7a - 2b))</td>
</tr>
</tbody>
</table>

Multiplying each term by -1 will remove the brackets from the second polynomial.
Add or subtract the following polynomials as indicated.

| 50. $(5x^2 - 4x - 2) + (8x^2 + 3x - 3)$ | 51. $(3m^2n + mn - 7n) - (5m^2n + 3mn - 8n)$ |
| 52. $(8y^2 + 5y - 7) - (9y^2 + 3y - 3)$ | 53. $(2x^2 - 6xy + 9) + (8x^2 + 3x - 3)$ |

Your notes here...
Lesson #2 – Multiplication Models

I. Rectangle Model

II. Breaking Numbers
   a) $16 \times 27$
   b) Area Model: $23 \times 14$

III. Algebra Tiles

   Legend:

   a) $x(x + 4)$
   b) $2x(-x + 2)$
c) \(-x(x - 2)\)  
d) \((x + 1)(2x + 3)\)

f) \((2x - 1)(x + 4)\)

g) Draw a model that has an area of \(x^2 + x\)

Write a quotient that represents this model.
Multiplication and the Area Model

Sometimes it is convenient to use a tool from one aspect of mathematics to study another.

To find the product of two numbers, we can consider the numbers as side lengths of a rectangle.

How are side lengths, rectangles, and products related? The Area Model

The product of the two sides is the area of a rectangle.  \( A = lw \)

Consider:

\[
\begin{array}{c}
+ & + & + & + \\
+ & + & + & + \\
+ & + & + & + \\
+ & + & + & + \\
\end{array}
\]

Length = ____  Width = ____

54. Show why \( 3 \times 3 = 9 \) using the area model.

Solution:

\[
\begin{array}{c}
+ & + & + & + \\
+ & + & + & + \\
+ & + & + & + \\
+ & + & + & + \\
\end{array}
\]

55. Show why \( 3 \times 4 = 12 \) using the area model.

56. Calculate \( 5 \times 4 \) using the area model.

57. How might we show \( -2 \times 4 = -8 \) using the area model?

\[
\begin{array}{c}
- & - & - & - \\
- & - & - & - \\
- & - & - & - \\
- & - & - & - \\
\end{array}
\]

58. Calculate \( -3 \times 4 \) using the area model.

59. Calculate \( -5 \times 4 \) using the area model.

60. How might we show \( -2 \times -4 = 8 \) using the area model?

\[
\begin{array}{c}
+ & + & + & + \\
+ & + & + & + \\
+ & + & + & + \\
+ & + & + & + \\
\end{array}
\]

61. Calculate \( -3 \times -4 \) using the area model.

62. Calculate \( -5 \times -4 \) using the area model.
There are some limitations when using the area model to show multiplication. The properties of multiplying integers \((+,+),(+,-),(-,-)\) need to be interpreted by the reader.

<table>
<thead>
<tr>
<th>63. Show how you could break apart the following numbers to find the product.</th>
<th>64. Show how you could break apart the following numbers to find the product.</th>
<th>65. Show how you could break apart the following numbers to find the product.</th>
</tr>
</thead>
<tbody>
<tr>
<td>[21 \times 12 = ]</td>
<td>[32 \times 14 = ]</td>
<td>[17 \times 24 = ]</td>
</tr>
<tr>
<td>[= (20 + 1) \times (10 + 2)]</td>
<td>[= 200 + 40 + 10 + 2]</td>
<td>[= 252]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>66. Draw a rectangle with side lengths of 21 units and 12 units. Model the multiplication above using the rectangle.</th>
<th>67. Draw a rectangle with side lengths of 32 units and 14 units. Model the multiplication above using the rectangle.</th>
<th>68. Draw a rectangle with side lengths of 17 units and 24 units. Model the multiplication above using the rectangle.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Rectangle 1" /></td>
<td><img src="image2.png" alt="Rectangle 2" /></td>
<td><img src="image3.png" alt="Rectangle 3" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>69. Use an area model to multiply the following without using a calculator.</th>
<th>70. Use an area model to multiply the following without using a calculator.</th>
<th>71. Use an area model to multiply the following without using a calculator.</th>
</tr>
</thead>
<tbody>
<tr>
<td>[23 \times 15]</td>
<td>[52 \times 48]</td>
<td>[73 \times 73]</td>
</tr>
</tbody>
</table>
Algebra tiles and the area model: Multiplication/Division of algebraic expressions.

First we must agree that the following shapes will have the indicated meaning.

1 - 1 x -x x^2 -x^2

We must also remember the result when we multiply:
- Two positives = Positive
- Two negatives = Positive
- One positive and one negative = Negative

72. Write an equation represented by the diagram below and then multiply the two monomials using the area model.

73. Write an equation represented by the diagram below and then multiply the two monomials using the area model.

74. Write an equation represented by the diagram below and then multiply the two monomials using the area model.

75. Write an equation represented by the diagram below and then multiply the two monomials using the area model.

76. If the shaded rectangle represents a negative value, find the product of the two monomials.

77. Write an equation represented by the diagram below and then multiply the two monomials using the area model.
<table>
<thead>
<tr>
<th>Problem</th>
<th>Equation</th>
<th>Diagram</th>
<th>Area</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>78.</td>
<td>Write an equation represented by the diagram below and then multiply the two polynomials using the area model.</td>
<td><img src="image1.png" alt="Diagram" /></td>
<td>$6x$</td>
<td><img src="image2.png" alt="Length" /></td>
</tr>
<tr>
<td>79.</td>
<td>Write an equation represented by the diagram below and then multiply the two polynomials using the area model.</td>
<td><img src="image3.png" alt="Diagram" /></td>
<td>$6x^2$</td>
<td><img src="image4.png" alt="Length" /></td>
</tr>
<tr>
<td>80.</td>
<td>Write an equation represented by the diagram below and then multiply the two polynomials using the area model.</td>
<td><img src="image5.png" alt="Diagram" /></td>
<td>$-6x^2$</td>
<td><img src="image6.png" alt="Length" /></td>
</tr>
<tr>
<td>81.</td>
<td>Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.</td>
<td><img src="image7.png" alt="Diagram" /></td>
<td>$6x$</td>
<td><img src="image8.png" alt="Length" /></td>
</tr>
<tr>
<td>82.</td>
<td>Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.</td>
<td><img src="image9.png" alt="Diagram" /></td>
<td>$6x^2$</td>
<td><img src="image10.png" alt="Length" /></td>
</tr>
<tr>
<td>83.</td>
<td>Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.</td>
<td><img src="image11.png" alt="Diagram" /></td>
<td>$-6x^2$</td>
<td><img src="image12.png" alt="Length" /></td>
</tr>
</tbody>
</table>
84. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.

85. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.

86. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.

87. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.

88. Draw and use an area model to find the product: $(2)(2x + 1)$

89. Draw and use an area model to find the product: $(2x)(x - 3)$
90. Draw and use an area model to find the product: 
\((x)(x + 3)\)

91. Draw and use an area model to find the product: 
\((-x)(x + 3)\)

92. Draw and use an area model to find the product: 
\((-3x)(2x + 3)\)

93. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

Area = \(x^2 + 3x\)

Length:__________

94. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

Area = \(-x^2 - 3x\)

Length:__________

95. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

Area = \(2x^2 - 8x\)

Length:__________
96. Find the area, length and width that can be represented by the diagram.

Area: 
Length: 
Width: 

97. Find the area, length and width that can be represented by the diagram.

Area: 
Length: 
Width: 

98. Find the area, length and width that can be represented by the diagram.

Area: 
Length: 
Width:
Multiplying & Dividing Monomials without TILES

When multiplying expressions that have more than one variable or degrees higher than 2, algebra tiles are not as useful.

Multiplying Monomials:

**Eg.1.**

\((2x^2)(7x)\)

Multiply numerical coefficients.

\(= 2 \times 7 \times x \times x^2\) Multiply variables using exponent laws.

\(= 14x^3\)

**Eg.2.**

\((-4a^2b)(3ab^3)\)

\(= -4 \times 3 \times a^2 \times a \times b \times b^3\)

\(= -12a^3b^4\)

Dividing Monomials:

**Eg.1.**

\(\frac{20x^3y^4}{-5x^2y^2}\)

Divide the numerical coefficients.

\(= \frac{20 \times 3 \times y^4}{-5 \times x^2 \times y^2}\) Divide variables using exponent laws.

\(= -4xy^2\)

**Eg.2.**

\(\frac{-36m^3n^4p^2}{9m^2n^p}\)

\(= \frac{-36 \times m^3 \times n^4 \times p^2}{9 \times m^2 \times n \times p}\)

\(= 4n^3p\)

Multiply or Divide the following.

99. \((-2ab^3)(-3ab^2)\)  
100. \((5x^2y^3)(-2x^3y^5)\)  
101. \(4x(-3x^2)\)

102. \(\left(\frac{1}{2}a^b\right)\left(\frac{3}{4}a^b\right)\)  
103. \(\frac{-75x^3t^3}{15x^2t^2}\)  
104. \(\frac{-45x^3y^2}{-9x^2y}\)

105. \(\frac{28x^3y^2}{18xy^3}\)  
106. \((2cd)(-2c^2d^3)(5c)\)  
107. \(\frac{(3xy)(4x^2y^3)}{2x^2y}\)

Revisit the exponent laws if necessary!
Lesson #3 - Multiplying Binomials

PART I: Algebra Tiles

(3x + 1)(x + 4)

PART II: Binomial x Binomial = F.O.I.L  (First Outside Inside Last)

1. (3x + 1)(x + 4) = 3x^2 + 12x + x + 4
   = 3x^2 + 13x + 4

2. (2x + 5)(x + 3) =

3. (5x + 6)(x - 2) =

4. (7x + 1)(7x - 1) =

5. (5x - 4)^2 =
PART III: Binomial x Trinomial (6 multiplication steps)

1. \((x + 2)(x^2 + 5x + 3) = \)

PART IV: Binomial x Binomial x Binomial

1. \((x + 2)(x + 3)(x + 4) = (x + 2)(x^2 + 7x + 12) \)

2. \(3(x + 10)(x - 2)(x + 2) \)
Multiplying Binomials

Challenge:
108. Which of the following are equal to $x^2 + 9x + 18$?

a) $(x + 3)(x + 6)$
b) $(x + 1)(x + 18)$
c) $(x - 3)(x - 6)$
d) $(x + 2)(x + 9)$

Challenge:
109. Multiply $(2x + 1)(x - 5)$

110. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.

111. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.

112. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.

113. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.

114. Draw and use an area model to find the product: $(x + 2)(2x + 1)$

115. Draw and use an area model to find the product: $(2x - 1)(x - 3)$
116. Draw and use an area model to find the product: \((2 - x)(x + 2)\)

117. Draw and use an area model to find the product: \((3 - x)(x - 1)\)

118. Draw and use an area model to find the product: \((3x + 1)(2x + 1)\)

119. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

\[\text{Area} = x^2 + 3x + 2\]

Length: 

120. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

\[\text{Area} = 2x^2 + 5x + 2\]

Length: 

121. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

\[\text{Area} = 4x^2 - 8x + 3\]

Length: 

122. Find the area, length and width that can be represented by the diagram.

Area:
Length:
Width:

123. Find the area, length and width that can be represented by the diagram.

Area:
Length:
Width:

124. Find the area, length and width that can be represented by the diagram.

Area:
Length:
Width:

125. Draw tiles that represent the multiplication of \((x + 1)(x - 3)\).

What is the product of \((x + 1)(x - 3)\)?

126. Draw tiles that represent the multiplication of \((2x + 1)(2x + 1)\).

What is the product of \((2x + 1)(2x + 1)\)?

127. Draw tiles that represent the multiplication of \((x - 4)(x + 4)\).

What is the product of \((x - 4)(x + 4)\)?
Multiplying Polynomials without TILES (also called expanding or distribution)

Multiplying Binomials: *use FOIL

Eg.1. 
\[(x + 3)(x + 6) = x^2 + 6x + 3x + 18 = x^2 + 9x + 18\]

\[\text{FOIL} \]
Firsts - Outsides - Insides - Lasts
\[(x)(x) + (x)(6) + (3)(x) + (3)(6)\]

Eg.2. 
\[(2x + 1)(x - 5) = 2x^2 - 10x + x - 5 = 2x^2 - 9x - 5\]

Multiplying a Binomial by a Trinomial:

Eg. 
\[(y - 3)(y^2 - 4y + 7) = y^3 - 4y^2 + 7y - 3y^2 + 12y - 21 = y^3 - 7y^2 + 19y - 21\]

Multiply each term in the first polynomial by each term in the second.

Multiplying: Binomial × Binomial × Binomial

Eg. 
\[(x + 2)(x - 3)(x + 4)\]
\[= (x^2 - 3x + 2x - 6)(x + 4)\]
\[= (x^2 - x - 6)(x + 4)\]
\[= x^3 + 4x^2 - x^2 - 4x - 6x - 24\]
\[= x^3 + 3x^2 - 10x - 24\]

Multiply the first two brackets (FOIL) to make a new trinomial.

Then multiply the new trinomial by the remaining binomial

Multiply the following as illustrated above.

128. 
\[(x + 2)(x - 5)\]

129. 
\[(2x + 1)(x - 3)\]

130. 
\[(x - 3)(x - 3)\]
Multiply the following.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>131. $(x + 2)(x + 2)$</td>
<td>132. $(2x + 1)(3x - 3)$</td>
<td>133. $(2x + 1)(2x - 1)$</td>
</tr>
<tr>
<td>134. $(x + 2)^2$</td>
<td>135. $(2x + 3)^2$</td>
<td>136. $(x - 1)(x - 1)(x + 4)$</td>
</tr>
<tr>
<td>137. $(x - 3)(x^2 - 5x + 1)$</td>
<td>138. $(2x - 3)(3x^2 + 2x + 1)$</td>
<td>139. $(x + 2)^3$</td>
</tr>
</tbody>
</table>
Lesson #4 – Conjugates and More Expanding

I. Conjugates

1. \((x + 3)(x - 3) = \)

2. \((x + 2)(x - 2) = \)

3. \((3m + 10)(3m - 10) = \)

4. \(\left(\frac{1}{2}x - y\right)\left(\frac{1}{2}x + y\right) = \)

5. \((m^3 + 1)(m^3 - 1) = \)

\[(a + b)(a - b) = a^2 - b^2\]

the product of conjugates is a binomial “a difference of squares”

II. Expand and Simplify

1. \((x + 5)(x - 1) + (x + 3)(x - 7)\)

2. \((x + 1) - (x - 4)(x + 4)\)

3. \(6 - 3(2x - 1)(2x + 1) - (x + 4)^2\)
Special Products: Follow the patterns

Conjugates: \[(a + b)(a - b)\]  
\[= a^2 + ab - ab - b^2\]  
\[= a^2 - b^2\]

140. Write an expression for the following diagram (do not simplify):

\[
\begin{array}{c|c|c|c}
\hline
& & & \\
\hline
& & & \\
\hline
& & & \\
\hline
\end{array}
\]

What two binomials are being multiplied above?

Write an equation using the binomials above and the simplified product.

141. Write an expression for the following diagram (do not simplify):

\[
\begin{array}{c|c|c|c}
\hline
& & & \\
\hline
& & & \\
\hline
& & & \\
\hline
\end{array}
\]

What two binomials are being multiplied above?

Write an equation using the binomials above and the simplified product.

QUESTION... Describe any patterns you observe in the two questions above.

Remember this pattern...it will be important when we factor "A Difference of Squares" later in this booklet.
142. Write an expression (polynomial) for the following diagram (do not simplify):

![Diagram for problem 142]

What two binomials are being multiplied above?

Write an equation using the binomials above and the simplified product.

Simplify the following.

144. \((x + 3)(x - 3)\)

145. \((2x + 3)(2x - 3)\)

146. \((3x - 1)(3x + 1)\)

147. \((x + \sqrt{2}y)(x - \sqrt{2}y)\)

143. Write an expression for the following diagram (do not simplify):

![Diagram for problem 143]

What two binomials are being multiplied above?

Write an equation using the binomials above and the simplified product.
### Simplify the following.

<table>
<thead>
<tr>
<th>148. $3(b - 7)(b + 7)$</th>
<th>149. $-2(c - 5)(c + 5)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>150. $(x + 6)(x + 4) + (x + 2)(x + 3)$</td>
<td>151. $3(x - 4)(x + 3) - 2(4x + 1)$</td>
</tr>
<tr>
<td>152. $5(3t - 4)(2t - 1) - (6t - 5)$</td>
<td>153. $10 - 2(2y + 1)(2y + 1) - (2y + 3)(2y + 3)$</td>
</tr>
</tbody>
</table>

### Some key points to master about the Distributive Property...

| FOIL | $(a + b)(a - b)$ | $(a + b)^2$ | $(a + b)^3$ |
Lesson #5 – (Greatest Common Factor) Factoring

I. Factoring

Factoring is the reverse of multiplying.

II. Factoring a Monomial Common Factor (with the GCF)

1. \(10x - 10 = \)

2. \(9x^2y^5 - 30x^4y = \)

3. \(8x^3 + 12x^2y - 20x = \)

4. \(3x + 11 = \)

III. Factoring a Binomial Common Factor (with the GCF)

1. \(3x(x + 2) + 7(x + 2) = \)

2. \(6a(a - 5) - 11(a - 5) = \)

3. \(2x(x - 3) + 9(3 - x) = \)
IV. Factoring by Grouping

1. \( mx + 2m + 3x + 6 = \)

2. \( 3a + 3b - ax - bx = \)

3. \( 4m^2 - 12m + 15t - 5mt = \)

4. \( xy + 10 + 2y + 5x = \)
Factoring:

When a number is written as a product of two other numbers, we say it is factored.

“Factor Fully” means to write as a product of **prime factors**.

<table>
<thead>
<tr>
<th>Eg.1. Write 15 as a product of its prime factors.</th>
<th>Eg.2. Write 48 as a product of its prime factors.</th>
<th>Eg.3. Write 120 as a product of its prime factors.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$15 = 5 \times 3$</td>
<td>$48 = 8 \times 6$</td>
<td>$120 = 10 \times 12$</td>
</tr>
<tr>
<td></td>
<td>$48 = 2 \times 2 \times 2 \times 3 \times 2$</td>
<td>$120 = 2 \times 5 \times 2 \times 2 \times 3$</td>
</tr>
<tr>
<td></td>
<td>$48 = 2^4 \times 3$</td>
<td>$120 = 2^3 \times 3 \times 5$</td>
</tr>
</tbody>
</table>

154. Write 18 as a product of its prime factors.

155. Write 144 as a product of its prime factors.

156. Write 64 as a product of its prime factors.

157. Find the greatest common factor (GCF) of 48 and 120.

Look at each factored form.

- $48 = 2^4 \times 3$
- $120 = 2^3 \times 3 \times 5$

Both contain $2 \times 2 \times 2 \times 3$, therefore this is the GCF,

GCF is 24.
We can also write algebraic expressions in factored form.

Eg. 4. Write $36x^2y^3$ as a product of its factors.

\[ 36x^2y^3 = 9 \times 4 \times x \times x \times y \times y \]
\[ 36x^2y^3 = 3^2 \times 2^2 \times x^2 \times y^3 \]

<table>
<thead>
<tr>
<th>160. Write $10a^2b$ as a product of its factors.</th>
<th>161. Write $18ab^2c^3$ as a product of its factors.</th>
<th>162. Write $12b^3c^2$ as a product of its factors.</th>
</tr>
</thead>
<tbody>
<tr>
<td>163. Find the greatest common factor (GCF) of $10a^2b$ and $18ab^2c^3$.</td>
<td>164. Find the greatest common factor (GCF) of $12b^3c^2$ and $18ab^2c^3$.</td>
<td>165. Find the greatest common factor (GCF) of $10a^2b$, $18ab^2c^3$, and $12b^3c^2$.</td>
</tr>
</tbody>
</table>
Factoring Polynomials:

Factoring means "write as a product of factors."

The method you use depends on the type of polynomial you are factoring.

**Challenge Question:**
Write a multiplication that would be equal to $5x + 10$.

**Challenge Question:**
Write a multiplication that would be equal to $3x^3 + 6x^2 - 12x$.

The answers to the above questions are called the “FACTORED FORM”.
Factoring: Look for a Greatest Common Factor

Hint: Always look for a GCF first.

Ask yourself: “Do all terms have a common integral or variable factor?”

Eg.1. Factor the expression.
5x + 10
Think…what factor do 5x and 10 have in common?
Both are divisible by 5...that is the GCF.
= 5(x) + 5(2) Write each term as a product using the GCF.
= 5(x + 2) Write the GCF outside the brackets, remaining factors inside.

You should check your answer by expanding. This will get you back to the original polynomial.

Eg.2. Factor the expression
3ax^3 + 6ax^2 - 12ax
GCF = 3ax
= 3ax(x^2 + 2x - 4)

Eg.3. Factor the expression 4x + 4 using algebra tiles.

I draw the expression as a rectangle using algebra tiles. Find length and width.

1 1 1 1
x +
1

4(x + 1) = 4x + 4
Factor the following polynomials.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>166. $5x + 25$</td>
<td>167. $4x + 13$</td>
<td>168. $8x + 8$</td>
</tr>
<tr>
<td>169. Model the expression above using algebra tiles.</td>
<td>170. Model the expression above using algebra tiles.</td>
<td>171. Model the expression above using algebra tiles.</td>
</tr>
<tr>
<td>172. $4ax + 8ay - 6az$</td>
<td>173. $24w^3 - 6w^3$</td>
<td>174. $3w^2xy + 12wxy^2 - wxy$</td>
</tr>
<tr>
<td>175. $27a^2b^3 + 9a^2b^2 - 16a^2b^2$</td>
<td>176. $6m^2n^2 + 18m^2n^2 - 12mn^2 + 24mn^3$</td>
<td></td>
</tr>
</tbody>
</table>
Factoring a Binomial Common Factor:  

Hint: There are brackets with identical terms.

The common factor **IS** the term in the brackets!

Eg.1. Factor. $4x(w + 1) + 5y(w + 1)$  
Eg.2. Factor. $3x(a + 7) - (a + 7)$

\[
\begin{align*}
4x(w + 1) + 5y(w + 1) & = (w + 1)(4x) + (w + 1)(5y) \\
& = (w + 1)(4x + 5y) \\
3x(a + 7) - (a + 7) & = (a + 7)(3x) - (a + 7)(1) \\
& = (a + 7)(3x - 1)
\end{align*}
\]

Sometimes it is easier to understand if we substitute a letter, such as $d$ where the common binomial is.

Consider Eg.1.

\[
\begin{align*}
4x(w + 1) + 5y(w + 1) & = (w + 1)(4x + 5y) \\
4xd + 5yd & = (w + 1)(4x + 5y)
\end{align*}
\]

Substitute $d$ for $(w + 1)$.

Now replace $(w + 1)$.

Factor the following, if possible.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Factor</th>
<th>Expression</th>
<th>Factor</th>
<th>Expression</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>177. $5x(a + b) + 3(a + b)$</td>
<td></td>
<td>178. $3m(x - 1) + 5(x - 1)$</td>
<td></td>
<td>179. $3t(x - y) + (x + y)$</td>
<td></td>
</tr>
<tr>
<td>180. $4t(m + 7) + (m + 7)$</td>
<td></td>
<td>181. $3t(x - y) + (y - x)$</td>
<td></td>
<td>182. $4y(p + q) - x(p + q)$</td>
<td></td>
</tr>
</tbody>
</table>

Challenge Question:  

Factor the expression $ac + bd + ad + bc$. 

Factoring: **Factor by Grouping.**

Hint: 4 terms!

Sometimes a polynomial with 4 terms but no common factor can be arranged so that grouping the terms into two pairs allows you to factor.

You will use the concept covered above...common binomial factor.

Eg.1. Factor \(ac + bd + ad + bc\)

\[
ac + bc \quad + ad + bd
\]

Group terms that have a common factor.

\[
c(a + b) + d(a + b)
\]

Notice the newly created binomial factor, \((a + b)\).

\[
= (a + b)(c + d)
\]

Factor out the binomial factor.

Eg.2. Factor \(5m^2t - 10m^2 + t^2 - 2t\)

\[
5m^2t - 10m^2 - t^2 + 2t
\]

Group.

\[
5m^2(t - 2) - t(t - 2)
\]

*Notice that I factored out a \(-t\) in the second group.

This made the binomials into common factors, \((t - 2)\).

\[
= (t - 2)(5m^2 - t)
\]

---

<table>
<thead>
<tr>
<th>183. (wx + wy + xz + yz)</th>
<th>184. (x^2 + x - xy - y)</th>
<th>185. (xy + 12 + 4x + 3y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>186. (2x^2 + 6y + 4x + 3xy)</td>
<td>187. (m^2 - 4m + 4m - mn)</td>
<td>188. (3a^2 + 6b^2 - 9a - 2ab^2)</td>
</tr>
</tbody>
</table>
Lesson #6 – Factoring Trinomials \((ax^2 + bx + c)\), where \(a = 1\)

**Type I**: \(x^2 + bx + c\)

1. \(x^2 + 7x + 12\)

2. \(x^2 + 9x + 20\)

3. \(2x^2 + 22x + 60\)

4. \(x^2 + 24xy + 44y^2\)

**Type II**: \(x^2 - bx + c\)

1. \(x^2 - 8x + 12\)

2. \(x^2 - 21x + 20\)

3. \(y^2 - 11y + 18\)

4. \(3x^2 - 18x + 27\)
Type III: \( x^2 \pm bx - c \)

1. \( x^2 + 2x - 24 \)

2. \( x^2 - 2x - 35 \)

3. \( x^4 + x^2 - 30 \)

4. \( 2x^3 - 6x^2 - 20x \)

5. \( x^2 - x - 90 \)
Factoring: \( ax^2 + bx + c \) (where \( a=1 \)) with tiles.

Hint: 3 terms, no common factor, leading coefficient is 1.

Eg.1. Consider \( x^2 + 3x + 2 \). The trinomial can be represented by the rectangle below.

Recall, the side lengths will give us the “factors”.

\[
\therefore x^2 + 3x + 2 = (x + 1)(x + 2)
\]

Eg.2. Factor \( x^2 - 5x - 6 \)

\[
\therefore x^2 - 5x - 6 = (x + 1)(x - 6)
\]

Factor the following trinomials using algebra tiles.

189. \( x^2 + 6x + 8 \) 

190. \( x^2 + 9x + 14 \)

191. \( x^2 - 7x + 6 \) 

192. \( x^2 + 9x - 10 \)
Factoring: \( ax^2 + bx + c \) (where \( a=1 \)) without tiles.

Did you see the pattern with the tiles?

If a trinomial in the form \( x^2 + bx + c \) can be factored, it will end up as \((x + \_)(x + \_)\). The trick is to find the numbers that fill the spaces in the brackets.

The Method...
If the trinomial is in the form: \( x^2 + bx + c \), look for two numbers that multiply to \( c \), and add to \( b \).

---

Eg.1.
Factor. \( x^2 + 6x + 8 \)

\[
(x + \_)(x + \_)
\]

What two numbers multiply to +8 but add to +6? 2 and 4

\[
= (x + 2)(x + 4)
\]

The numbers 2 and 4 fill the spaces inside the brackets.

---

Eg.2. Factor. \( x^2 - 11x + 18 \)

\[
(x + \_)(x + \_)
\]

What two numbers multiply to +18 but add to -11? -2 and -9

\[
= (x - 2)(x - 9)
\]

The numbers -2 and -9 fill the spaces inside the brackets.

---

Eg.3. Factor. \( x^2 - 7xy - 60y^2 \) The \( y \)'s can be ignored temporarily to find the two numbers. Just write them in at the end of each bracket.

\[
(x + \_y)(x + \_y)
\]

What two numbers multiply to -60 but add to -7? -12 and +5

\[
= (x - 12y)(x + 5y)
\]

The numbers -12 and +5 fill the spaces in front of the \( y \)'s.

---

Factor the trinomials if possible.

| 193. \( a^2 + 6a + 5 \) | 194. \( n^2 + 7n + 10 \) | 195. \( x^2 - x - 30 \) |

---
Factor the trinomials if possible.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>196. $q^2 + 2q - 15$</td>
<td>197. $k^2 + k - 56$</td>
<td>198. $t^2 + 11t + 24$</td>
</tr>
<tr>
<td>199. $y^2 - 7y - 30$</td>
<td>200. $g^2 - 11g + 10$</td>
<td>201. $s^2 - 2s - 80$</td>
</tr>
<tr>
<td>202. $m^2 - 12m + 27$</td>
<td>203. $x^2 - 27 - 6x$</td>
<td>204. $p^2 + 3p - 54$</td>
</tr>
<tr>
<td>205. $2a^2 - 16a + 32$</td>
<td>206. $a^2 - 14a + 45$</td>
<td>207. $6x + 2x^2 - 20$</td>
</tr>
</tbody>
</table>
Factor the trinomials if possible.

<table>
<thead>
<tr>
<th>208. $x^4 - 3x^2 - 10$</th>
<th>209. $w^6 + 7w^3 + 12$</th>
<th>210. $p^8 - 4p^4 - 21$</th>
</tr>
</thead>
<tbody>
<tr>
<td>211. $56x + x^2 - x^3$</td>
<td>212. $x^4 + 11x^2 - 80$</td>
<td>213. $x^2 - 3x + 7$</td>
</tr>
<tr>
<td>--------------------------</td>
<td>--------------------------</td>
<td>--------------------------</td>
</tr>
<tr>
<td>214. $x^2 - 6xy + 5y^2$</td>
<td>215. $x^2 + 5xy - 36y^2$</td>
<td>216. $a^2b^2 - 5ab + 6$</td>
</tr>
</tbody>
</table>

Challenge Question
Factor $2x^2 + 7x + 6$. 
Lesson #7 – Factoring Trinomials \((ax^2 + bx + c), \text{ where } a \neq 1\)

Lesson Focus:
- To use an algebraic method to factor a trinomial of the form \(ax^2 + bx + c\), using one of two strategies:
  1. Strategy #1: The Decomposition Method
  2. Strategy #2: The X-Method (or The Trial & Error Method)

Review Example: Factor \(3x^2 - 9x - 12\) completely.

Note: In this example, after we remove the GCF, the coefficient on the “a” term (the \(x^2\) term) is 1.

What if \(a \neq 1\), even after common factoring??

(ONLY use these two strategies if \(a \neq 1\). If \(a = 1\), look back at Lesson #6)
### Strategy #1: The Decomposition Method

<table>
<thead>
<tr>
<th>Steps</th>
<th>Example: Factor $9x^2 + 21x - 8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Mentally figure out two numbers that multiply to “ac” and add to “b”.</td>
<td></td>
</tr>
<tr>
<td>2. <strong>Decompose</strong> the middle term (ie the “b” term) using the answer from step #1. (NOTE: The order that you list the decomposed middle terms doesn’t matter)</td>
<td></td>
</tr>
<tr>
<td>3. Now you have four terms, so let’s factor by grouping!</td>
<td></td>
</tr>
<tr>
<td>4. Check your answer using FOIL</td>
<td></td>
</tr>
</tbody>
</table>
### Strategy #2: The X Method (or The Trial & Error Method)

<table>
<thead>
<tr>
<th>Steps</th>
<th>Example: Factor $2x^2 + 3x - 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Draw a large $\times$ under the trinomial, leaving one line of space in between.</td>
<td></td>
</tr>
<tr>
<td>2. On the LHS of the $\times$, write two numbers that multiply to “a” (ie. two factors of “a”)</td>
<td></td>
</tr>
<tr>
<td>3. On the RHS of the $\times$, write two numbers that multiply to “c” (ie. two factors of “c”)</td>
<td></td>
</tr>
<tr>
<td>4. Cross multiply, and check to see if the two numbers can add to “b”. Keep trying new combinations of numbers until you find the “winning” numbers. <strong>Put “+” or “-“ signs on the RIGHT HAND SIDE ONLY. Put the variable on the LEFT HAND SIDE ONLY.</strong></td>
<td></td>
</tr>
<tr>
<td>5. Write the numbers in the $\times$ as factors. The top two numbers form one factor. The bottom two numbers form the other factor.</td>
<td></td>
</tr>
<tr>
<td>6. Check your answer using FOIL</td>
<td></td>
</tr>
</tbody>
</table>
Example. Factor the following completely, using one of the two strategies

a) $6x^2 + 5x - 6$

b) $2x^2 + 5x + 2$

Final Thoughts on Trinomial Factoring:
- Only 2 methods have been outlined in this section. There are even more, but these are the ones I like! You may have learned an alternative method last year, in fact.
- Every teacher has their preferred method.
- Every student has their preferred method.
- YOU MAY CHOOSE WHICHEVER METHOD YOU WISH. YOU ONLY NEED TO KNOW ONE METHOD. PICK ONE AND MASTER IT!

Do not recycle the Polynomials notes!* It is absolutely imperative that you remember how to factor next year and years to come. You will not be taught again, but you will be expected to know how to do it. *I wouldn’t recycle any of Math 10, if I were you, but especially not Chapter 3.
Factoring $ax^2 + bx + c$ where $a \neq 1$

When the trinomial has an $x^2$ term with a coefficient other than 1 on the $x^2$ term, you cannot use the same method as you did when the coefficient is 1.

We will discuss 3 other methods:
1. Trial & Error   2. Decomposition   3. Algebra Tiles

**Trial & Error:**

Eg.1. Factor $2x^2 + 5x + 3$.

$2x^2 + 5x + 3 = (\_\_)(\_\_)$  

We know the first terms in the brackets have product of $2x^2$.

$2x^2 + 5x + 3 = (2x)(x)$

$2x$ and $x$ have a product of $2x^2$, place them at front of brackets.

The product of the second terms is 3. (1, 3 or -1, -3).

These will fill in the second part of the binomials.

List the possible combinations of factors.

\[
\begin{align*}
(2x + 1)(x + 3) \\
(2x + 3)(x + 1) \\
(2x - 1)(x - 3) \\
(2x - 3)(x - 1)
\end{align*}
\]

IF $2x^2 + 5x + 3$ is factorable, one of these must be the solution.

Expand each until you find the right one.

\[
(2x + 3)(x + 1) = 2x^2 + 2x + 3x + 3 = 2x^2 + 5x + 3.
\]

This is the factored form.

**Decomposition:**

Using this method, you will break apart the middle term in the trinomial, then factor by grouping.

To factor $ax^2 + bx + c$, look for two numbers with a product of $ac$ and a sum of $b$.

Eg.1. Factor. $3x^2 - 10x + 8$

1. We see that $ac = 3 \times 8 = 24$; and $b = -10$

   We need two numbers with a product of 24, but add to -10...

   -6 and -4.

2. Break apart the middle term.

3. Factor by grouping.

\[
\begin{align*}
3x^2 - 6x - 4x + 8 \\
3x(x - 2) - 4(x - 2)
\end{align*}
\]

\[
= (x - 2)(3x - 4)
\]
Eg.2. Factor. \(3a^2 - 22a + 7\) We need numbers that multiply to 21, but add to -22…

-21 and -1

Decompose middle term.

\(3a^2 - 21a - 1a + 7\) Factor by grouping.

\(= (a - 7)(3a - 1)\)

Eg.3. Factor \(2x^2 + 7x + 6\) using algebra tiles.

Arrange the tiles into a rectangle (notice the “ones” are again grouped together at the corner of the \(x^2\) tiles)

Side lengths are \((2x + 3)\) and \((x + 2)\) \(\therefore 2x^2 + 7x + 6 = (2x + 3)(x + 2)\)

Your notes here...

Factor the following if possible.

| 217. \(2a^2 + 11a + 12\) | 218. \(5a^2 - 7a + 2\) | 219. \(3x^2 - 11x + 6\) |
Factor the following if possible.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>220. $2y^2 + 9y + 9$</td>
<td>221. $5y^2 - 14y - 3$</td>
<td>222. $10x^2 - 17x + 3$</td>
</tr>
<tr>
<td>223. $2x^2 + 3x + 1$</td>
<td>224. $6k^2 - 5k - 4$</td>
<td>225. $6y^2 + 11y + 3$</td>
</tr>
<tr>
<td>226. $3x^2 - 16x - 12$</td>
<td>227. $3x^3 - 5x^2 - 2x$</td>
<td>228. $9x^2 + 15x + 4$</td>
</tr>
</tbody>
</table>
Factor the following if possible.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Expression</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>229. $21x^2 + 37x + 12$</td>
<td>230. $6x^3 - 15x - x^2$</td>
<td>231. $4t + 10t^2 - 6$</td>
</tr>
<tr>
<td>232. $3x^2 - 22xy + 7y^2$</td>
<td>233. $4c^2 - 4cd + d^2$</td>
<td>234. $2x^4 + 7x^2 + 6$</td>
</tr>
</tbody>
</table>

Challenge Question

Write a simplified expression for the following diagram of algebra tiles.

What two binomials are being multiplied in the diagram above?

Write an equation using the binomials above and the simplified product.
Lesson #8 – Factoring Special Polynomials

Lesson Focus:
- To learn the shortcut for expanding \((a \pm b)^2\)
- To learn to identify and factor the following special polynomials: Perfect Square Trinomials and Difference of Squares Binomials

Expanding \((a \pm b)^2\)

<table>
<thead>
<tr>
<th>((a + b)^2)</th>
<th>((a - b)^2)</th>
</tr>
</thead>
</table>

Note: A polynomial of the form \((a \pm b)^2\) is called a Perfect Square Trinomial.

Example: Expand the following polynomials.

a) \((5x - 2)^2\)  
b) \((6x + 7)^2\)

Factoring Perfect Square Trinomials

- All perfect square trinomials (PSTs) can be factored into: \((a \pm b)^2\)
- Therefore, if you can identify one, you can skip ALL FACTORING STEPS and go straight to the final answer. NO WORK NEED BE SHOWN.

Algebraically, we can spot one by first noticing the following: \(ax^2 + bx + c\)

Note: “a” and “c” MUST be positive for the polynomial to be a perfect square trinomial. WHY?
Example. Factor the following completely. Check your answer by expanding (FOIL).

a) \( x^2 - 6x + 9 \)  
b) \( 121d^2 + 66d + 9 \)

Factoring Difference of Squares Binomials

A difference of squares binomial is a binomial in the form \( a^2 - b^2 \).
For example: \( x^2 - 81 \)

Some other examples:

Note: It must be a DIFFERENCE (\(-\)) NOT a sum (\(+\)).

The ONLY ways to factor a binomial are:
1. Common Factor (Remove the GCF)
2. Difference of Squares

Example: Factor \( x^2 - 81 \) completely.

Don’t forget the Golden Rule of factoring! The first step of any factoring process is to ALWAYS...

Let’s Play..... **SPOT THE DIFFERENCE OF SQUARES!!!!!**

Circle the numbers of questions that are differences of squares. If it isn’t, circle the part of the binomial that is preventing it from being a difference of squares. Lastly, factor each question.

1. \( x^2 - 4y^2 \)  
2. \( 49x^3 - 16 \)  
3. \( 25n^2 + 100 \)
4. \( 18k^2 - 98 \)  
5. \( \frac{x^4}{9} - y^6 \)
6. \( x^4 - 16 \)
A Difference of Squares

235. Write a simplified expression for the following diagram.

\[
\begin{array}{c}
\text{Solution: } x^2 - 2x + 2x - 4 \\
\end{array}
\]

What two binomials are being multiplied in the diagram above?
\((x - 2)(x + 2)\)

Write an equation using the binomials above and the simplified product.
\[x^2 - 4 = (x - 2)(x + 2)\]

Factored Form

236. Write a simplified expression for the following diagram.

What two binomials are being multiplied above?

Write an equation using the binomials above and the simplified product.

237. Write a simplified expression for the following diagram.

What two binomials are being multiplied above?

Factor the polynomial represented above by writing the binomials as a product (multiplication).

238. Write a simplified expression for the following diagram.

What two binomials are being multiplied above?

Factor the polynomial represented above by writing the binomials as a product (multiplication).
Factoring a Difference of Squares: \( a^2 - b^2 \)

**Conjugates:** Sum of two terms and a difference of two terms.

Learn the pattern that exists for multiplying conjugates.

\[(x + 2)(x - 2) = x^2 - 2x + 2x - 4 = x^2 - 4\] The two middle terms cancel each other out.

We can use this knowledge to quickly factor polynomials that look like \( x^2 - 4 \).

**Eg.1.** Factor \( x^2 - 9 \).

\[= (x + 3)(x - 3) \quad \text{Square root each term, place them in 2 brackets with opposite signs ( + and -).} \]

**Eg.2.** Factor \( 100a^2 - 81b^2 \)

\[= (10a + 9b)(10a - 9b) \quad \text{Square root each term, place them in 2 brackets with opposite signs ( + and -).} \]

Factor the following completely.

<table>
<thead>
<tr>
<th>239. ( a^2 - 25 )</th>
<th>240. ( x^2 - 144 )</th>
<th>241. ( 1 - c^2 )</th>
</tr>
</thead>
</table>

I recognize a polynomial is a difference of squares because____________________________________________________
________________________________________________________________________________________________________________________
________________________________________________________________________________________________________________________
Factor the following completely.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>242. $4x^2 - 36$</td>
<td>243. $9x^2 - y^2$</td>
<td>244. $25a^4 - 36$</td>
</tr>
<tr>
<td>245. $49t^2 - 36u^2$</td>
<td>246. $7x^2 - 28y^2$</td>
<td>247. $-18a^2 + 2b^2$</td>
</tr>
<tr>
<td>248. $-9 + d^4$</td>
<td>249. $\frac{a^2}{9} - \frac{b^2}{16}$</td>
<td>250. $\frac{x^2 + y^2}{49} - 1$</td>
</tr>
</tbody>
</table>
## Factoring a Perfect Square Trinomial

<table>
<thead>
<tr>
<th>251. Write an expression for the following diagram (do not simplify):</th>
<th>252. Write an expression for the following diagram (do not simplify):</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram" /></td>
<td><img src="image2.png" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>What two binomials are being multiplied above?</strong></td>
<td><strong>What two binomials are being multiplied above?</strong></td>
</tr>
<tr>
<td><strong>Write an equation using the binomials above and the simplified product.</strong></td>
<td><strong>Write an equation using the binomials above and the simplified product.</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>253. Write an expression for the following diagram (do not simplify):</th>
<th>254. Write an expression for the following diagram (do not simplify):</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3.png" alt="Diagram" /></td>
<td><img src="image4.png" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>What two binomials are being multiplied above?</strong></td>
<td><strong>What two binomials are being multiplied above?</strong></td>
</tr>
<tr>
<td><strong>Write an equation using the binomials above and the simplified product.</strong></td>
<td><strong>Write an equation using the binomials above and the simplified product.</strong></td>
</tr>
</tbody>
</table>
PERFECT SQUARE TRINOMIALS
You may use the methods for factoring trinomials to factor *trinomial squares* but recognizing them could make factoring them quicker and easier.

Eg.1. Factor.
\[ x^2 + 6x + 9 \]
Recognize that the first and last terms are both perfect squares.
\[(x + 3)^2 \]
Guess by taking the square root of the first and last terms and put them in two sets of brackets.
Check: Does \(2(x)(3) = 6x\)
Yes! Trinomial Square!
\[(x + 3)^2 \]
Answer in simplest form.

Eg.2. Factor.
\[ 121m^2 - 22m + 1 \]
\[(11m - 1)^2 \]
Guess & Check. \(2(11m \times -1) = -22m.\)
Since the middle term is negative, binomial answer will be a subtraction.

Factor the following.
<table>
<thead>
<tr>
<th>255. (x^2 + 14x + 49)</th>
<th>256. (4x^2 - 4x + 1)</th>
<th>257. (9b^2 - 24b + 16)</th>
</tr>
</thead>
<tbody>
<tr>
<td>258. (64m^2 - 32m + 4)</td>
<td>259. (81n^2 + 90n + 25)</td>
<td>260. (81x^2 - 144xy + 64y^2)</td>
</tr>
</tbody>
</table>
Create a Factoring Flowchart.
Start with the first thing you should do... *collect like terms.*
### Combined Factoring
Factor the following completely.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>261. $3a^2 - 3b^2$</td>
<td>262. $4x^2 + 28x + 48$</td>
<td>263. $x^4 - 16$</td>
</tr>
<tr>
<td>264. $2y^2 - 2y - 24$</td>
<td>265. $16 - 28x + 20x^2$</td>
<td>266. $m^4 - 5m^2 - 36$</td>
</tr>
<tr>
<td>267. $x^8 - 1$</td>
<td>268. $x^2 - xy^2$</td>
<td>269. $x^4 - 5x^2 + 4$</td>
</tr>
</tbody>
</table>

### Higher Difficulty...
For some of the following questions, you may try substituting a variable in the place of the brackets to factor first, and then replace brackets.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>270. $(a + b)^2 - c^2$</td>
<td>271. $(c - d)^2 - (c + d)^2$</td>
<td>272. $(m + 7)^2 + 7(m + 7) + 12$</td>
</tr>
</tbody>
</table>
273. Factor.
\[(x + 2)^2 - (x - 3)^2\]

274. Find all the values of \(k\) so that \(x^2 + kx - 12\) can be factored.

275. For which integral values of \(k\) can \(3x^2 + kx - 3\) be factored.

276. What value of \(k\) would make \(kx^2 + 24xy + 16y^2\) a perfect square trinomial?

277. What value of \(k\) would make \(2kx^2 - 24xy + 9y^2\) a perfect square trinomial?

278. For which integral values of \(k\) can \(6x^2 + kx + 1\) be factored.
   a. 5, 7
   b. ±5, ±7
   c. –5, –7
   d. all integers from 5 to 7.

279. Expand and simplify.
\[-2(3m + 4)^2\]

280. If \(a = 2x + 3\), write \(a^2 - 5a + 3\) in terms of \(x\).
### 281. Incorrect Multiplication
Lindsay was helping Anya with her math homework. She spotted an error in Anya's multiplication below. Find and correct any errors.

Multiply:
\[ 5x(2x+1)+2(2x+1) \]
\[ = 10x+1+4x+2 \]
\[ = 14x+3 \]

### 282. Factoring Assistance
When asked to factor the following polynomial, Timmy was a little unsure where to start.
Factor: \( 10x + 5 + 2xy + y \)
What type of factoring could you tell him to perform to help him along?

### 283. Error in Factoring
Find and correct any errors in the following factoring.

\[ 2x^2 - 5x - 12 \]
\[ = 2x^2 - 12x + 2x - 12 \]
\[ = 2x(x-6) + 2(x-6) \]
\[ = (2x+2)(x-6) \]

### 284. Error in Multiplication
Explain why \( 3x^2 - 17x + 10 \neq (3x + 1)(x + 10) \)

### 285. Incorrect Multiplication
Find and correct any errors in the following multiplication.

\[ (x^2 + 2)^2 \]
\[ = x^4 + 4 \]

### 286. Algebra Tiles
Explain why it is uncommon to use algebra tiles to multiply the following.

\[ (x + 1)^3 \]

### 287. Expression Multiplication
Multiply the expression above.
Answers:

1. 5, -7
2. 13
3. x, y
4. no, negative exponent
5. yes
6. no, negative exponent
7. no, exponent not a whole number
8. yes
9. no, exponent not a whole number
10. 1, binomial
11. 3, trinomial
12. 7, polynomial
13. 0, monomial
14. Many possibilities
15. Many possibilities
16. 5x
17. \(-3x^2\)
18. \(x^2 + 3x + 4\)
19. \(-4x^2 - 2x - 3\)
20. \(3x^2 + 3x + 4\)

21. 

22. 

23. The two terms cancel each other, resulting in a sum of 0.

24. The two expressions cancel each other, resulting in a sum of 0.

25. 0

26. \(-x^2 + x - 1\)

27. 

28. 

29. \(-3x + 4\)

30. \(-x^2 + 5x + 2\)

31. 0

32. 0

33. 

34. You cannot subtract / take away, or cancel the "negative-x" tile from the first expression because there was not one there. The same problem arises with the "+2".

35. Raj added "zero" in the form of opposite tiles so that he could then subtract the \((-x + 2)\) from the first expression.

36. \(7x - 6\)

37. \(5x^2 + 5x - 8\)

38. \(x^2 - 4x - 8\)

39. Same shape.

40. Same letter, same exponent (degree).

41. \(-9x + 9y, -45\)

42. \(3x^3 - 5x^2 - 6, 30\)

43. \(11x^3y^2 - 5, -797\)

44. \(6x + 17\)

45. \(12a + 4b\)

46. \(4x + 4\)

47. 7a

48. \(12x - 5y\)

49. \(19a - 3b\)

50. \(13x^2 - x - 5\)

51. \(-2mn^2 - 2mn + n\)

52. \(-y^2 + 2y - 4\)

53. \(10x^2 - 6xy + 3x + 6\)

54. A rectangle that is 3 by 3 has an area of 9 square units.

55. A rectangle that is 3 by 4 has an area of 12 square units.

56. 20

57. Colour one side differently. The \((-2)\) could be shaded.

58. \(-12\)

59. \(-2a\)

60. Both edges would be shaded to represent negatives.

61. 12

62. 20

63. 252

64. \((30 + 2)(10 + 4)\)

65. 408

66. 252

67. \(= 448\)

68. \(= 408\)

69. 345

70. 2496

71. 5329

72. \((4)(5) = 20\)

73. \((-3)(6) = -18\)

74. \((x)(5) = 5x\)

75. \((3)(x) = x^2\)

76. \((x)(-x) = -x^2\)

77. \((x)(2x) = 2x^2\)

78. \((3)(2x) = 6x\)

79. \((-3)(2x) = -6x\)

80. \((2)(-3x) = -6x\)

81. \(\frac{6a}{3x} = 6, \text{ length is 6 units.}\)

82. \(\frac{6a}{3x} = 2x, \text{ length is 2x units.}\)

83. \(-\frac{6a}{3x} = -2x, \text{ length is -2x units.}\)

84. \((2x)(x + 1) = 2x^2 + 2x\)

85. \((2x)(-x + 1) = -2x^2 + 2x\)

86. \((2x)(-x - 2) = 2x^2 - 4x\)

87. \((-2x)(x - 3) = -2x^2 + 6x\)

88. 

89. 

90. 

91. \(-x^2 - 3x\)

92. \(-6x^2 - 9x\)

93. \(\frac{x^2 + 3x}{x} \text{ or } (x^2 + 3x)(x)\)

94. \(\frac{3x}{x} \text{ or } (-x^2 - 3x)(x)\)

95. \(\text{length is } x + 3\)

96. \(-x^2 - 3x\)

97. \(-6x^2 - 9x\)

98. \(\frac{x^2 + 3x}{x} \text{ or } (x^2 + 3x)(x)\)

99. \(\frac{3x}{x} \text{ or } (-x^2 - 3x)(x)\)

100. \(\text{length is } x + 3\)
95. \( \frac{2x^2 - 8x}{2x} \) or \((2x^2 - 8x) / (2x)\\ length \ is \ x - \frac{4}{x}\\ 96. \ 2x^2 + 6x\\ \ x + 3\\ \ 2x\\ 97. \ 6x + 18\\ \ 6\\ \ x + 3\\ 98. \ 2x^2 + 3x\\ \ 2x + 3\\ \ x\\ 99. \ 6ax^2b^3\\ 100. \ -10x^2y^3\\ 101. \ -12x^3\\ \ \frac{3}{\frac{1}{a}b^3} \ or \ \frac{3ab^3}{a}\\ 103. \ -5t^4\\ 104. \ 5zx^2\\ \ \frac{x^2}{y}\\ 106. \ -20cd^4\\ 107. \ 6x^2y^2\\ 108. \ a\\ 109. \ 2x^2 - 9x - 5\\ 110. \ 2x(x + 1) = 2x^2 + 2x\\ 111. \ 2x(2x + 1) = 4x^2 + 2x\\ 112. \ 2x(x - 2) = 2x^2 - 4x\\ 113. \ -2x(x - 3) = -2x^2 + 6x\\ 114.\\ \frac{\text{Length: } x + 2}{\text{Area: } (x + 2)(x - 2) = x^2 - 4}\\ 115.\\ \frac{\text{Area: } x^2 + 6x + 9}{\text{Length: } x + 3\\ \text{Width: } x + 2}\\ 116. \ 4 - x^2\\ See \ solutions \ guide \ for \ area \ model.\\ 117. \ -x^2 + 4x - 3\\ See \ solutions \ guide \ for \ area \ model.\\ 118. \ 6x^2 + 5x + 1\\ See \ solutions \ guide \ for \ area \ model.\\ 119. \ A = lw\\ l = \frac{A}{w}\\ \frac{x^2 + 3x + 2}{x + 1}\\ 120. \ \frac{2x^2 + 5x + 2}{2x + 1}\\ \frac{\text{Length: } x + 2}{\text{Area: } (x + 2)(x - 2) = x^2 - 4}\\ 121. \ \\ \frac{\text{Length: } x + 2}{\text{Area: } (x + 2)(x - 2) = x^2 - 4}\\ 122. Area: x^2 + 5x + 6\\ Lengthx + 3\\ Width: x + 2\\ 123. a: x^2 + 6x + 9\\ Lengthx + 3\\ Width: x + 2\\ 124. Area: 2x^2 + 7x + 6\\ Length: 2x + 3\\ Width: x + 2\\ 125. x^2 - 2x - 3\\ 126. 4x^2 + 4x + 1\\ 127. x^2 - 16\\ 128. x^2 = 3x - 10\\ 129. 2x^2 - 5x - 3\\ 130. x^2 - 6x + 9\\ 131. x^2 + 6x + 4\\ 132. 6x^2 - 3x - 3\\ 133. 4x^2 - 1\\ 134. x^2 + 4x + 4\\ 135. 4x^2 + 20x + 25\\ 136. x^2 + 2x - 7x + 4\\ 137. x^2 - 10x + 26x - 5\\ 138. 6x^2 - 5x^2 - 4x - 3\\ 139. x^2 + 6x^2 + 12x + 8\\ 140. x^2 + 2x - 2x - 4\\ (x + 2)(x - 2) = x^2 - 4\\ 141. x^2 - 3x - 9\\ (x + 3)(x - 3) = x^2 - 9\\ (x + 3)(x - 3) = x^2 - 9\\ 142. 4x^2 + 4x + 4 = 4x^2 + 4x + 4\\ (2x + 2)(2x - 2) = 4x^2 - 4\\ (2x + 2)(2x - 2) = 4x^2 - 4\\ 143. 9x^2 + 12x - 12x - 16\\ (3x + 4)(3x - 4) = 9x^2 - 16\\ (3x + 4)(3x - 4) = 9x^2 - 16\\ 144. x^2 - 9\\ 145. 4x^2 - 9\\ 146. 9x^2 - 1\\ 147. x^2 - 2y\\ 148. 3b^2 - 147\\ 149. -2x^2 + 50\\ 150. 2x^2 + 15x + 30\\ 151. 3x^2 - 11x - 38\\ 152. 30x^2 + 61x + 25\\ 153. -12y^2 - 20y - 1\\ 154. 3x^2 + 2\\ 155. 3x^2 - x^4\\ 156. z^2\\ 157. z^2 + 3 = 24\\ 158. 2z^2 = 16\\ 159. 2x^2 = 18\\ 160. 5x^2xaxab
193. \((a + 5)(a + 1)\)
194. \((n + 5)(n + 2)\)
195. \((x - 6)(x + 5)\)
196. \((q + 5)(q - 3)\)

197. \((k - 7)(k + 8)\)
198. \((x + 8)(t + 3)\)
199. \((y - 10)(y + 3)\)
200. \((g - 10)(g - 1)\)
201. \((s - 10)(s + 8)\)
202. \((m - 3)(m - 9)\)
203. \((x - 9)(x + 3)\)
204. \((p + 9)(p - 6)\)
205. \(2(y - 4)^2\)
206. \((a - 9)(a - 5)\)
207. \((2x + 5)(x - 2)\)
208. \((x^2 - 5)(x^2 + 2)\)
209. \((w^2 + 4)(w^2 + 3)\)
210. \((p^4 - 7)(p^4 + 3)\)
211. \(x(8 - x)(7 + x)\)
212. \((x^2 + 16)(x^2 - 5)\)
213. Not factorable.
214. \((x - 5y)(x - y)\)
215. \((x + 9y)(x - 4y)\)
216. \((ab - 3)(ab - 2)\)

Challenge \((2x + 3)(x + 2)\)

217. \((a + 4)(2a + 3)\)
218. \((5a - 2)(a - 1)\)
219. \((3x - 2)(x - 3)\)
220. \((2y + 3)(y + 3)\)
221. \((5y + 1)(y - 3)\)
222. \((2x - 3)(5x - 1)\)
223. \((2x + 1)(x + 1)\)
224. \((3k - 4)(2k + 1)\)
225. \((2y + 3)(3y + 1)\)
226. \((3x + 2)(x - 6)\)
227. \((x^3 + 1)(x - 2)\)
228. \((3x + 1)(3x + 4)\)
229. \((7x + 3)(3x + 4)\)
230. \((x^3 - 5)(2x + 3)\)
231. \(2(5t - 3)(t + 1)\)
232. \((3x - y)(x - 7y)\)
233. \((2c - d)(2c - d)\)
234. \((x^2 + 2)(2x^2 + 3)\)

Challenge

\[
\begin{align*}
(x^2 - 4) \\
(x^2 + 2)(x - 2) \\
x^2 - 4 &= (x + 2)(x - 2) \\
\end{align*}
\]

235. Answered on page.
236. \(x^2 - 9\)
\[
\begin{align*}
(x + 3)(x - 3) \\
x^2 - 9 &= (x + 3)(x - 3) \\
\end{align*}
\]

237. \(4x^2 - 4\)
\[
\begin{align*}
(2x + 2)(2x - 2) \\
4x^2 - 4 &= (2x + 2)(2x - 2) \\
\end{align*}
\]

238. \(9x^2 - 16\)
\[
\begin{align*}
(3x + 4)(3x - 4) \\
9x^2 - 16 &= (3x + 4)(3x - 4) \\
\end{align*}
\]

239. \((a + 5)(a - 5)\)
240. \((x + 12)(x - 12)\)
241. \((1 + c)(1 - c)\)
242. \(4(x + 3)(x - 3)\)

Note:

\[
\begin{align*}
(2x + 6)(2x - 6) &\text{ is not fully} \\
&\text{ factored because there is GCF} \\
&\text{ that can be removed.} \\
\end{align*}
\]

243. \((3x + y)(3x - y)\)
244. \((5a + 6)(5a - 6)\)
245. \((7t + 6u)(7t - 6u)\)
246. \((7x + 2y)(x - 2y)\)
247. \(-2(3a + b)(3a - b)\)
248. \((d^2 + 3)(d^2 - 3)\)
249. \((\frac{2}{3} + \frac{3}{4})(\frac{2}{3} - \frac{3}{4})\)
250. \((\frac{x + y}{1})(\frac{x - y}{1})\)
251. \(x^2 + 4x + 4\)
\[
\begin{align*}
&= (x + 2)(x + 2) \\
&= x^2 + 4x + 4 = (x + 2)(x + 2) \\
&= \text{Factored Form: } (x + 2)^2 \\
\end{align*}
\]

252. \(x^2 - 3x - 3x + 9\)
\[
\begin{align*}
&= (x - 3)(x - 3) \\
&= x^2 - 6x + 9 = (x - 3)(x - 3) \\
&= \text{Factored Form: } (x - 3)^2 \\
\end{align*}
\]

253. \(9x^2 + 12x + 12x + 16\)
\[
\begin{align*}
&= (3x + 4)(3x + 4) \\
&= 9x^2 + 24x + 16 = (3x + 4)(3x + 4) \\
&= \text{Factored Form: } (3x + 4)^2 \\
\end{align*}
\]

254. \(4x^2 - 2x - 2x + 1\)
\[
\begin{align*}
&= (2x - 1)(2x - 1) \\
&= 4x^2 - 4x + 1 = (2x - 1)(2x - 1) \\
&= \text{Factored Form: } (2x - 1)^2 \\
\end{align*}
\]

255. \((x + 7)^2\)
256. \((2x - 1)^2\)
257. \((3b - 4)^2\)
258. \(4(m - 1)^2\)

Additional Material:

288. \(x = \pm 6\)
289. \(x = \pm 4\)
290. \(x = \pm \frac{1}{2}\)
291. \(x = \pm \sqrt{7}\)
292. \(x = -8 0r 7\)
293. \(x = -3 0r 7\)
294. \(x = \frac{1}{2}\)
295. \(n = 3 0r \frac{1}{2}\)
296. \(a = b 0r a = -b\)
297. \(x^2 + 2x + 3 + 2 = (x + 1)(x^2 + x + 2)\)
298. \(t^2 + 3t^2 - 5t - 4 = (t + 4)(t^2 - t - 1)\)
299. \(m^3 + 2m^2 - m - 4 = (m + 1)(m^2 - m - 1)\)
300. \(x^3 - 4x^2 - 2x + 8 = (x - 2)(x^2 + 2x - 4)\)
301. \(m^3 + 3m^2 - 4 = (m + 2)(m^2 + m - 2)\)
302. \(a^3 - 3a + 6 = (a + 1)(a^2 - a + 2) + 6\)
303. \(x^3 + 2x^2 - n - 2 = (n - 1)(x + 2)\)
304. \(a^2 + 25a + 14 = (3x - 5y)(x^2 + 1)\)
305. \(12a + 3a^2 - 20a - 5 = (3a^2 - 5y)(a + 1)\)
306. \(4y^2 - 29 = (2y - 5)(2y + 5) - 4\)

282. Factor by grouping.
283. The first step in decomposition should have read \(2x^2 - 8x + 3x - 12\)
\[
\begin{align*}
2x(x - 4) &+ 3(x - 4) \\
(2x + 3)(x - 4) \\
\end{align*}
\]
284. If we expand the two binomials, the middle term will not equal -17.
285. \((x^2 + 2)(x^2 + 2)\)
\[
\begin{align*}
x^4 + 2x^2 + 2x^2 + 4 \\
x^4 + 4x^2 + 4 \\
\end{align*}
\]
286. We would need to describe the tiles in 3-dimensions.
287. \(x^2 + 3x^2 + 3x + 1\)

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