Foundations & Pre-Calculus 10
Homework & Notebook

Name:______________________________ **Block:**__________
Teacher: Miss Zukowski **Date Submitted:** ____/______/ 2018

Unit # 4 : Polynomials

Submission Checklist: *(make sure you have included all components for full marks)*

- ❏ Cover page & Assignment Log
- ❏ Class Notes
- ❏ Homework *(attached any extra pages to back)*
- ❏ Quizzes *(attached original quiz + corrections made on separate page)*
- ❏ Practice Test/ Review Assignment

Assignment Rubric: Marking Criteria			
Excellent (5) - Good (4) - Satisfactory (3) - Needs Improvement (2) - Incomplete (1) - NHI (0)		**Self Assessment**	**Teacher Assessment**
Notebook	• *All* teacher notes complete • Daily homework assignments have been recorded & completed *(front page)* • Booklet is neat, organized & well presented *(ie: name on, no rips/stains, all pages, no scribbles/doodles, etc)*	/5	/5
Homework	• *All* questions attempted/completed • *All* questions **marked** *(use answer key, correct if needed)*	/5	/5
Quiz *(1mark/dot point)*	• Corrections have been made **accurately** • Corrections made in a *different colour pen/pencil* *(+½ mark for each correction on the quiz)*	/2	/2
Practice Test *(1mark/dot point)*	• Student has completed all questions • Mathematical working out leading to an answer is shown • Questions are **marked** *(answer key online)*	/3	/3
Punctuality	• *All checklist items were submitted, and completed on the day of the unit test. (-1 each day late)*	/5	/5
Comments:		/20	/20

Homework Assignment Log

& Textbook Pages:____________________________________

Date	Assignment/Worksheet	Due Date	Completed?

Quizzes & Tests:

What?	When?	Completed?
Quiz 1		
Quiz 2		
Unit/ Chapter test		

HW Mark: 10	9	8	7	6	RE-Submit

Polynomials

This booklet belongs to:___________________Period____

LESSON #	DATE	QUESTIONS FROM NOTES	Questions that I find difficult
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		REVIEW	
		TEST	

Your teacher has important instructions for you to write down below.

Polynomials: Key Terms

Term	Definition	Example
Term		
Coefficient		
Variable		
Constant		
Monomial		
Binomial		
Trinomial		
Polynomial		
Degree of a term		
Degree of a Polynomial		
Algebra Tiles		
Combine like-terms		
Area Model		
Distribution (Expanding)		
FOIL		
GCF vs LCM		
Factoring using a GCF		
Factoring by Grouping		
Factoring $ax^2 + bx + c$ when $a = 1$		
Factoring $ax^2 + bx + c$ when $a \neq 1$		
Difference of Squares		
Perfect Square Trinomial		

Lesson #1 – <u>Intro to Polynomials & Addition/Subtraction</u>

Term: A number and or variable connected by ______________ or _____________ (also called a monomial)

**Coefficients must be _________________ and exponents must be ______________________

ie.

	# of Terms	Example
Monomial		
Binomial		
Trinomial		
Polynomial*		

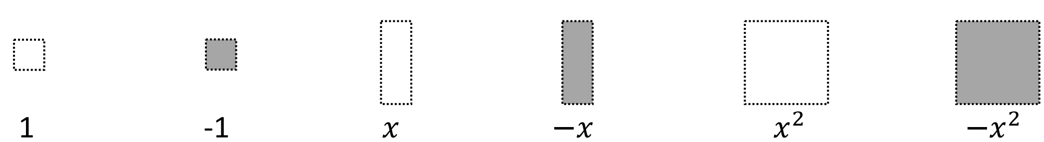
* is a general name for an expression with 1 or more terms.

Degree of a…

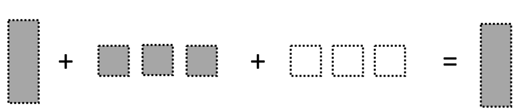
Term: ie.

Polynomial: ie.

Algebra Tile Legend:



Zero Principle: the idea that the addition of opposites cancel each other out and the result is zero



example: Subtract the following using algebra tiles $(2x-1)-(-x+2)$

I. Simplify the following

- you can simplify expressions by collecting ______________ terms (terms with __________________ variables and exponents)

1. $7x + 3y + 5x - 2y$

2. $3x^2 + 4xy - 6xy + 8x^2 - 3yx$

II. Add/Subtract the following

3. $(x^2 + 4x - 2) + (2x^2 - 6x + 9)$

4. $(4x^2 - 2x + 3) - (3x^2 + 5x - 2)$

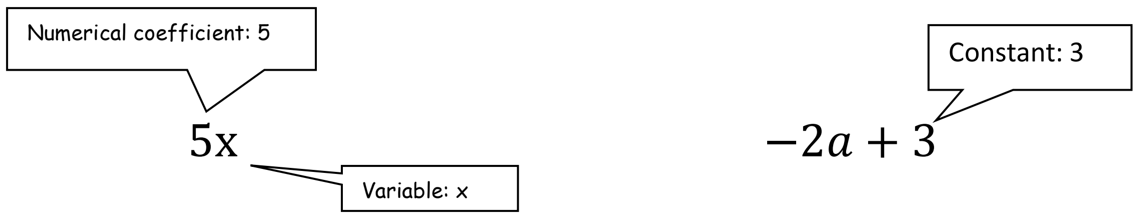
ASSIGNMENT # 1

pages 4- 11 Questions #1-53

What is a Polynomial?

What is a Term?

A **term** is a number and/or variable connected by multiplication or division. One term is also called a **monomial**.



The following are terms: 5, x, $3x$, $5x^2$, $\frac{3x}{4}$, $-2xy^2z^3$

Each term may have a **coefficient, variable(s) and exponents.** One term is also called a **monomial.**

If there is no variable present…we call the term a **constant.**

Answer the questions below.

1. What is/are the coefficients below? $5xy^2 - 7x + 3$	2. What is/are the constant(s) below? $12x^2 - 5x + 13$	3. What is/are the variable(s) below? $5xy^2 + 3$

A **polynomial** is an expression made up of **one or more terms** connected to the next by addition or subtraction.

We say a polynomial is any expression where the **coefficients are real** numbers and all **exponents are whole** numbers. That is, no <u>variables</u> under radicals (rational exponents), no <u>variables</u> in denominators (negative exponents).

The following are polynomials:

x, $2x - 5$, $5 + 3x^2 - 12y^3$, $\frac{x^2+3x+2}{2}$, $\sqrt{3}x^2 + 5y - z$

The following are **NOT** polynomials:

x^{-2}, $3\sqrt{x}$, $4xy + 3xy^{-3}$, $12xz + 3^x$

Which of the following are not polynomials? Indicate why.

4. $3xyz - \frac{2}{x}$	5. $\frac{1}{-5}x^3 - 5y$	6. $2x - 4y^{-2}$
7. $(3x+2)^{\frac{1}{3}}$	8. $\sqrt{3} + x^2 - 5$	9. $\frac{5}{3}x - 2^x$

Classifying polynomials:

By Number of Terms:

- **Monomial**: one term. Eg. $7x$, 5, $-3xy^3$
- **Binomial**: two terms Eg. $x+2$, $5x - 3y, y^3 + \frac{5x}{3}$
- **Trinomial**: three terms Eg. $x^2 + 3x + 1$, $5xy - 3x + y^2$
- **Polynomial**: four terms Eg. $7x + y - z + 5$, $x^4 - 3x^3 + x^2 - 7x$

By Degree:
To find the degree of a *term*, add the exponents within that term.

Eg. $-3xy^3$ is a 4th degree term because the sum of the exponents is 4.
$5z^4y^2x^3$ is a 9th degree term because the sum of the exponents is 9.

To find the degree of a polynomial first calculate the degree of each term. The highest degree amongst the terms is the degree of the polynomial.

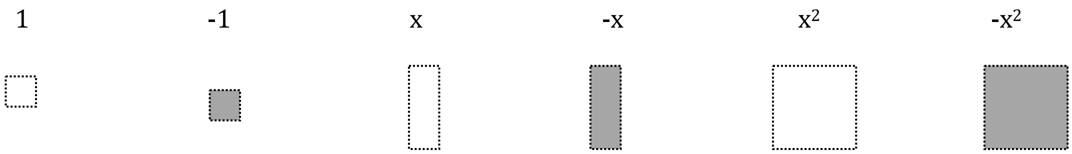
Eg. $x^4 - 3x^3 + x^2 - 7x$ is a 4th degree polynomial. The highest degree term is x^4.
$3xyz^4 - 2x^2y^3$ is a 6th degree binomial. The highest degree term is $3xyz^4$ (6th degree)

Classify each of the following by degree and by number of terms.

10. $2x + 3$ Degree: __1__ Name: _Binomial__	11. $x^3 - 2x^2 + 7$ Degree: ______ Name: ______________	12. $2a^3b^4 + a^2b^4 - 27c^5 + 3$ Degree: ______ Name: ______________
13. 7 Degree: ______ Name: ______________	14. Write a polynomial with one term that is degree 3.	15. Write a polynomial with three terms that is degree 5.

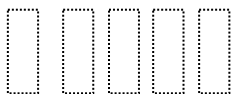
Algebra Tiles

The following will be used as a legend for algebra tiles in this guidebook.

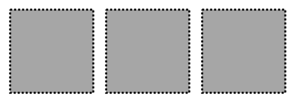


Write an expression that can be represented by each of the following diagrams.

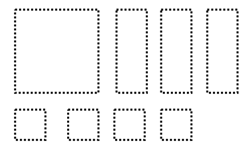
16.



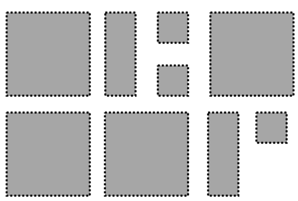
17.



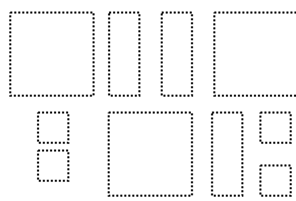
18.



19.



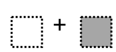
20.



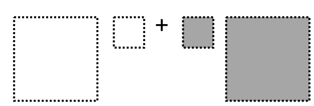
21. Draw a diagram to represent the following polynomial.
$3x^2 - 5x + 6$

22. Draw a diagram to represent the following polynomial.
$-3x^2 + x - 2$

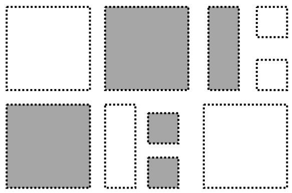
23. What happens when you add the following?



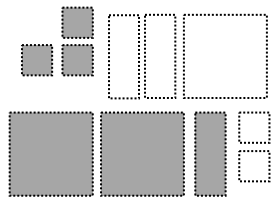
24. What happens when you add the following?



25. Simplify by cancelling out tiles that add to zero. Write the remaining expression.



26. Simplify by cancelling out tiles that add to zero. Write the remaining expression.



27. Represent the following addition using algebra tiles. Do not add.
$x + (x - 1)$

28. Represent the following addition using algebra tiles. Do not add.

$(5x + 3) + (2x + 1)$

29. Use Algebra tiles to add the following polynomials.
(collect like-terms)

$(2x - 1) + (-5x + 5)$

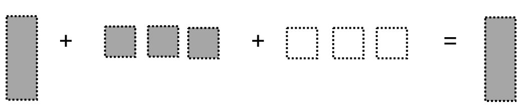
30. Use Algebra tiles to add the following polynomials.
(collect like-terms)

$(2x^2 + 5x - 3) + (-3x^2 + 5)$

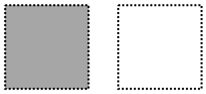
<u>The Zero Principle</u>:

The idea that opposites cancel each other out and the result is zero.

Eg. $x + 3 + (-3) = x$ The addition of opposites did not change the initial expression.

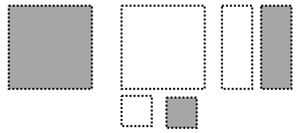


31. What is the sum of the following tiles?



Sum__________

32. If you add the following to an expression, what have you increased the expression by?

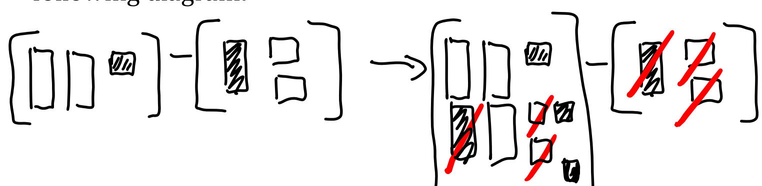


33. Represent the following subtraction using algebra tiles.

$(2x - 1) - (-x + 2)$

34. Why can you not simply "collect like-terms" when subtracting the two binomials in the previous question?

35. When asked to subtract $(2x - 1) - (-x + 2)$, Raj drew the following diagram:



Explain how Raj applied the zero principle to subtract the polynomials.

36. Use Algebra tiles to subtract the following polynomials.

$$(2x - 1) - (-5x + 5)$$

37. Use Algebra tiles to subtract the following polynomials.

$$(2x^2 + 5x - 3) - (-3x^2 + 5)$$

38. Use Algebra tiles to subtract the following polynomials.

$$(-2x^2 - 4x - 3) - (-3x^2 + 5)$$

Like Terms

39. When considering algebra tiles, what makes two tiles "alike"?

40. What do you think makes two algebraic terms alike? (Remember, tiles are used to represent the parts of an expression.)

Collecting Like Terms without tiles:

Exactly the same variable & exponents.

You have previously been taught to combine like terms in algebraic expressions.

Terms that have the same variable factors, such as 7*x* and 5*x*, are called ***like terms***.

Simplify any expression containing like terms by adding their coefficients.

Eg.1. Simplify:

$7x + 3y + 5x - 2y$
$7x + 5x + 3y - 2y$
$= 12x + y$

Eg.2. Simplify

$3x^2 + 4xy - 6xy + 8x^2 - 3yx$
$3x^2 + 8x^2 + 4xy - 6xy - 3xy$
$= 11x^2 - 5xy$

Remember … $3yx$ is the same

Simplify by collecting like terms. Then evaluate each expression for $x = 3, y = -2$.

41. $3x + 7y - 12x + 2y$

42. $2x^2 + 3x^3 - 7x^2 - 6$

43. $5x^2y^3 - 5 + 6x^2y^3$

Adding & Subtracting Polynomials without TILES.

ADDITION
To <u>add</u> polynomials, collect like terms.

Eg.1. $(x^2 + 4x - 2) + (2x^2 - 6x + 9)$

Horizontal Method:

$=x^2 + 4x - 2 + 2x^2 - 6x + 9$
$=x^2 + 2x^2 + 4x - 6x - 2 + 9$

$=3x^2 - 2x + 7$

Vertical Method:

$x^2 + 4x - 2$
$\underline{2x^2 - 6x + 9}$

$= 3x^2 - 2x + 7$

SUBTRACTION
It is important to remember that the subtraction refers to all terms in the bracket immediately after it.

To <u>subtract</u> a polynomial, determine the opposite and add.

Eg.2. $(4x^2 - 2x + 3) - (3x^2 + 5x - 2)$

Multiplying each term by -1 will remove the brackets from the **second** polynomial.

This question means the same as:

$(4x^2 - 2x + 3) - \mathbf{1}(3x^2 + 5x - 2)$
$= 4x^2 - 2x + 3 - 3x^2 - 5x + 2$
$= 4x^2 - 3x^2 - 2x - 5x + 3 + 2$

$= x^2 - 7x + 5$

We could have used vertical addition once the opposite was determined if we chose.

Add or subtract the following polynomials as indicated.

44. $(4x + 8) + (2x + 9)$	45. $(3a + 7b) + (9a - 3b)$	46. $(7x + 9) - (3x + 5)$
47. Add. $(4a - 2b)$ $\underline{+(3a + 2b)}$	48. Subtract. $(7x - 3y)$ $\underline{-(-5x + 2y)}$	49. Subtract. $(12a - 5b)$ $\underline{-(-7a - 2b)}$

Add or subtract the following polynomials as indicated.

50. $(5x^2 - 4x - 2) + (8x^2 + 3x - 3)$

51. $(3m^2n + mn - 7n) - (5m^2n + 3mn - 8n)$

52. $(8y^2 + 5y - 7) - (9y^2 + 3y - 3)$

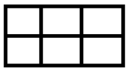
53. $(2x^2 - 6xy + 9) + (8x^2 + 3x - 3)$

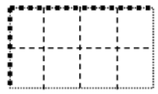
Your notes here…

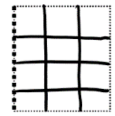
Name: ________________________

Lesson #2 – Multiplication Models

I. Rectangle Model







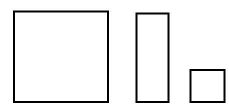
II. Breaking Numbers

a) 16×27

b) Area Model: 23×14

III. Algebra Tiles

Legend:



x^2 x 1

a) $x(x+4)$

b) $2x(-x+2)$

c) $-x(x-2)$

d) $(x+1)(2x+3)$

f) $(2x-1)(x+4)$

g) Draw a model that has an area of x^2+x

Write a quotient that represents this model

Homework

ASSIGNMENT # 2

pages 12-19 Questions #54-107

Multiplication and the Area Model

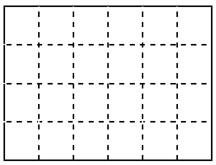
Sometimes it is convenient to use a tool from one aspect of mathematics to study another.

To find the product of two numbers, we can consider the numbers as side lengths of a rectangle.

How are side lengths, rectangles, and products related? The Area Model

The product of the two sides is the area of a rectangle. $A = lw$

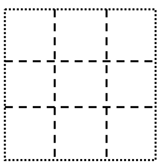
Consider:



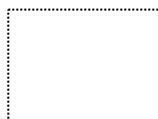
Length =___ Width =___

54. Show why $3 \times 3 = 9$ using the area model.

Solution:

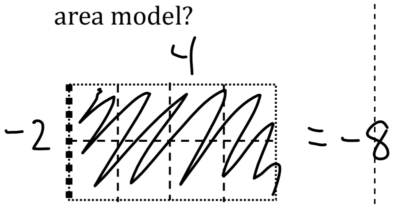


55. Show why $3 \times 4 = 12$ using the area model.

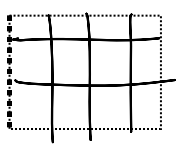


56. Calculate 5×4 using the area model.

57. How might we show $-2 \times 4 = -8$ using the area model?

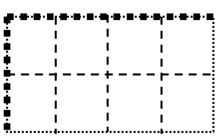


58. Calculate -3×4 using the area model.

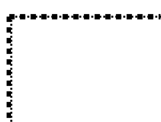


59. Calculate -5×4 using the area model.

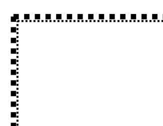
60. How might we show $-2 \times -4 = 8$ using the area model?



61. Calculate -3×-4 using the area model.



62. Calculate -5×-4 using the area model.



There are some limitations when using the area model to show multiplication. The properties of multiplying integers (+,+), (+,-), (-,-) need to be interpreted by the reader.

63. Show how you could break apart the following numbers to find the product.

$21 \times 12 =$

$= (20 + 1) \times (10 + 2)$
$= 200 + 40 + 10 + 2$
$= 252$

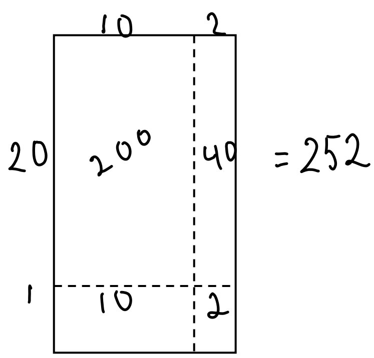
64. Show how you could break apart the following numbers to find the product.

$32 \times 14 =$

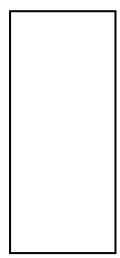
65. Show how you could break apart the following numbers to find the product.

$17 \times 24 =$

66. Draw a rectangle with side lengths of 21 units and 12 units. Model the multiplication above using the rectangle.



67. Draw a rectangle with side lengths of 32 units and 14 units. Model the multiplication above using the rectangle.



68. Draw a rectangle with side lengths of 17 units and 24 units. Model the multiplication above using the rectangle.

69. Use an area model to multiply the following without using a calculator.
23×15

70. Use an area model to multiply the following without using a calculator.
52×48

71. Use an area model to multiply the following without using a calculator.
73×73

Algebra tiles and the area model: Multiplication/Division of algebraic expressions.

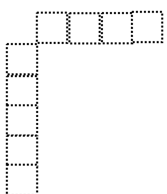
First we must agree that the following shapes will have the indicated meaning.

1 -1 x -x x^2 $-x^2$

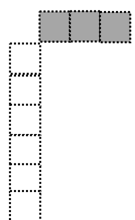
We must also remember the result when we multiply:

- Two positives = Positive
- Two negatives = Positive
- One positive and one negative = Negative

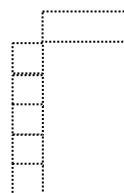
72. Write an equation represented by the diagram below and then multiply the two monomials using the area model.



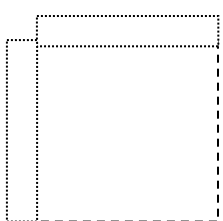
73. Write an equation represented by the diagram below and then multiply the two monomials using the area model.



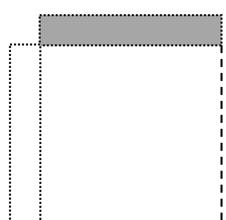
74. Write an equation represented by the diagram below and then multiply the two monomials using the area model.



75. Write an equation represented by the diagram below and then multiply the two monomials using the area model.

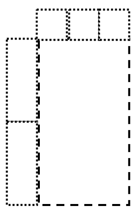


76. If the shaded rectangle represents a negative value, find the product of the two monomials.

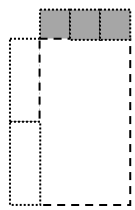


77. Write an equation represented by the diagram below and then multiply the two monomials using the area model.

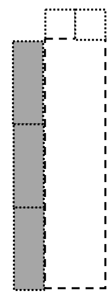
78. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.



79. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.

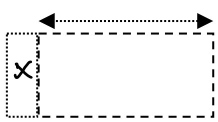


80. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.



81. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

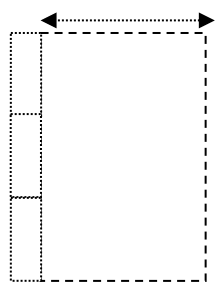
Area $= 6x$



Length:___________

82. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

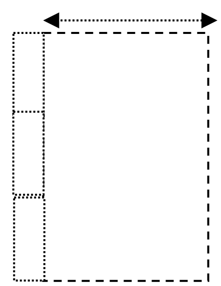
Area $= 6x^2$



Length:___________

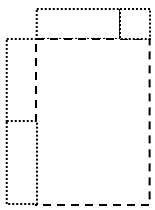
83. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

Area $= -6x^2$

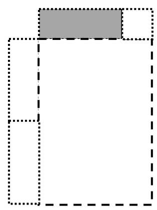


Length:___________

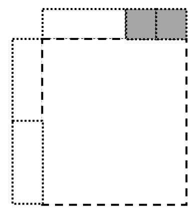
84. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.



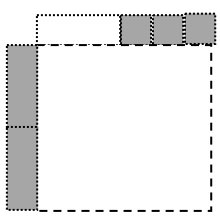
85. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.



86. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.



87. Write an equation represented by the diagram below and then multiply the two expressions using the area model.



88. Draw and use an area model to find the product:
$(2)(2x + 1)$

89. Draw and use an area model to find the product:
$(2x)(x - 3)$

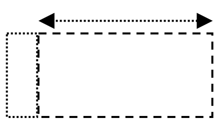
90. Draw and use an area model to find the product:
$(x)(x+3)$

91. Draw and use an area model to find the product:
$(-x)(x+3)$

92. Draw and use an area model to find the product:
$(-3x)(2x+3)$

93. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

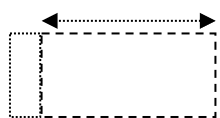
Area $= x^2 + 3x$



Length:___________

94. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

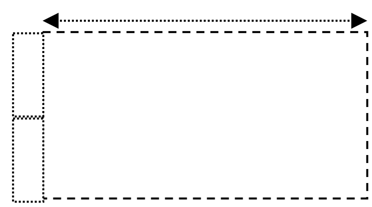
Area $= -x^2 - 3x$



Length:___________

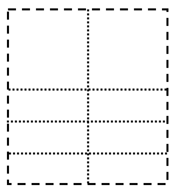
95. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

Area $= 2x^2 - 8x$



Length:___________

96. Find the area, length and width that can be represented by the diagram.

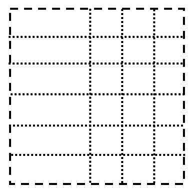


Area:

Length:

Width:

97. Find the area, length and width that can be represented by the diagram.

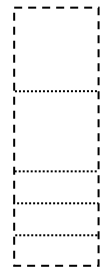


Area:

Length:

Width:

98. Find the area, length and width that can be represented by the diagram.



Area:

Length:

Width:

Multiplying & Dividing Monomials without TILES

When multiplying expressions that have more than one variable or degrees higher than 2, algebra tiles are not as useful.

Multiplying Monomials:

Eg.1.

$(2x^2)(7x)$ Multiply numerical coefficients.

$= 2 \times 7 \times x \times x^2$ Multiply variables using exponent laws.

$= 14x^3$

Eg.2.

$(-4a^2b)(3ab^3)$

$= -4 \times 3 \times a^2 \times a \times b \times b^3$

$= -12a^3b^4$

Dividing Monomials:

Eg.1.

$\frac{20x^3y^4}{-5x^2y^2}$ Divide the numerical coefficients.

$= \frac{20}{-5}\frac{x^3}{x^2}\frac{y^4}{y^2}$ Divide variables using exponent laws.

$= -4xy^2$

Eg.2.

$\frac{-36m^3n^4p^2}{-9m^3np}$

$= \frac{-36}{-9}\frac{m^3}{m^3}\frac{n^4}{n}\frac{p^2}{p}$

$= 4n^3p$

Revisit the exponent laws if

Multiply or Divide the following.

99. $(-2ab^3)(-3ab^5)$

100. $(5x^2y^3)(-2x^3y^5)$

101. $4x(-3x^3)$

102. $\left(\frac{1}{2}ab^2\right)\left(\frac{3}{4}a^3b\right)$

103. $\frac{-75s^2t^5}{15s^2t^2}$

104. $\frac{-45x^3yz^2}{-9x^2y}$

105. $\frac{24x^3y^2}{18xy^3}$

106. $(2cd)(-2c^2d^3)(5c)$

107. $\frac{(3xy)(4x^3y^2)}{2x^2y}$

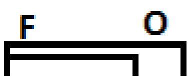
Name: ______________________

Lesson #3 - Multiplying Binomials

PART I: Algebra Tiles

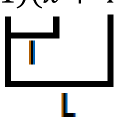
$(3x + 1)(x + 4)$

PART II: Binomial x Binomial = F.O.I.L (First Outside Inside Last)

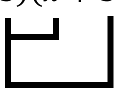


1. $(3x + 1)(x + 4) = 3x^2 + 12x + x + 4$

   $= 3x^2 + 13x + 4$



2. $(2x + 5)(x + 3) =$



3. $(5x + 6)(x - 2) =$

4. $(7x + 1)(7x - 1) =$

5. $(5x - 4)^2 =$

PART III: Binomial x Trinomial (6 multiplication steps)

1. $(x + 2)(x^2 + 5x + 3) =$

PART IV: Binomial x Binomial x Binomial

1. $(x + 2)(x + 3)(x + 4) = (x + 2)(x^2 + 7x + 12)$

2. $3(x + 10)(x - 2)(x + 2)$

ASSIGNMENT # 3

pages 20-24 Questions #108-139

Multiplying Binomials

Challenge:

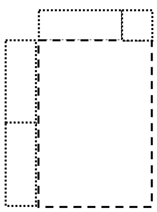
108. Which of the following are equal to $x^2 + 9x + 18$?

a) $(x+3)(x+6)$
b) $(x+1)(x+18)$
c) $(x-3)(x-6)$
d) $(x+2)(x+9)$

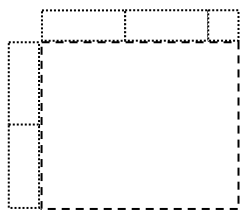
Challenge:

109. Multiply $(2x+1)(x-5)$

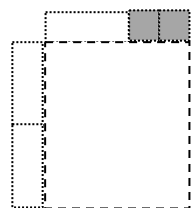
110. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.



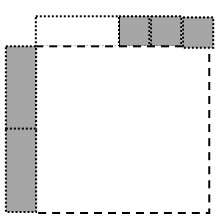
111. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.



112. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.



113. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.



114. Draw and use an area model to find the product: $(x+2)(2x+1)$

115. Draw and use an area model to find the product: $(2x-1)(x-3)$

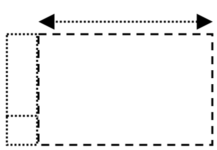
116. Draw and use an area model to find the product:
$(2-x)(x+2)$

117. Draw and use an area model to find the product:
$(3-x)(x-1)$

118. Draw and use an area model to find the product:
$(3x+1)(2x+1)$

119. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

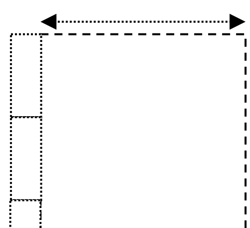
Area $= x^2+3x+2$



Length:___________

120. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

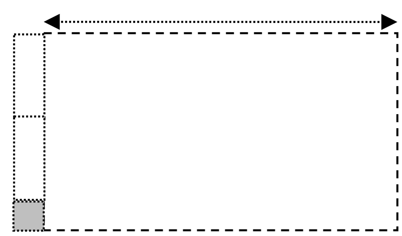
Area $= 2x^2+5x+2$



Length:___________

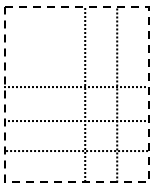
121. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

Area $= 4x^2-8x+3$



Length:___________

122. Find the area, length and width that can be represented by the diagram.

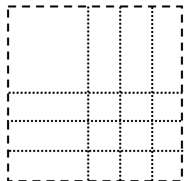


Area:

Length:

Width:

123. Find the area, length and width that can be represented by the diagram.

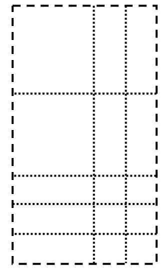


Area:

Length:

Width:

124. Find the area, length and width that can be represented by the diagram.



Area:

Length:

Width:

125. Draw tiles that represent the multiplication of $(x+1)(x-3)$.

What is the product of $(x+1)(x-3)$?

126. Draw tiles that represent the multiplication of $(2x+1)(2x+1)$.

What is the product of $(2x+1)(2x+1)$?

127. Draw tiles that represent the multiplication of $(x-4)(x+4)$.

What is the product of $(x-4)(x+4)$?

Multiplying Polynomials without TILES (also called expanding or distribution)

Multiplying Binomials: *use FOIL

Eg.1. $(x+3)(x+6) = x^2 + 6x + 3x + 18 = \mathbf{x^2 + 9x + 18}$

FOIL
Firsts - Outsides-Insides-Lasts
(x)(x) + (x)(6) + (3)(x) + (3)(6)

Eg.2. $(2x+1)(x-5) = 2x^2 - 10x + x - 5 = \mathbf{2x^2 - 9x - 5}$

Multiplying a Binomial by a Trinomial:

Eg. $(y-3)(y^2 - 4y + 7) = y^3 - 4y^2 + 7y - 3y^2 + 12y - 21 = \mathbf{y^3 - 7y^2 + 19y - 21}$

Multiply each term in the first polynomial by each term in the second.

Multiplying: Binomial × Binomial × Binomial

Eg.
$$(x+2)(x-3)(x+4)$$
$$= (x^2 - 3x + 2x - 6)(x+4)$$
$$= (x^2 - x - 6)(x+4)$$
$$= x^3 + 4x^2 - x^2 - 4x - 6x - 24$$
$$= \mathbf{x^3 + 3x^2 - 10x - 24}$$

Multiply the first two brackets (FOIL) to make a new trinomial.

Then multiply the new trinomial by the remaining binomial

Multiply the following as illustrated above.

128. $(x+2)(x-5)$

129. $(2x+1)(x-3)$

130. $(x-3)(x-3)$

Multiply the following.

131. $(x + 2)(x + 2)$

132. $(2x + 1)(3x - 3)$

133. $(2x + 1)(2x - 1)$

134. $(x + 2)^2$

135. $(2x + 5)^2$

136. $(x - 1)(x - 1)(x + 4)$

137. $(x - 5)(x^2 - 5x + 1)$

138. $(2x - 3)(3x^2 + 2x + 1)$

139. $(x + 2)^3$

Lesson #4 – Conjugates and More Expanding

I. Conjugates

1. $(x+3)(x-3) =$

2. $(x+2)(x-2) =$

3. $(3m+10)(3m-10) =$

4. $\left(\frac{1}{2}x - y\right)\left(\frac{1}{2}x + y\right) =$

5. $(m^3+1)(m^3-1) =$

$$(a+b)(a-b) = a^2 - b^2$$

the product of conjugates is a binomial "*a difference of squares*"

II. Expand and Simplify

1. $\mathbf{(x+5)(x-1)+(x+3)(x-7)}$

2. $\mathbf{(x+1)-(x-4)(x+4)}$

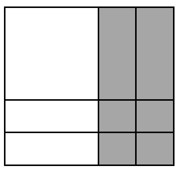
3. $\mathbf{6-3(2x-1)(2x+1)-(x+4)^2}$

ASSIGNMENT # 4
pages 25-27 Questions #140-153

Special Products: Follow the patterns

Conjugates: $(a+b)(a-b)$
$= a^2 + ab - ab - b^2$
$= a^2 - b^2$

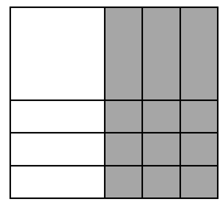
140. Write an expression for the following diagram (do not simplify):



What two binomials are being multiplied above?

Write an equation using the binomials above and the simplified product.

141. Write an expression for the following diagram (do not simplify):



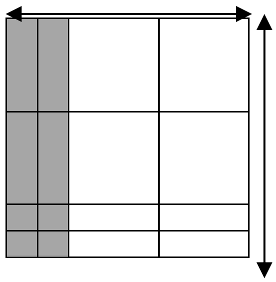
What two binomials are being multiplied above?

Write an equation using the binomials above and the simplified product.

QUESTION… Describe any patterns you observe in the two questions above.

Remember this pattern…it will be important when we factor "A Difference of Squares" later in this booklet.

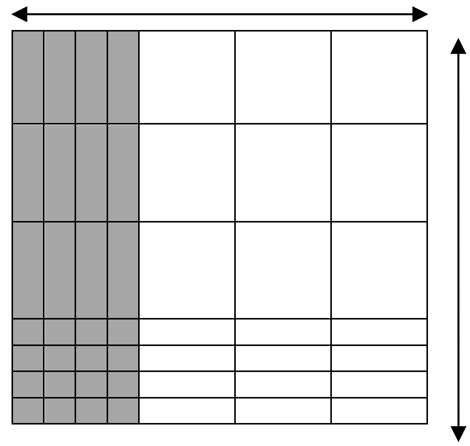
142. Write an expression (polynomial) for the following diagram (do not simplify):



What two binomials are being multiplied above?

Write an equation using the binomials above and the simplified product.

143. Write an expression for the following diagram (do not simplify):



What two binomials are being multiplied above?

Write an equation using the binomials above and the simplified product.

Simplify the following.

144. $(x+3)(x-3)$

145. $(2x+3)(2x-3)$

146. $(3x-1)(3x+1)$

147. $(x+\sqrt{2y})(x-\sqrt{2y})$

Simplify the following.

148. $3(b-7)(b+7)$

149. $-2(c-5)(c+5)$

150. $(x+6)(x+4)+(x+2)(x+3)$

151. $3(x-4)(x+3)-2(4x+1)$

152. $5(3t-4)(2t-1)-(6t-5)$

153. $10-2(2y+1)(2y+1)-(2y+3)(2y+3)$

Some key points to master about the Distributive Property…

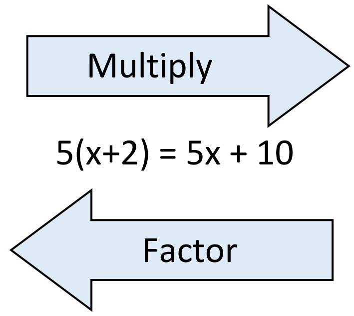
FOIL	$(a+b)(a-b)$	$(a+b)^2$	$(a+b)^3$

Name: ______________________

Lesson #5 – (Greatest Common Factor) Factoring

I. Factoring

Factoring is the reverse of multiplying.



II. Factoring a Monomial Common Factor (with the GCF)

1. $10x - 10 =$

2. $9x^2y^5 - 30x^4y =$

3. $8x^3 + 12x^2y - 20x =$

4. $3x + 11 =$

III. Factoring a Binomial Common Factor (with the GCF)

1. $3x(x+2) + 7(x+2) =$

2. $6a(a-5) - 11(a-5) =$

3. $2x(x-3) + 9(3-x) =$

IV. Factoring by Grouping

1. $mx + 2m + 3x + 6 =$

2. $3a + 3b - ax - bx =$

3. $4m^2 - 12m + 15t - 5mt =$

4. $xy + 10 + 2y + 5x =$

ASSIGNMENT # 5
Pages 28-34 Questions #154-188

Factoring:

When a number is written as a product of two other numbers, we say it is factored.

"Factor Fully" means to write as a product of **prime factors**.

Eg.1.
Write 15 as a product of its prime factors.

$$15 = 5 \times 3$$

5 and 3 are the prime factors.

Eg.2.
Write 48 as a product of its prime factors.

$$48 = 8 \times 6$$
$$48 = 2 \times 2 \times 2 \times 3 \times 2$$
$$48 = 2^4 \times 3$$

Eg.3.
Write 120 as a product of its prime factors.

$$120 = 10 \times 12$$
$$120 = 2 \times 5 \times 2 \times 2 \times 3$$
$$120 = 2^3 \times 3 \times 5$$

154. Write 18 as a product of its prime factors.

155. Write 144 as a product of its prime factors.

156. Write 64 as a product of its prime factors.

157. Find the greatest common factor (GCF) of 48 and 120.

Look at each factored form.

$$48 = 2^4 \times 3$$
$$120 = 2^3 \times 3 \times 5$$

Both contain $2 \times 2 \times 2 \times 3$, therefore this is the GCF,

GCF is 24.

158. Find the greatest common factor (GCF) of 144 and 64.

159. Find the greatest common factor (GCF) of 36 and 270.

We can also write algebraic expressions in factored from.

Eg.4. Write $36x^2y^3$ as a product of its factors.

$$36x^2y^3 = 9 \times 4 \times x \times x \times y \times y \times y$$
$$36x^2y^3 = 3^2 \times 2^2 \times x^2 \times y^3$$

160. Write $10a^2b$ as a product of its factors.

161. Write $18ab^2c^3$ as a product of its factors.

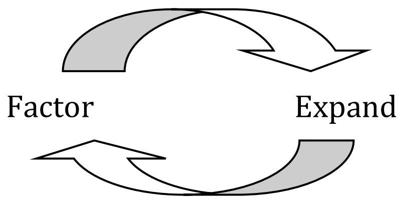
162. Write $12b^3c^2$ as a product of its factors.

163. Find the greatest common factor (GCF) of $10a^2b$ and $18ab^2c^3$.

164. Find the greatest common factor (GCF) of $12b^3c^2$ and $18ab^2c^3$.

165. Find the greatest common factor (GCF) of $10a^2b$, $18ab^2c^3$, and $12b^3c^2$.

Factoring Polynomials:



The process of factoring "undoes" the process of expanding, and vice versa.

They are opposites.

You must be able to interchange a polynomial between these two forms.

Factoring means "*write as a product of factors*."

The method you use depends on the type of polynomial you are factoring.

Challenge Question:

Write a multiplication that would be equal to $5x + 10$.

Challenge Question:

Write a multiplication that would be equal to $3x^3 + 6x^2 - 12x$.

The answers to the above questions are called the "FACTORED FORM".

Factoring: Look for a <u>Greatest Common Factor</u>

Hint: Always look for a GCF first.

Ask yourself: "Do all terms have a common integral or variable factor?"

Eg.1. Factor the expression.

$5x + 10$

Think…what factor do 5x and 10 have in common?
Both are divisible by 5…that is the GCF.

$= 5(x) + 5(2)$ Write each term as a product using the GCF.

$= \mathbf{5(x+2)}$ Write the GCF outside the brackets, remaining factors inside.

You should check your answer by expanding. This will get you back to the original polynomial.

Eg.2.
Factor the expression

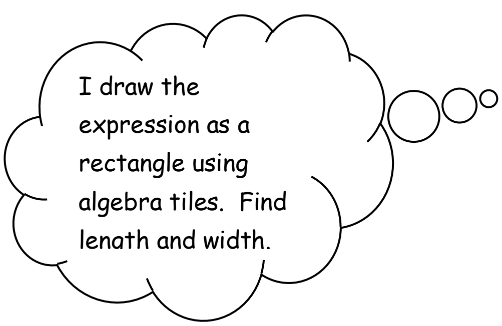
$3ax^3+6ax^2-12ax$

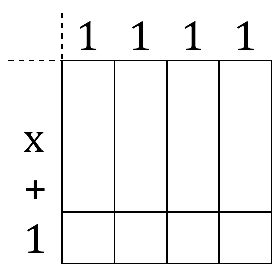
GCF = 3ax

$=3ax(x^2)+3ax(2x)+3ax(-4)$

$=3ax(x^2+2x-4)$

Eg.3. Factor the expression $4x + 4$ using algebra tiles.





$4(x + 1) = 4x + 4$

Factor the following polynomials.

166. $5x + 25$

167. $4x + 13$

168. $8x + 8$

169. Model the expression above using algebra tiles.

170. Model the expression above using algebra tiles.

171. Model the expression above using algebra tiles.

172. $4ax + 8ay - 6az$

173. $24w^5 - 6w^3$

174. $3w^3xy + 12wxy^2 - wxy$

175. $27a^2b^3 + 9a^2b^2 - 18a^3b^2$

176. $6m^3n^2 + 18m^2n^3 - 12mn^2 + 24mn^3$

Factoring a Binomial Common Factor: Hint: There are brackets with identical terms.

The common factor **IS** the term in the brackets!

Eg.1. Factor. $4x(w+1)+5y(w+1)$

$$4x(w+1)+5y(w+1)$$
$$=(w+1)(4x)+(w+1)(5y)$$
$$=(w+1)(4x+5y)$$

Eg.2. Factor. $3x(a+7)-(a+7)$

$$3x(a+7)-(a+7)$$
$$=(a+7)(3x)-(a+7)(1)$$
$$=(a+7)(3x-1)$$

Sometimes it is easier to understand if we substitute a letter, such as *d* where the common binomial is.

Consider Eg.1.

$4x(w+1)+5y(w+1)$ Substitute d for $(w+1)$.

$4xd+5yd$

$d(4x+5y)$ Now replace $(w+1)$.

$=(w+1)(4x+5y)$

Factor the following, if possible.

177. $5x(a+b)+3(a+b)$	178. $3m(x-1)+5(x-1)$	179. $3t(x-y)+(x+y)$
180. $4t(m+7)+(m+7)$	181. $3t(x-y)+(y-x)$	182. $4y(p+q)-x(p+q)$

Challenge Question:

Factor the expression $ac+bd+ad+bc$.

Factoring: *Factor by Grouping*. Hint: 4 terms!

Sometimes a polynomial with 4 terms but no common factor can be arranged so that grouping the terms into two pairs allows you to factor.

You will use the concept covered above…common binomial factor.

Eg.1. Factor $ac + bd + ad + bc$

$ac + bc \quad + ad + bd$ Group terms that have a common factor.

$c(a + b) + d(a + b)$ Notice the newly created binomial factor, $(a + b)$.

$= (\boldsymbol{a} + \boldsymbol{b})(\boldsymbol{c} + \boldsymbol{d})$ Factor out the binomial factor.

Eg.2. Factor $5m^2t - 10m^2 + t^2 - 2t$

$5m^2t - 10m^2 \quad - t^2 + 2t$ Group.

$5m^2(t - 2) - t(t - 2)$ *Notice that I factored out a $-t$ in the second group. This made the binomials into common factors, $(t - 2)$.

$= (\boldsymbol{t} - \mathbf{2})(\mathbf{5}\boldsymbol{m}^{\mathbf{2}} - \boldsymbol{t})$

183. $wx + wy + xz + yz$

184. $x^2 + x - xy - y$

185. $xy + 12 + 4x + 3y$

186. $2x^2 + 6y + 4x + 3xy$

187. $m^2 - 4n + 4m - mn$

188. $3a^2 + 6b^2 - 9a - 2ab^2$

Name: ______________________

Lesson #6 – Factoring Trinomials ($ax^2 + bx + c$), where $a = 1$

Type I: $x^2 + bx + c$

1. $x^2 + 7x + 12$

2. $x^2 + 9x + 20$

3. $2x^2 + 22x + 60$

4. $x^2 + 24xy + 44y^2$

Type II: $x^2 - bx + c$

1. $x^2 - 8x + 12$

2. $x^2 - 21x + 20$

3. $y^2 - 11y + 18$

4. $3x^2 - 18x + 27$

Type III: $\boldsymbol{x^2 \pm bx - c}$

1. $x^2 + 2x - 24$

2. $x^2 - 2x - 35$

3. $x^4 + x^2 - 30$

4. $2x^3 - 6x^2 - 20x$

5. $x^2 - x - 90$

ASSIGNMENT # 6

pages 35-38 Questions #189-216

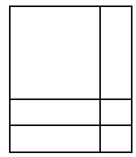
Factoring: $ax^2 + bx + c$ (where a=1) with tiles.

Hint: 3 terms, no common factor, leading coefficient is 1.

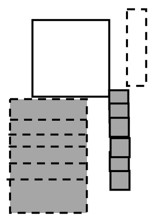
Eg.1. Consider $x^2 + 3x + 2$. The trinomial can be represented by the rectangle below.

Recall, the side lengths will give us the "factors".

$\therefore\ x^2 + 3x + 2 = (x + 1)(x + 2)$



Eg.2. Factor $x^2 - 5x - 6$



Start by placing the "x^2 tile" and the six "-1 tiles" at the corner. Then you can fill in the "x tiles". You'll need one x tile and six –x tiles.

$\therefore\ x^2 - 5x - 6 = (x + 1)(x - 6)$

Factor the following trinomials using algebra tiles.

189. $x^2 + 6x + 8$

190. $x^2 + 9x + 14$

191. $x^2 - 7x + 6$

192. $x^2 + 9x - 10$

Factoring: $ax^2 + bx + c$ (where a=1) without tiles.

Did you see the pattern with the tiles?

If a trinomial in the form $x^2 + bx + c$ can be factored, it will end up as $(x + \_\_)(x + \_\_)$.

The trick is to find the numbers that fill the spaces in the brackets.

The Method…
If the trinomial is in the form: $x^2 + bx + c$, look for two numbers that multiply to c, and add to b.

Eg.1.
Factor. $x^2 + 6x + 8$

$(x + \_\_)(x + \_\_)$ What two numbers multiply to +8 but add to +6? 2 and 4

$= (x + 2)(x + 4)$ The numbers 2 and 4 fill the spaces inside the brackets.

Eg.2. Factor. $x^2 - 11x + 18$

$(x + \_\_)(x + \_\_)$ What two numbers multiply to +18 but add to -11? -2 and -9

$= (x - 2)(x - 9)$ The numbers -2 and -9 fill the spaces inside the brackets.

Eg.3. Factor. $x^2 - 7xy - 60y^2$ The y's can be ignored temporarily to find the two numbers. Just write them in at the end of each bracket.

$(x + \_\_y)(x + \_\_y)$ What two numbers multiply to -60 but add to -7? -12 and +5

$= (x - 12y)(x + 5y)$ The numbers -12 and +5 fill the spaces in front of the y's.

Factor the trinomials if possible.

193. $a^2 + 6a + 5$

194. $n^2 + 7n + 10$

195. $x^2 - x - 30$

Factor the trinomials if possible.

196. $q^2 + 2q - 15$	197. $k^2 + k - 56$	198. $t^2 + 11t + 24$
199. $y^2 - 7y - 30$	200. $g^2 - 11g + 10$	201. $s^2 - 2s - 80$
202. $m^2 - 12m + 27$	203. $x^2 - 27 - 6x$	204. $p^2 + 3p - 54$
205. $2a^2 - 16a + 32$	206. $a^2 - 14a + 45$	207. $6x + 2x^2 - 20$

Factor the trinomials if possible.

208. $x^4 - 3x^2 - 10$	209. $w^6 + 7w^3 + 12$	210. $p^8 - 4p^4 - 21$
211. $56x + x^2 - x^3$	212. $x^4 + 11x^2 - 80$	213. $x^2 - 3x + 7$
214. $x^2 - 6xy + 5y^2$	215. $x^2 + 5xy - 36y^2$	216. $a^2b^2 - 5ab + 6$

Challenge Question

Factor $2x^2 + 7x + 6$.

Name: ______________________

Lesson #7 – Factoring Trinomials ($ax^2 + bx + c$), where $a \neq 1$

Lesson Focus:

- To use an algebraic method to factor a trinomial of the form $ax^2 + bx + c$, using one of two strategies:
    1. Strategy #1: The Decomposition Method
    2. Strategy #2: The X-Method (or The Trial & Error Method)

Review Example: Factor $3x^2 - 9x - 12$ completely.

Note: In this example, after we remove the GCF, the coefficient on the "a" term (the x^2) term is 1.

What if $a \neq 1$, even after common factoring??

(ONLY use these two strategies if $a \neq 1$. If $a = 1$, look back at Lesson #6)

Strategy #1: The Decomposition Method

Steps	**Example:** Factor $9x^2 + 21x - 8$
1. Mentally figure out two numbers that multiply to "ac" and add to "b".	
2. **Decompose** the middle term (ie the "b" term) using the answer from step #1. (NOTE: The order that you list the decomposed middle terms doesn't matter)	
3. Now you have four terms, so let's factor by grouping!	
4. Check your answer using FOIL	

Strategy #2: The X Method (or The Trial & Error Method)

Steps	**Example:** Factor $2x^2 + 3x - 2$
1. Draw a large X under the trinomial, leaving one line of space in between. 2. On the LHS of the X, write two numbers that multiply to "a" (ie. two factors of "a") 3. On the RHS of the X, write two numbers that multiply to "c" (ie. two factors of "c") 4. Cross multiply, and check to see if the two numbers can add to "b". Keep trying new combinations of numbers until you find the "winning" numbers. **Put "+" or "-" signs on the RIGHT HAND SIDE ONLY. Put the variable on the LEFT HAND SIDE ONLY.** 5. Write the numbers in the X as factors. The top two numbers form one factor. The bottom two numbers form the other factor. 6. Check your answer using FOIL	

Example. Factor the following completely, using **one of the two** strategies

a) $6x^2 + 5x - 6$

b) $2x^2 + 5x + 2$

Final Thoughts on Trinomial Factoring:

- Only 2 methods have been outlined in this section. There are even more, but these are the ones I like! You may have learned an alternative method last year, in fact.
- Every teacher has their preferred method.
- Every student has their preferred method.
- YOU MAY CHOOSE WHICHEVER METHOD YOU WISH. YOU ONLY NEED TO KNOW ONE METHOD. PICK ONE AND MASTER IT!

Do not recycle the Polynomials notes!* It is absolutely imperative that you remember how to factor next year and years to come. You will not be taught again, but you will be expected to know how to do it. *I wouldn't recycle any of Math 10, if I were you, but especially not Chapter 3.

ASSIGNMENT # 7
pages 39-42 Questions #217-234

Factoring $ax^2 + bx + c$ where $a \neq 1$

When the trinomial has an x^2 term with a coefficient other than 1 on the x^2 term, you cannot use the same method as you did when the coefficient is 1.

We will discuss 3 other methods:
1. Trial & Error 2. Decomposition 3. Algebra Tiles

Trial & Error:

Eg.1. Factor $2x^2 + 5x + 3$.

$2x^2 + 5x + 3 = (\quad)(\quad)$ We know the first terms in the brackets have product of $2x^2$

$2x^2 + 5x + 3 = (2x \quad)(x \quad)$ $2x$ and x have a product of $2x^2$, place them at front of brackets.

The product of the second terms is 3. (1, 3 or -1, -3).
These will fill in the second part of the binomials.

List the possible combinations of factors.

$(2x + 1)(x + 3)$
$(2x + 3)(x + 1)$
$(2x - 1)(x - 3)$
$(2x - 3)(x - 1)$

> IF $2x^2 + 5x + 3$ is factorable, one of these must be the solution.
>
> Expand each until you find the right one.

$(2x + 3)(x + 1) = 2x^2 + 2x + 3x + 3 = 2x^2 + 5x + 3$. This is the factored form.

Decomposition:
Using this method, you will break apart the middle term in the trinomial, then factor by grouping.

To factor $\boldsymbol{ax^2 + bx + c}$, look for two numbers with a product of ***ac*** and a sum of ***b***.

Eg.1. Factor. $3x^2 - 10x + 8$

1. We see that $ac = 3 \times 8 = 24$; and $b = -10$
   We need two numbers with a product of 24, but add to -10…
   -6 and -4.

$3x^2 - \mathbf{6}x - \mathbf{4}x + 8$ 2. Break apart the middle term.

$3x(x - 2) - 4(x - 2)$ 3. Factor by grouping.

$= (\boldsymbol{x - 2})(\boldsymbol{3x - 4})$

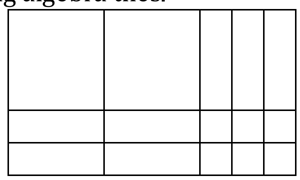
Eg.2. Factor. $3a^2 - 22a + 7$ — We need numbers that multiply to 21, but add to -22… **-21** and **-1**

$3a^2 - \mathbf{21}a - \mathbf{1}a + 7$ — Decompose middle term.

$3a(a - 7) - 1(a - 7)$ — Factor by grouping.

$= (\boldsymbol{a} - \mathbf{7})(\mathbf{3}\boldsymbol{a} - \mathbf{1})$

Eg.3. Factor $2x^2 + 7x + 6$ using algebra tiles.



Arrange the tiles into a rectangle (notice the "ones" are again grouped together at the corner of the x^2 tiles)

Side lengths are $(2x + 3)$ and $(x + 2)$ $\therefore 2x^2 + 7x + 6 = (2x + 3)(x + 2)$

Your notes here…

Factor the following if possible.

217. $2a^2 + 11a + 12$

218. $5a^2 - 7a + 2$

219. $3x^2 - 11x + 6$

Factor the following if possible.

220. $2y^2 + 9y + 9$

221. $5y^2 - 14y - 3$

222. $10x^2 - 17x + 3$

223. $2x^2 + 3x + 1$

224. $6k^2 - 5k - 4$

225. $6y^2 + 11y + 3$

226. $3x^2 - 16x - 12$

227. $3x^3 - 5x^2 - 2x$

228. $9x^2 + 15x + 4$

Factor the following if possible.

229. $21x^2 + 37x + 12$

230. $6x^3 - 15x - x^2$

231. $4t + 10t^2 - 6$

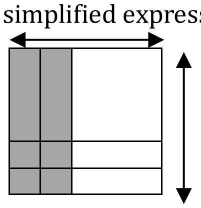
232. $3x^2 - 22xy + 7y^2$

233. $4c^2 - 4cd + d^2$

234. $2x^4 + 7x^2 + 6$

Challenge Question

Write a simplified expression for the following diagram of algebra tiles.



What two binomials are being multiplied in the diagram above?

Write an equation using the binomials above and the simplified product.

Lesson #8 – Factoring Special Polynomials

Lesson Focus:

- To learn the shortcut for expanding $(a \pm b)^2$
- To learn to identify and factor the following special polynomials: Perfect Square Trinomials and Difference of Squares Binomials

Expanding $(a \pm b)^2$

$(a+b)^2$	$(a-b)^2$

Note: A polynomial of the form $(a \pm b)^2$ is called a Perfect Square Trinomial.

Example: Expand the following polynomials.

a) $(5x-2)^2$

b) $(6x+7)^2$

Factoring Perfect Square Trinomials

- All perfect square trinomials (PSTs) can be factored into: $(a \pm b)^2$
- Therefore, if you can identify one, you can skip ALL FACTORING STEPS and go straight to the final answer. NO WORK NEED BE SHOWN.

Algebraically, we can spot one by first noticing the following: $ax^2 + bx + c$

Note: "a" and "c" MUST be positive for the polynomial to be a perfect square trinomial. **WHY?**

Example. Factor the following completely. Check your answer by expanding (FOIL).

a) $x^2 - 6x + 9$

b) $121d^2 + 66d + 9$

<u>Factoring Difference of Squares Binomials</u>

A difference of squares binomial is a binomial in the form $a^2 - b^2$.
For example: $x^2 - 81$

Some other examples:

Note: It must be a DIFFERENCE (-) NOT a sum (+).

The ONLY ways to factor a binomial are:

1. Common Factor (Remove the GCF)
2. Difference of Squares

Example: Factor $x^2 - 81$ completely.

Difference of Squares Rule

$$a^2 - b^2 = (a - b)(a + b)$$

Don't forget the Golden Rule of factoring! ***The first step of any factoring process is to ALWAYS…***

Let's Play….. *SPOT THE DIFFERENCE OF SQUARES!!!!!*

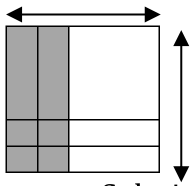
Circle the numbers of questions that are differences of squares. If it isn't, circle the part of the binomial that is preventing it from being a difference of squares. Lastly, factor each question.

1. $x^2 - 4y^2$
2. $49x^3 - 16$
3. $25n^2 + 100$
4. $18k^2 - 98$
5. $\frac{x^4}{9} - y^6$
6. $x^4 - 16$

ASSIGNMENT # 8
Pages 43-47 Questions #235-260

A Difference of Squares

235. Write a simplified expression for the following diagram.



Solution: $x^2 - 2x + 2x - 4$

What two binomials are being multiplied in the diagram above?

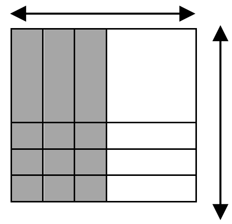
$(x - 2)(x + 2)$

Write an equation using the binomials above and the simplified product.

$x^2 - 4 = (x - 2)(x + 2)$

Factored Form

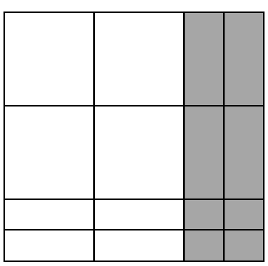
236. Write a simplified expression for the following diagram.



What two binomials are being multiplied above?

Write an equation using the binomials above and the simplified product.

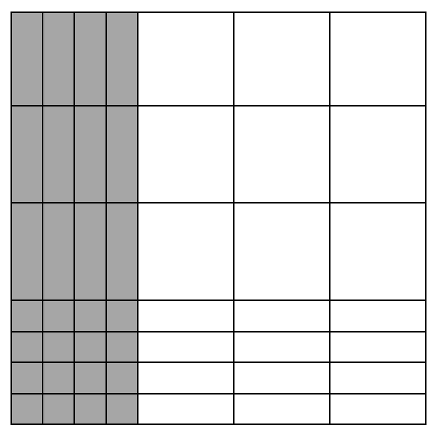
237. Write a simplified expression for the following diagram.



What two binomials are being multiplied above?

Factor the polynomial represented above by writing the binomials as a product (multiplication).

238. Write a simplified expression for the following diagram.



What two binomials are being multiplied above?

Factor the polynomial represented above by writing the binomials as a product (multiplication).

Factoring a Difference of Squares: $a^2 - b^2$

Conjugates: Sum of two terms and a difference of two terms.

Learn the pattern that exists for multiplying conjugates.

$(x+2)(x-2) = x^2 - 2x + 2x - 4 = x^2 - 4$ The two middle terms cancel each other out.

We can use this knowledge to quickly factor polynomials that look like $x^2 - 4$.

Eg.1. Factor $x^2 - 9$.

$= (x+3)(x-3)$ Square root each term, place them in 2 brackets with opposite signs (+ and -).

Eg.2. Factor $100a^2 - 81b^2$

$= (10a+9b)(10a-9b)$ Square root each term, place them in 2 brackets with opposite signs (+ and -).

Factor the following completely.

239. $a^2 - 25$	240. $x^2 - 144$	241. $1 - c^2$

I recognize a polynomial is a difference of squares because____

____

____

Factor the following completely.

242. $4x^2 - 36$

243. $9x^2 - y^2$

244. $25a^4 - 36$

245. $49t^2 - 36u^2$

246. $7x^2 - 28y^2$

247. $-18a^2 + 2b^2$

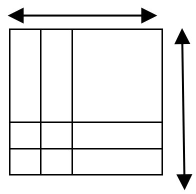
248. $-9 + d^4$

249. $\frac{a^2}{9} - \frac{b^2}{16}$

250. $\frac{x^2y^2}{49} - 1$

Factoring a Perfect Square Trinomial

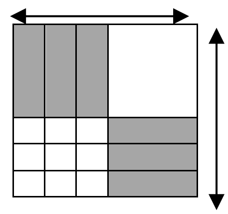
251. Write an expression for the following diagram (do not simplify):



What two binomials are being multiplied above?

Write an equation using the binomials above and the simplified product.

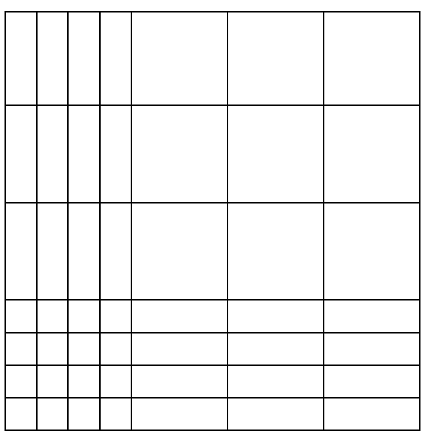
252. Write an expression for the following diagram (do not simplify):



What two binomials are being multiplied above?

Write an equation using the binomials above and the simplified product.

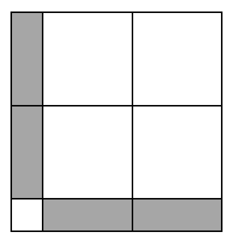
253. Write an expression for the following diagram (do not simplify):



What two binomials are being multiplied above?

Write an equation using the binomials above and the simplified product.

254. Write an expression for the following diagram (do not simplify):



What two binomials are being multiplied above?

Write an equation using the binomials above and the simplified product.

PERFECT SQUARE TRINOMIALS

You may use the methods for factoring trinomials to factor ***trinomial squares*** but recognizing them could make factoring them quicker and easier.

Eg.1. Factor.

$x^2 + 6x + 9$ — Recognize that the first and last terms are both perfect squares.

$(x + 3)^2$ — Guess by taking the square root of the first and last terms and put them in two sets of brackets.

Check: Does $2(x)(3) = 6x$
Yes! Trinomial Square!

In a trinomial square, the middle term will be double the product of the square root of first and last terms. Wow, that's a mouthful!

$(x + 3)^2$ — Answer in simplest form.

Eg.2. Factor.

$121m^2 - 22m + 1$

$(11m - 1)^2$ — Guess & Check. $2(11m \times -1) = -22m$.
Since the middle term is negative, binomial answer will be a subtraction.

Factor the following.

255. $x^2 + 14x + 49$	256. $4x^2 - 4x + 1$	257. $9b^2 - 24b + 16$
258. $64m^2 - 32m + 4$	259. $81n^2 + 90n + 25$	260. $81x^2 - 144xy + 64y^2$

Create a Factoring Flowchart.

Start with the first thing you should do….collect like terms.

ASSIGNMENT # 9

pages 49-51 Questions #261-286

Combined Factoring. Factor the following completely.

261. $3a^2 - 3b^2$	262. $4x^2 + 28x + 48$	263. $x^4 - 16$
264. $2y^2 - 2y - 24$	265. $16 - 28x + 20x^2$	266. $m^4 - 5m^2 - 36$
267. $x^8 - 1$	268. $x^3 - xy^2$	269. $x^4 - 5x^2 + 4$

HIGHER DIFFICULTY…

For some of the following questions, you may try substituting a variable in the place of the brackets to factor first, and then replace brackets.

270. $(a+b)^2 - c^2$	271. $(c-d)^2 - (c+d)^2$	272. $(m+7)^2 + 7(m+7) + 12$

273. Factor.
$(x+2)^2-(x-3)^2$

274. Find all the values of k so that $x^2+kx-12$ can be factored.

275. For which integral values of k can $3x^2+kx-3$ be factored.

276. What value of k would make $kx^2+24xy+16y^2$ a perfect square trinomial?

277. What value of k would make $2kx^2-24xy+9y^2$ a perfect square trinomial?

278. For which integral values of k can $6x^2+kx+1$ be factored.

a. $5,7$

b. $\pm 5, \pm 7$

c. $-5, -7$

d. all integers from 5 to 7.

279. Expand and simplify.
$-2(3m+4)^2$

280. If $a=2x+3$, write a^2-5a+3 in terms of x.

281. Lindsay was helping Anya with her math homework. She spotted an error in Anya's multiplication below. Find and correct any errors.

Multiply:

$5x(2x+1)+2(2x+1)$

$=10x+1+4x+2$

$=14x+3$

282. When asked to factor the following polynomial, Timmy was a little unsure where to start.

Factor: $10x + 5 + 2xy + y$

What type of factoring could you tell him to perform to help him along?

283. Find and correct any errors in the following factoring.

$2x^2 - 5x - 12$

$=2x^2 -12x+2x - 12$

$=2x(x-6) +2(x-6)$

$=(2x+2)(x-6)$

284. Explain why

$$3x^2 - 17x + 10 \neq (3x + 1)(x + 10)$$

285. Find and correct any errors in the following multiplication.

$$(x^2 + 2)^2$$

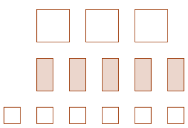
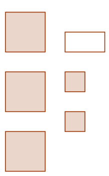
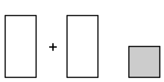
$$= x^4 + 4$$

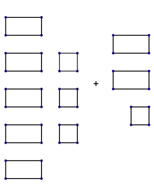
286. Explain why it is uncommon to use algebra tiles to multiply the following

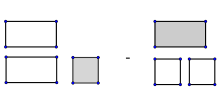
$$(x + 1)^3$$

287. Multiply the expression above.

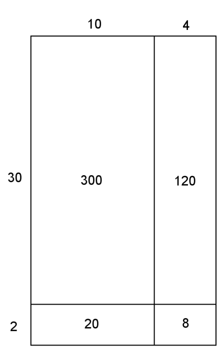
Answers:

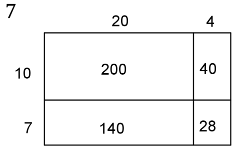
1. 5,-7
2. 13
3. x, y
4. no, negative exponent
5. yes
6. no, negative exponent
7. no, exponent not a whole number
8. yes
9. no, exponent not a whole number
10. 1, binomial
11. 3, trinomial
12. 7, polynomial
13. 0, monomial
14. Many possibilities
15. Many possibilities
16. 5x
17. $-3x^2$
18. $x^2 + 3x + 4$
19. $-4x^2 - 2x - 3$
20. $3x^2 + 3x + 4$
21. 
22. 
23. The two terms cancel each other, resulting in a sum of 0.
24. The two expressions cancel each other, resulting in a sum of 0.
25. 0
26. $-x^2 + x - 1$
27. 

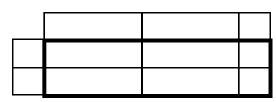
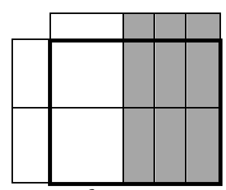
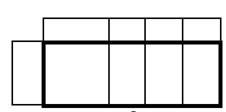
28. 

29. $-3x + 4$
30. $-x^2 + 5x + 2$
31. 0
32. 0
33. 

34. You cannot subtract / take away, or cancel the "negative-x" tile from the first expression because there was not one there. The same problem arises with the "+2".
35. Raj added "zero" in the form of opposite tiles so that he could then subtract the $(-x + 2)$ from the first expression.
36. $7x - 6$
37. $5x^2 + 5x - 8$
38. $x^2 - 4x - 8$
39. Same shape.
40. Same letter, same exponent (degree).
41. $-9x + 9y,\ -45$
42. $3x^3 - 5x^2 - 6,\ 30$
43. $11x^2y^3 - 5, -797$
44. $6x + 17$
45. $12a + 4b$
46. $4x + 4$
47. 7a
48. $12x - 5y$
49. $19a - 3b$
50. $13x^2 - x - 5$
51. $-2m^2n - 2mn + n$
52. $-y^2 + 2y - 4$
53. $10x^2 - 6xy + 3x + 6$
54. A rectangle that is 3 by 3 has an area of 9 square units.
55. A rectangle that is 3 by 4 has an area of 12 square units.
56. 20
57. Colour one side differently. The (-2) could be shaded.
58. -12
59. -20
60. Both edges would be shaded to represent negatives.
61. 12
62. 20
63. 252
64. $(30 + 2)(10 + 4)$
$300 + 120 + 20 + 8$
448
65. 408
66. 252
67. =448



68. =408
7


69. 345
70. 2496
71. 5329
72. $(4)(5) = 20$
73. $(-3)(6) = -18$
74. $(x)(5) = 5x$
75. $(x)(x) = x^2$
76. $(x)(-x) = -x^2$
77. $(x)(2x) = 2x^2$
78. $(3)(2x) = 6x$
79. $(-3)(2x) = -6x$
80. $(2)(-3x) = -6x$
81. $\frac{6x}{x} = 6$, length is 6 units.
82. $\frac{6x^2}{3x} = 2x$,
length is 2x units.
83. $\frac{-6x^2}{3x} = -2x$,
length is $-2x$ units.
84. $(2x)(x + 1) = 2x^2 + 2x$
85. $(2x)(-x + 1) = -2x^2 + 2x$
86. $(2x)(x - 2) = 2x^2 - 4x$
87. $(-2x)(x - 3) = -2x^2 + 6x$
88. 
$= 4x + 2$
89. 
$= 2x^2 - 6x$
90. 
$= x^2 + 3x$
91. $-x^2 - 3x$
92. $-6x^2 - 9x$
93. $\frac{x^2+3x}{x}$ or $(x^2 + 3x) \div (x)$
length is $x + 3$
94. $\frac{-x^2-3x}{x}$ or $(-x^2 - 3x) \div (x)$
length is $-x - 3$

95. $\frac{2x^2-8x}{2x}$ or $(2x^2-8x) \div (2x)$
length is $x-4$

96. $2x^2+6x$
$x+3$
$2x$

97. $6x+18$
6
$x+3$

98. $2x^2+3x$
$2x+3$
x

99. $6a^2b^8$

100. $-10x^5y^8$

101. $-12x^4$

102. $\frac{3}{8}a^4b^3$ or $\frac{3a^4b^3}{8}$

103. $-5t^3$

104. $5xz^2$

105. $\frac{4x^2}{3y}$

106. $-20c^4d^4$

107. $6x^2y^2$

108. a

109. $2x^2-9x-5$

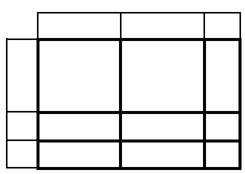
110. $2x(x+1)=2x^2+2x$

111. $2x(2x+1)=4x^2+2x$

112. $2x(x-2)=2x^2-4x$

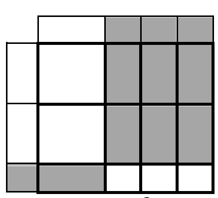
113. $-2x(x-3)=-2x^2+6x$

114.



$=2x^2+5x+2$

115.



$=2x^2-7x+3$

116. $4-x^2$
See solutions guide for area model.

117. $-x^2+4x-3$
See solutions guide for area model.

118. $6x^2+5x+1$
See solutions guide for area model.

119. $A=lw$

$$l=\frac{A}{w}$$

$$\frac{x^2+3x+2}{x+1}$$

length: $x+2$

120. $\frac{2x^2+5x+2}{2x+1}$

length: $x+2$

121. $\frac{4x^2-8x+3}{2x-1}$

length: $2x-3$

122. Area: x^2+5x+6
Length: $x+3$
Width: $x+2$

123. a: x^2+6x+9
Length: $x+3$
Width: $x+3$

124. Area: $2x^2+7x+6$
Length: $2x+3$
Width: $x+2$

125. x^2-2x-3

126. $4x^2+4x+1$

127. x^2-16

128. $x^2-3x-10$

129. $2x^2-5x-3$

130. x^2-6x+9

131. x^2+4x+4

132. $6x^2-3x-3$

133. $4x^2-1$

134. x^2+4x+4

135. $4x^2+20x+25$

136. x^3+2x^2-7x+4

137. $x^3-10x^2+26x-5$

138. $6x^3-5x^2-4x-3$

139. $x^3+6x^2+12x+8$

140. $x^2+2x-2x-4$
$(x+2)(x-2)$
$(x+2)(x-2)=x^2-4$

141. $x^2+3x-3x-9$
$(x+3)(x-3)$
$(x+3)(x-3)=x^2-9$

142. $4x^2+4x-4x-4$
$(2x+2)(2x-2)$
$(2x+2)(2x-2)=4x^2-4$

143. $9x^2+12x-12x-16$
$(3x+4)(3x-4)$
$(3x+4)(3x-4)=9x^2-16$

144. x^2-9

145. $4x^2-9$

146. $9x^2-1$

147. x^2-2y

148. $3b^2-147$

149. $-2c^2+50$

150. $2x^2+15x+30$

151. $3x^2-11x-38$

152. $30t^2-61t+25$

153. $-12y^2-20y-1$

154. $3^2\times 2$

155. $3^2\times 2^4$

156. 2^6

157. $2^3\times 3=24$

158. $2^4=16$

159. $2\times 3^2=18$

160. $5\times2\times a\times a\times b$

161. $2\times3\times3\times a\times b\times b\times c\times c\times c$

162. $2\times2\times3\times b\times b\times b\times c\times c$

163. 2ab

164. $6b^2c^2$

165. 2b

Challenge: $5(x+2)$

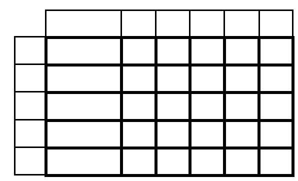
Challenge: $3x(x^2+2x-4)$

166. $5(x+5)$

167. Not factorable.

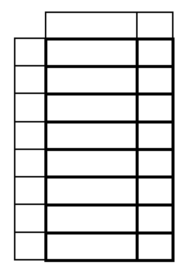
168. $8(x+1)$

169.



170. Cannot be represented as a rectangle using the tiles we have established, therefore it is not factorable.

171.



172. $2a(2x+4y-3z)$

173. $6w^3(2w-1)(2w+1)$

174. $wxy(3w^2+12y-1)$

175. $9a^2b^2(3b+1-2a)$

176. $6mn^2(m^2+3mn-2+4n)$

177. $(5x+3)(a+b)$

178. $(3m+5)(x-1)$

179. Not factorable

180. $(4t+1)(m+7)$

181. $(3t-1)(x-y)$

182. $(4y-x)(p+q)$

Challenge: $(a+b)(c+d)$

183. $(w+z)(x+y)$

184. $(x+1)(x-y)$

185. $(x+3)(y+4)$

186. $(2x+3y)(x+2)$

187. $(m+4)(m-n)$

188. $(3a-2b^2)(a-3)$

Refer to solutions guide to see algebra tiles for questions 189-192.

189. $(x+4)(x+2)$

190. $(x+7)(x+2)$

191. $(x-6)(x-1)$

192. $(x-1)(x+10)$

193. $(a+5)(a+1)$
194. $(n+5)(n+2)$
195. $(x-6)(x+5)$
196. $(q+5)(q-3)$
197. $(k-7)(k+8)$
198. $(t+8)(t+3)$
199. $(y-10)(y+3)$
200. $(g-10)(g-1)$
201. $(s-10)(s+8)$
202. $(m-3)(m-9)$
203. $(x-9)(x+3)$
204. $(p+9)(p-6)$
205. $2(y-4)^2$
206. $(a-9)(a-5)$
207. $2(x+5)(x-2)$
208. $(x^2-5)(x^2+2)$
209. $(w^3+4)(w^3+3)$
210. $(p^4-7)(p^4+3)$
211. $x(8-x)(7+x)$
212. $(x^2+16)(x^2-5)$
213. Not factorable.
214. $(x-5y)(x-y)$
215. $(x+9y)(x-4y)$
216. $(ab-3)(ab-2)$

Challenge: $(2x+3)(x+2)$

217. $(a+4)(2a+3)$
218. $(5a-2)(a-1)$
219. $(3x-2)(x-3)$
220. $(2y+3)(y+3)$
221. $(5y+1)(y-3)$
222. $(2x-3)(5x-1)$
223. $(2x+1)(x+1)$
224. $(3k-4)(2k+1)$
225. $(2y+3)(3y+1$
226. $(3x+2)(x-6)$
227. $x(3x+1)(x-2)$
228. $(3x+1)(3x+4)$
229. $(7x+3)(3x+4)$
230. $x(3x-5)(2x+3)$
231. $2(5t-3)(t+1)$
232. $(3x-y)(x-7y)$
233. $(2c-d)(2c-d)$
234. $(x^2+2)(2x^2+3)$

Challenge:

(x^2-4)
$(x+2)(x-2)$
$x^2-4=(x+2)(x-2)$

235. Answered on page.
236. x^2-9
    $(x+3)(x-3)$
    $x^2-9=(x+3)(x-3)$
237. $4x^2-4$
    $(2x+2)(2x-2)$
    $4x^2-4=(2x+2)(2x-2)$
238. $9x^2-16$
    $(3x+4)(3x-4)$
    $9x^2-16=(3x+4)(3x-4)$
239. $(a+5)(a-5)$
240. $(x+12)(x-12)$
241. $(1+c)(1-c)$
242. $4(x+3)(x-3)$
    Note: $(2x+6)(2x-6)$ is not fully factored because there is GCF that can be removed.

243. $(3x+y)(3x-y)$
244. $(5a^2+6)(5a^2-6)$
245. $(7t+6u)(7t-6u)$
246. $7(x+2y)(x-2y)$
247. $-2(3a+b)(3a-b)$
248. $(d^2+3)(d^2-3)$
249. $(\frac{a}{3}+\frac{b}{4})(\frac{a}{3}-\frac{b}{4})$
250. $(\frac{xy}{7}+1)(\frac{xy}{7}-1)$
251. x^2+4x+4
    $(x+2)(x+2)$
    $x^2+4x+4=(x+2)(x+2)$

    Factored Form: $(x+2)^2$

252. $x^2-3x-3x+9$
    $(x-3)(x-3)$
    $x^2-6x+9=(x-3)(x-3)$

    Factored Form: $(x-3)^2$

253. $9x^2+12x+12x+16$
    $(3x+4)(3x+4)$
    $9x^2+24x+16=(3x+4)(3x+4)$

    Factored Form: $(3x+4)^2$

254. $4x^2-2x-2x+1$
    $(2x-1)(2x-1)$
    $4x^2-4x+1=(2x-1)(2x-1)$

    Factored Form: $(2x-1)^2$

255. $(x+7)^2$
256. $(2x-1)^2$
257. $(3b-4)^2$
258. $4(4m-1)^2$
    Careful. Look for the GCF first.
259. $(9n+5)^2$
260. $(9x-8y)^2$

261. $3(a+b)(a-b)$
262. $4(x+4)(x+3)$
263. $(x^2+4)(x+2)(x-2)$
264. $2(y-4)(y+3)$
265. $4(5x^2-7x+4)$
266. $(m+3)(m-3)(m^2+4)$
267. $(x+1)(x-1)(x^2+1)(x^4+1)$
268. $x(x+y)(x-y)$
269. $(x+2)(x-2)(x+1)(x-1)$
270. $(a+b+c)(a+b-c)$
271. $-4dc$
272. $(m+11)(m+10)$
273. $5(2x-1)$
274. $\pm 1, \pm 4, \pm 11$
275. $\pm 8, 0, 3$
276. $k=9$
277. $k=8$
278. b
279. $-18m^2-48m-32$
280. $4x^2+2x$
281. The second line should read $10x^2+5x+4x+2$. The simplified answer would then be $10x^2+9x+2$.
282. Factor by grouping.
283. The first step in decomposition should have read
    $2x^2-8x+3x-12$
    $2x(x-4)+3(x-4)$
    $(2x+3)(x-4)$
284. If we expand the two binomials, the middle term will not equal -17.
285. $(x^2+2)(x^2+2)$
    $x^4+2x^2+2x^2+4$
    x^4+4x^2+4
286. We would need to describe the tiles in 3-dimensions.
287. x^3+3x^2+3x+1

Additional Material:

288. $x=\pm 6$
289. $x=\pm 4$
290. $x=\pm\frac{3}{2}$
291. $x=\pm\sqrt{7}$
292. $x=-8 \text{ or } 7$
293. $x=-3 \text{ or } 7$
294. $x=\frac{3}{2}$
295. $n=3 \text{ or } \frac{2}{3}$
296. $a=b \text{ or } a=-b$
297. $x^3+2x^2+3x+2=(x+1)(x^2+x+2)$
298. $t^3+3t^2-5t-4=(t+4)(t^2-t-1)$
299. $m^3+2m^2-m-4=(m+1)(m^2+m-2)-2$
300. $x^3-4x^2-2x+8=(x-4)(x^2-2)$
301. $m^3+3m^2-4=(m+2)(m^2+m-2)$
302. $a^3-3a+6=(a+1)(a^2-a+2)+8$
303. $n^3+2n^2-n-2=(n^2-1)(n+2)$
304. $6r^2-25r+14=(3r-2)(2r-7)$
305. $12s^3+3s^2-20s-5=(3s^2-5)(4s+1)$
306. $4y^2-29=(2y-5)(2y+5)-4$