Name: ____________________________

Teacher: Miss Zukowski

Date Submitted: __ / __ / 2018

Unit #____: __________________________________________

Submission Checklist: (make sure you have included all components for full marks)

☑ Cover page & Assignment Log
☑ Class Notes
☑ Homework (attached any extra pages to back)
☑ Quizzes (attached original quiz + corrections made on separate page)
☑ Practice Test/ Review Assignment

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**Assignment Rubric: Marking Criteria**

<table>
<thead>
<tr>
<th>Component</th>
<th>Self Assessment</th>
<th>Teacher Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Notebook</strong></td>
<td></td>
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</tr>
<tr>
<td>● All teacher notes complete</td>
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<tr>
<td>● Daily homework assignments have been recorded &amp; completed (front page)</td>
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<tr>
<td>● Booklet is neat, organized &amp; well presented (ie: name on, no rips/stains, all pages, no scribbles/doodles, etc)</td>
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<tr>
<td><strong>Homework</strong></td>
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<tr>
<td>● All questions attempted/completed</td>
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<tr>
<td>● All questions marked (use answer key, correct if needed)</td>
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<tr>
<td><strong>Quiz</strong> (1mark/dot point)</td>
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<tr>
<td>● Corrections have been made accurately</td>
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<tr>
<td>● Corrections made in a different colour pen/pencil (+½ mark for each correction on the quiz)</td>
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<tr>
<td><strong>Practice Test</strong> (1mark/dot point)</td>
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<tr>
<td>● Student has completed all questions</td>
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<td>● Mathematical working out leading to an answer is shown</td>
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<td>● Questions are marked (answer key online)</td>
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<tr>
<td><strong>Punctuality</strong></td>
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<tr>
<td>● All checklist items were submitted, and completed on the day of the unit test. (-1 each day late)</td>
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</tbody>
</table>
# Homework Assignment Log

& Textbook Pages: ____________________________

<table>
<thead>
<tr>
<th>Date</th>
<th>Assignment/Worksheet</th>
<th>Due Date</th>
<th>Completed?</th>
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</thead>
<tbody>
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</table>

**Quizzes & Tests:**

<table>
<thead>
<tr>
<th>What?</th>
<th>When?</th>
<th>Completed?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quiz 1</td>
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<tr>
<td>Quiz 2</td>
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<tr>
<td>Unit/ Chapter test</td>
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</tbody>
</table>
Example: Place the following numbers on the number line below:

A \(\frac{6}{2}\)  B \(-5.6\)  C \(\sqrt{20}\)  D \(\frac{0}{12}\)  E \(10.325\)  F \(\sqrt[4]{4}\)  G \(\sqrt[3]{4913}\)  H \(3\pi\)
# Real Numbers & Radicals

## Key Terms

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
<th>Example</th>
</tr>
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<tbody>
<tr>
<td>Real Number (R)</td>
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<tr>
<td>Rational Number (Q)</td>
<td></td>
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<tr>
<td>Irrational Number ((\bar{Q}))</td>
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<tr>
<td>Integer (Z)</td>
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<tr>
<td>Whole Number (W)</td>
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<tr>
<td>Natural Number (N)</td>
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<td>Factor</td>
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<td>Factor Tree</td>
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<td>Prime Number</td>
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<td>Prime Factorization</td>
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<td>GCF</td>
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<td>Multiple</td>
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<td>LCM</td>
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<td>Radical</td>
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<td>Index</td>
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<td>Root</td>
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<td>Square root</td>
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<td>Cube root</td>
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<td>Power</td>
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<td>Entire Radical</td>
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<tr>
<td>Mixed Radical</td>
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</table>
The Real Number System

Real numbers are the set of numbers that we can place on the number line.

Real numbers may be positive, negative, decimals that repeat, decimals that stop, decimals that don’t repeat or stop, fractions, square roots, cube roots, other roots. Most numbers you encounter in high school math will be real numbers.

The square root of a negative number is an example of a number that does not belong to the Real Numbers.

There are 5 subsets we will consider.

<table>
<thead>
<tr>
<th>Natural (N)</th>
<th>Whole (W)</th>
<th>Integers (Z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>{1, 2, 3,...}</td>
<td>{0, 1, 2, 3,...}</td>
<td>{...,-3,-2,-1, 0, 1, 2, 3,...}</td>
</tr>
</tbody>
</table>

Real Numbers

Rational Numbers (Q)

Numbers that can be written in the form \( \frac{m}{n} \) where \( m \) and \( n \) are both integers and \( n \) is not 0.

Rational numbers will be terminating or repeating decimals.

Eg. 5, -2.3, \( \frac{4}{3} \), 2\( \frac{3}{8} \)

Irrational Numbers (\( \bar{Q} \))

Cannot be written as \( \frac{m}{n} \).

Decimals will not repeat, will not terminate.

Eg. \( \sqrt{3} \), \( \sqrt{7} \), \( \pi \), 53.123423656787659...

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### Name all of the sets to which each of the following belong?

1. 8  
2. \( \frac{4}{5} \)  
3. \( \frac{15}{5} \)  
4. \( \sqrt{7} \)  
5. \( \sqrt{0.5} \)  
6. 12.34  
7. \(-17\)  
8. \(-\left(\frac{2}{7}\right)^3\)  
9. 2.7328769564923 ...  

### Write each of the following Real Numbers in decimal form. Round to the nearest thousandth if necessary. Label each as Rational or Irrational.

10. \( \frac{5}{9} \)  
11. \(-3 \frac{3}{7} \)  
12. \( \sqrt{8} \)  
13. \( \sqrt{9} \)  
14. \( \sqrt{256} \)  
15. \( \sqrt{25} \)  

16. Fill in the following diagram illustrating the relationship among the subsets of the real number system. (Use descriptions on previous page)

- A
- B
- C
- D
- E
- F
17. Place the following numbers into the appropriate set, rational or irrational.

\[ 5, \sqrt{2}, \sqrt[3]{2}, \sqrt[6]{6}, \frac{1}{2}, \sqrt[7]{7}, \sqrt[8]{8}, \sqrt[25]{25} \]

18. Which of the following is a rational number?

a. \( \frac{\sqrt{2}}{2} \)

b. \( \sqrt[6]{6} \)

c. \( \frac{5}{7} \)

d. 12.356528349875 ...

19. Which of the following is an irrational number?

a. \( \sqrt[4]{16} \)

b. \( \pi \)

c. \( \frac{3}{7} \)

12. \( \sqrt[27]{27} \)

20. To what sets of numbers does -4 belong?

a. natural and whole

b. irrational and real

c. integer and whole

d. rational and integer

21. To what sets of numbers does \(-\frac{4}{3}\) belong?

a. natural and whole

b. irrational and real

c. integer and whole

d. rational and real
The Real Number Line

All real numbers can be placed on the number line. We could never list them all, but they all have a place.

Estimation:
It is important to be able to estimate the value of an irrational number. It is one tool that allows us to check the validity of our answers.

Without using a calculator, estimate the value of each of the following irrational numbers. Show your steps!

22. \( \sqrt{7} \)
Find the perfect squares on either side of 7.
\( \rightarrow 4 \) and 9
Square root 4 = 2
Square root 9 = 3

Guess & Check:
2.6 \( \times \) 2.6 = 6.76
2.7 \( \times \) 2.7 = 7.29
\( \therefore \) \( \sqrt{7} \) is about 2.6

23. \( \sqrt{14} \)

24. \( \sqrt{75} \)

25. \( \sqrt{11} \)

26. \( \sqrt{90} \)

27. \( \sqrt{150} \)

28. Place the corresponding letter of the following Real Numbers on the number line below.

A. \(-6\) B. \(\frac{2}{3}\) C. \(-\frac{2}{3}\) D. \(5\frac{1}{4}\) E. \(\sqrt{2}\) F. \(-\sqrt{7}\) G. \(\frac{\sqrt{5}}{2}\) H. \(-\frac{\sqrt{10}}{3}\)
A. **Factor (noun):**
Example: List the factors of 24.

B. **Factor (verb):**
Example: Factor 24.

C. **Greatest Common Factor (GCF) [think: largest into all]**
TO FIND GCF: List the primes that are in both numbers and multiply them.

Example #1: Find the GCF of 36 & 126.

Example #2: Find the GCF of 42, 90, & 84.

D. **Lowest Common Multiple (LCM)**
Example #1: List the first 6 multiples of 20:

24:

LCM of 20 & 24 is the lowest number that they both divide evenly into.

TO FIND LCM: List the largest power of each prime number & multiply them.
Example #2: Find the LCM of 45 & 60.

Example #3: Find the LCM of 84, 28, & 72.
Factors, Factoring, and the Greatest Common Factor

We often need to find factors and multiples of integers and whole numbers to perform other operations.

For example, we will need to find common multiples to add or subtract fractions. For example, we will need to find common factors to reduce fractions.

**Factor**: (NOUN)

Factors of 20 are \{1,2,4,5,10,20\} because 20 can be evenly divided by each of these numbers.
Factors of 36 are \{1,2,3,4,6,9,12,18,36\}
Factors of 198 are \{1,2,3,6,9,11,18,22,33,66,99,198\}

**To Factor**: (VERB) The act of writing a number (or an expression) as a product.

To factor the number 20 we could write \(2 \times 10\) or \(4 \times 5\) or \(1 \times 20\) or \(2 \times 2 \times 5\) or \(2^2 \times 5\).
When asked to factor a number it is most commonly accepted to write as a product of prime factors. Use powers where appropriate.

Eg. \(20 = 2^2 \times 5\) Eg. \(36 = 2^2 \times 3^2\) Eg. \(198 = 2 \times 3^2 \times 11\)

A factor tree can help you “factor” a number.

```
  36
 / \   /
\  \ /\  \
2  2 9
 / \ / \
3 3
```

\(\therefore\) \(36 = 2^2 \times 3^2\)

<table>
<thead>
<tr>
<th>29. 100</th>
<th>30. 120</th>
<th>31. 250</th>
</tr>
</thead>
</table>

Write each of the following numbers as a product of their prime factors.

Prime:
When a number is only divisible by 1 and itself, it is considered a prime number.

Use division to find factors of a number. Guess and check is a valuable strategy for numbers you are unsure of.
| 32. 324 | 33. 1200 | 34. 800 |

**Greatest Common Factor**

At times it is important to find the largest number that divides evenly into two or more numbers...the **Greatest Common Factor (GCF)**.

**Challenge:**

35. Find the GCF of 36 and 198.

**Challenge:**

36. Find the GCF of 80, 96 and 160.

**Some Notes...**
Find the GCF of each set of numbers.

<table>
<thead>
<tr>
<th>37. 36, 198</th>
<th>38. 98, 28</th>
<th>39. 80, 96, 160</th>
</tr>
</thead>
<tbody>
<tr>
<td>36 = $2^2 \times 3^2$</td>
<td>98 = $2 \times 3^2 \times 11$</td>
<td>80 = $2^4 \times 5$</td>
</tr>
<tr>
<td>198 = $2 \times 3^2 \times 11$</td>
<td></td>
<td>96 = $2^5 \times 3$</td>
</tr>
<tr>
<td>-Prime factors in common are 2 and $3^2$.</td>
<td></td>
<td>160 = $2^5 \times 5$</td>
</tr>
<tr>
<td>-GCF is $2 \times 3^2 = 18$</td>
<td>-Prime factors in common are $2^4$.</td>
<td>-GCF is $2^4=16$</td>
</tr>
<tr>
<td>-Alternate method: List all factors...choose largest in both lists.</td>
<td>-Alternate method: List all factors...choose largest in both lists.</td>
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</table>

40. 24, 108 41. 126, 189, 735, 1470 42. 504, 1050, 1386

Multiples and Least Common Multiple

Challenge

43. Find the first seven multiples of 8.

Challenge

44. Find the least common multiple of 8 and 28.
Multiples of a number

Multiples of a number are found by multiplying that number by \( \{1, 2, 3, 4, 5, \ldots\} \).

Find the first five multiples of each of the following numbers.

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<thead>
<tr>
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<tbody>
<tr>
<td>45.</td>
<td>8</td>
<td></td>
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<tr>
<td>46.</td>
<td>28</td>
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<td>47.</td>
<td>12</td>
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Find the least common multiple of each of the following sets of numbers.

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<tbody>
<tr>
<td>48.</td>
<td>8, 28</td>
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<tr>
<td>49.</td>
<td>72, 90</td>
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<td>50.</td>
<td>25, 220</td>
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</table>

\[ 8 = 2^3 \]
\[ 28 = 2^2 \times 7 \]
- Look for largest power of each prime factor...
- In this case, \( 2^3 \) and 7.

- \( LCM = 2^3 \times 7 \)
  \[ LCM = 56 \]

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<tbody>
<tr>
<td>51.</td>
<td>8, 12, 22</td>
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<tr>
<td>52.</td>
<td>4, 15, 25</td>
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<tr>
<td>53.</td>
<td>18, 20, 24, 36</td>
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</tbody>
</table>

54. Use the least common multiple of 2, 6, and 8 to add:
\[
\frac{3}{8} + \frac{5}{6} + \frac{1}{2}
\]

55. Use the least common multiple of 2, 5, and 7 to evaluate:
\[
\frac{3}{5} - \frac{2}{7} + \frac{3}{2}
\]

56. Use the least common multiple of 3, 8, and 9 to evaluate:
\[
\frac{7}{9} - \frac{1}{3} - \frac{1}{8}
\]
1. $\sqrt{4 + 5} =$

2. $\sqrt{2 + 2 \times 7} =$

3. $\frac{\sqrt{49}}{\sqrt{81}} =$

4. $\sqrt{-576} =$

5. $\sqrt[3]{-512} =$

6. $\sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 11 \cdot 11 \cdot 11 \cdot 11} =$

7. $\sqrt{25x^2} =$

8. $\sqrt{100x^6} =$

9. $\sqrt[3]{27x^6} =$

10. Use the prime factorization of 1728 to determine if it is a perfect cube. If so, determine $\sqrt[3]{1728}$. 
Radicals:
Radicals are the name given to square roots, cube roots, quartic roots, etc.

\[ n \sqrt[ ]{x} \]

The parts of a radical:
- Radical sign \( \sqrt{ } \) (Operations under the radical are evaluated as if inside brackets.)
- Index \( n \) (tells us what type of root we are looking for, if blank…index is 2)
- Radicand \( x \) (the number to be “rooted”)

Square Roots
Square root of 81 looks like \( \sqrt{81} \). It means to find what value must be multiplied by itself twice to obtain the number we began with.

\[ \sqrt{81} \text{ we think } 81 = 9 \times 9 \rightarrow \sqrt{81} = 9 \]

\[ \sqrt{a^4} \text{ we think } a^4 = a^2 \times a^2 \rightarrow \sqrt{a^4} = a^2 \]

PERFECT SQUARE NUMBER: A number that can be written as a product of two equal factors.

\[ 81 = 9 \times 9 \] 81 is a perfect square. Its square root is 9.

First 15 Perfect Square Numbers:
1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, ...

Your notes here...
### Evaluate the following.

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<tbody>
<tr>
<td>57. $\sqrt{49}$</td>
<td>58. $\sqrt{-25}$</td>
<td>59. $-\sqrt{36}$</td>
</tr>
<tr>
<td>60. Finish the statement:</td>
<td>61. Finish the statement:</td>
<td>62. Finish the statement:</td>
</tr>
<tr>
<td>I know that $\sqrt{16} = 4$ because...</td>
<td>I know that $\sqrt{64 \over 81} = \frac{8}{9}$ because...</td>
<td>I know that $\sqrt{-36} \neq -6$ because...</td>
</tr>
<tr>
<td>63. $\sqrt{121}$</td>
<td>64. $\sqrt{45 - 20}$</td>
<td>65. $2\sqrt{40} - (-9)$</td>
</tr>
<tr>
<td>66. Simplify. $\sqrt{x^2}$</td>
<td>67. Simplify. $\sqrt{4x^2}$</td>
<td>68. Simplify. $\sqrt{16x^4}$</td>
</tr>
</tbody>
</table>

**Operations inside a $\sqrt{}$ must be considered as if they were inside brackets...do them**
**Cube Roots:**

PERFECT CUBE NUMBER: A number that can be written as a product of three equal factors.

Cube root of 64 looks like $\sqrt[3]{64}$.

The index is 3. So we need to multiply our answer by itself 3 times to obtain $64 \times 4 \times 4 = 64$

**First 10 Perfect Cube Numbers:** 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, ...

Evaluate or simplify the following.

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<table>
<thead>
<tr>
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<tbody>
<tr>
<td>69. $\sqrt[3]{8}$ Explain what the small 3 in this problem means.</td>
<td>70. $\sqrt[3]{8}$</td>
<td>71. How could a factor tree be used to help find $\sqrt[3]{125}$?</td>
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<tr>
<td>73. $\sqrt[3]{-27}$</td>
<td>74. $\sqrt[3]{1000}$</td>
<td>75. $\sqrt[3]{-8}$</td>
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<tr>
<td>76. Show how prime factorization can be used to evaluate $\sqrt[3]{27}$.</td>
<td>77. $\sqrt[3]{343}$</td>
<td>78. $\sqrt[3]{-216}$</td>
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<tr>
<td>79. $\sqrt[3]{27} \times \sqrt[3]{20 \times 5}$</td>
<td>80. $\sqrt[3]{64 \times 45 - 20}$</td>
<td>81. $\sqrt[3]{-125}$</td>
</tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>82. $\sqrt[3]{a^{12}}$</td>
<td>83. $\sqrt[3]{a^6}$</td>
<td>84. $\sqrt[3]{8x^3}$</td>
</tr>
</tbody>
</table>
### Other Roots.

<table>
<thead>
<tr>
<th>Question</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>85. How does $\sqrt[3]{729}$ differ from $\sqrt[4]{729}$? Explain, do not simply evaluate.</td>
<td>$\sqrt[3]{729}$ differs from $\sqrt[4]{729}$ because the cube root is smaller than the fourth root.</td>
</tr>
<tr>
<td>86. Evaluate if possible. $\sqrt[3]{16}$</td>
<td>$\sqrt[3]{16} = 2$</td>
</tr>
<tr>
<td>87. Evaluate if possible. $\sqrt[4]{-16}$</td>
<td>$\sqrt[4]{-16}$ is not a real number.</td>
</tr>
<tr>
<td>88. Evaluate if possible. $\sqrt{32}$</td>
<td>$\sqrt{32} = 4\sqrt{2}$</td>
</tr>
<tr>
<td>89. Evaluate if possible. $\sqrt[3]{81}$</td>
<td>$\sqrt[3]{81} = 3$</td>
</tr>
<tr>
<td>90. Evaluate if possible. $\sqrt{64}$</td>
<td>$\sqrt{64} = 8$</td>
</tr>
<tr>
<td>91. Evaluate if possible. $\sqrt[2]{24} - 16$</td>
<td>$\sqrt[2]{24} - 16$</td>
</tr>
<tr>
<td>92. Evaluate if possible. $\sqrt[3]{2}(32 - 24)$</td>
<td>$\sqrt[3]{2}(32 - 24)$</td>
</tr>
<tr>
<td>93. Evaluate if possible. $\sqrt[4]{25 - 3}$</td>
<td>$\sqrt[4]{25 - 3}$</td>
</tr>
</tbody>
</table>

Using a calculator, evaluate the following to two decimal places.

<table>
<thead>
<tr>
<th>Question</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>94. $\sqrt{27} - \sqrt{27}$</td>
<td>$\sqrt{27} - \sqrt{27} = 0$</td>
</tr>
<tr>
<td>95. $2\sqrt{10} + \sqrt{64}$</td>
<td>$2\sqrt{10} + \sqrt{64} = 10.92$</td>
</tr>
<tr>
<td>96. $\sqrt{-32} - \sqrt{16}$</td>
<td>$\sqrt{-32} - \sqrt{16}$</td>
</tr>
<tr>
<td>97. $19 - \sqrt{18}$</td>
<td>$19 - \sqrt{18}$</td>
</tr>
<tr>
<td>98. $\frac{\sqrt{12} - \sqrt{7}}{2}$</td>
<td>$\frac{\sqrt{12} - \sqrt{7}}{2}$</td>
</tr>
<tr>
<td>99. $\frac{\sqrt{18} - \sqrt{27}}{3}$</td>
<td>$\frac{\sqrt{18} - \sqrt{27}}{3}$</td>
</tr>
</tbody>
</table>

100. Describe the difference between radicals that are rational numbers and those that are irrational numbers.

Describe the difference between radicals that are rational numbers and those that are irrational numbers.
Evaluate or simplify the following.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>101.</td>
<td>(\sqrt[3]{125})</td>
<td>102.</td>
</tr>
<tr>
<td>104.</td>
<td>(\sqrt{0.16})</td>
<td>105.</td>
</tr>
<tr>
<td>107.</td>
<td>(\sqrt{\frac{1}{4}})</td>
<td>108.</td>
</tr>
<tr>
<td>110.</td>
<td>(\sqrt{a^4})</td>
<td>111.</td>
</tr>
</tbody>
</table>
Evaluate or simplify the following.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>113.</td>
<td>( \sqrt{5^2} )</td>
<td>114.</td>
</tr>
<tr>
<td>116.</td>
<td>( (\sqrt{49} - \sqrt{64})^3 )</td>
<td>117.</td>
</tr>
<tr>
<td>119.</td>
<td>( (\sqrt{16})^3 )</td>
<td>120.</td>
</tr>
</tbody>
</table>

122. Use the prime factors of 324 to determine if 324 is a perfect square. If so, find \( \sqrt{324} \).

Answer:

\[ 324 = 2^2 \times 3^4 \] if fully factored

\[ \therefore \sqrt{324} = \sqrt{2^2 \times 3^2 \times 3^2} \]

\[ \therefore \sqrt{324} = \sqrt{(2 \times 3^2) \times (2 \times 3^2)} \]

\[ \therefore \sqrt{324} = (2 \times 3^2) \]

\[ \therefore \sqrt{324} = 18 \]

123. Use the prime factors of 576 to determine if 576 is a perfect square. If so, find \( \sqrt{576} \).

124. Use the prime factors of 1728 to determine if it is a perfect cube. If so, find \( \sqrt[3]{1728} \).

125. Use the prime factors of 5832 to determine if it is a perfect cube. If so, find \( \sqrt[3]{5832} \).
Unit 1: Real Numbers and Radicals

Lesson 4: pages 18-19

Part 1: Undefined Roots

What values of square roots are UNDEFINED? (ie: NO real solution)

What values of x make these roots undefined?
1. \( \sqrt{x} + 4 \)
2. \( \sqrt{10 - 5x} \)

Part 2: Pythagoras \( (a^2 + b^2 = c^2) \) can only be used if a triangle has a _________ angle!

Calculate the perimeter of the following triangles.
1. \( \sqrt{51} \text{cm} \)
2. \( \sqrt{30} \text{mm} \)

Part 3: Squares and Cubes

1. Is this a perfect square? \( \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 3 \cdot 5} \)

2. Is this a perfect cube? \( \sqrt[3]{3 \cdot 7 \cdot 3 \cdot 7 \cdot 3 \cdot 7} \)

3. The volume of a cube is 729 cm\(^3\). Find the surface area of the cube.
126. An engineering student developed a formula to represent the maximum load, in tons, that a bridge could hold. The student used 1.7 as an approximation for $\sqrt{3}$ in the formula for his calculations. When the bridge was built and tested in a computer simulation, it collapsed. The student had predicted the bridge would hold almost three times as much.

The formula was:
$$5000 \left( 140 - 80\sqrt{3} \right)$$

What weight did the student think the bridge would hold?

Calculate the weight the bridge would hold if he used $\sqrt{3}$ in his calculator instead.

127. For what values of $x$ is $\sqrt{x - 2}$ not defined?

128. For what values of $x$ is $\sqrt{x + 3}$ not defined?

129. For what values of $x$ is $\sqrt{5 - x}$ not defined?

130. Calculate the perimeter to the nearest tenth.

The two smaller triangles are right triangles.

131. To the nearest tenth:

As an expression using radicals:
(you may need to come back to this one)

132. Calculate the area of the shaded region.
133. Consider the square below. Why might you think $\sqrt{\text{36 cm}^2}$ is called a square root?

134. Consider the diagram below. Why do you think $\sqrt[3]{\text{64 cm}^3}$ is called a cube root?

135. Find the side length of the square above.

136. Find the edge length of the cube above.

137. Why do you think 81 is called a "perfect square" number?

138. Why do you think 729 is called a "perfect cube" number?

139. Find the surface area of the following cube.

140. Find the surface area of the following cube.

141. A cube has a surface area of 294 m$^2$. Find its edge length in centimetres.

142. A cube has a surface area of 1093.5 m$^2$. Find its edge length in centimetres.
The Real Number System Answer Key

1. Q, Z, W, N
2. Q
3. Q, Z, W, N
4. Q
5. Q
6. Q
7. Q, Z
8. Q
9. Q
10. Q
11. −3.429, Q (rounded, actually 3.428571)
12. 2.828, √2 (√2)
13. 2.080, √2 (√2)
14. Q
15. 1.904, Q
16. A: real numbers
   B: whole numbers
   C: natural numbers
   D: rational numbers
   E: irrational numbers
   F: integers
17. Rational:
   5, 2, $\sqrt{3}$, $\sqrt{16}$, $\frac{1}{2}$, $\sqrt{8}$
   Irrational:
   $\sqrt{2}$, $\sqrt{5}$, $\sqrt{13}$, $\sqrt{16}$, $\sqrt{25}$
18. c
19. b
20. d
21. d
22. answered on page
23. 3.7
24. 8.7
25. 2.2
26. 4.5
27. 5.3
28. From left to right:
   A, F, C/H, B, G, E, D
29. $5^2 \times 2^1$
30. $2^2 \times 3 \times 5$
31. $5^3 \times 2$
32. $2^2 \times 3^4$
33. $2^4 \times 3 \times 5^2$
34. $2^5 \times 5^2$
35. 18
36. 16
37. 18
38. 14
39. 16
40. 12
41. 21
42. 42
43. 8, 16, 24, 32, 40, 48, 56
44. 56
45. 8, 16, 24, 32, 40
46. 28, 56, 84, 112, 140
47. 12, 24, 36, 48, 60
48. $3^2 \times 2 = 56$
49. $3^2 \times 3^2 \times 5 = 360$
50. 1100
51. 264
52. 300
53. 360
54. $\frac{12}{24}$
55. $\frac{12}{24}$
56. $\frac{12}{24}$
57. 7
58. no real solution
59. −6
60. $4 \times 4 = 16$ or $4^2 = 16$
61. $\frac{5}{9} \times 9 = \frac{45}{9}$
62. There is no real number that can be multiplied by itself to produce a negative number.
63. 11
64. 5
65. 14
66. $x$
67. $2x$
68. $4x^2$
69. Cube or Third root of 8.
   Which means find a number that if multiplied by itself 3
times would have a product of 8. You could also think:
   $2^3 = 8$
70. 2
71. $125 = 5 \times 5 \times 5 = 5^3$ A
times power and a third
   (cube) root are inverse
   operations.
72. 5
73. −3
74. 10
75. −2
76. $27 = 3 \times 3 \times 3 = 3^3$
   $\therefore \sqrt[3]{27} = \sqrt[3]{3^3} = 3$
77. 7
78. −6
79. 30
80. 20
81. −5
82. $a^2$
83. $a^2$
84. $2x$
85. $\sqrt[6]{729}$ means sixth root
   $729 = 3^6$
   $\therefore \sqrt[6]{729} = 3$
   $\sqrt[3]{729}$ means third root
   $729 = 9^3$
   $\therefore \sqrt[3]{729} = 9$
86. 2
87. no real solution
88. 2
89. 3
90. 2
91. 2
92. 2
93. 2
94. 1.07
95. 9.15
96. −4.00
97. 16.38
98. 0.78
99. −0.31
100. Radicals that are rational
   numbers contain radicands
   that are perfect squares,
   cubes, etc. Radicals that are
   irrational numbers do not.
101. 5
102. 6
103. 2
104. 0.4
105. 0.01
106. 7
107. $\frac{1}{2}$
108. $\frac{1}{2}$
109. $\frac{1}{2}$
110. $a^2$
111. $a^2$
112. $2x$
113. 5
114. 5
115. 5
116. −1
117. 3
118. 12 cm
119. 8
120. −2
121. 2
122. Yes,$\sqrt[3]{27} = \sqrt[3]{3^3}$
   $= 2 \times 3^2 = 18$
123. Yes,$\sqrt[3]{729} = \sqrt[3]{9^3}$
   $= 2 \times 3 \times 3 = 24$
124. Yes,$\sqrt[3]{1728} = \sqrt[3]{2^6 \times 3^3}$
   $= 2 \times 3 \times 3 = 12$
125. Yes,
\[ \sqrt{5832} = \sqrt{2^3 \times 3^6} \]
\[ \sqrt{5832} = 2 \times 3^2 = 18 \]

- Student calculated 20 000 tons. The student would have calculated 7179.7 tons if he did not round \( \sqrt{3} \) to \( \pm 7 \).
- \( \sqrt{x} - 2 \) is not defined for values of \( x \) less than 2. That is, if \( x < 2 \).

- \( x < -3 \)
- \( x > 5 \)
- 21.7 units
- 2.8 cm²
- \( 10\sqrt{5} \times \sqrt{5} = \sqrt{3} \times \sqrt{6} = ' \)
- \( 2\sqrt{2} \text{cm}^2 \)

**Perhaps because:** the side length of a square is the square root of that square’s area.

**Perhaps because:** the edge length of a cube is the cube root of that cube’s volume.

- 6 cm
- 4 cm
- "Its square root is an integer.
- "Its cube root is an integer.
- "150 cm²
- "216 cm²
- "700 cm
- "1350 cm
# Exponents: Integral & Rational

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Power</strong></td>
<td>$2^1, 2^2, 2^3, 2^4, \ldots$ are powers of 2.</td>
<td>$2^4 = 2 \times 2 \times 2 \times 2$</td>
</tr>
<tr>
<td></td>
<td>A power is made up of a base and an exponent.</td>
<td></td>
</tr>
<tr>
<td><strong>Exponent</strong></td>
<td>The smaller number written to the upper right of the base that tells you how many times to multiply the base by itself.</td>
<td>$4$ is the exponent.</td>
</tr>
<tr>
<td><strong>Base</strong></td>
<td>The &quot;larger&quot; number that the exponent is applied to. (The bottom number in a power)</td>
<td>$2^4 = 2 \times 2 \times 2 \times 2$</td>
</tr>
<tr>
<td></td>
<td>$2$ is the base.</td>
<td></td>
</tr>
<tr>
<td><strong>Rational number</strong></td>
<td>Numbers that can be written as fractions.</td>
<td></td>
</tr>
<tr>
<td><strong>Rational Exponent</strong></td>
<td>The exponent on a power is a rational number (fraction).</td>
<td>$27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = (3)^2 = 9$</td>
</tr>
<tr>
<td><strong>Integral number</strong></td>
<td>An integer ${-3,-2,-1,0,1,2,3,\ldots}$.</td>
<td></td>
</tr>
<tr>
<td><strong>Integral Exponent</strong></td>
<td>The exponent on a power is an integer. Such as $x^2, x^{-3}$.</td>
<td></td>
</tr>
<tr>
<td><strong>Coefficient</strong></td>
<td>The numbers in front of the letters in mathematical expressions.</td>
<td>In $3x^2$, $3$ is the coefficient.</td>
</tr>
<tr>
<td><strong>Variable</strong></td>
<td>The letters in mathematical expressions.</td>
<td>In $3x^2$, $x$ is the variable.</td>
</tr>
<tr>
<td><strong>Undefined</strong></td>
<td>If there is no good way to describe something, we say it is undefined.</td>
<td>$\frac{3}{0}$ is undefined because we cannot divide by zero.</td>
</tr>
<tr>
<td><strong>Radical form</strong></td>
<td>$(\sqrt{8})^\frac{2}{3}$ is in radical form.</td>
<td></td>
</tr>
<tr>
<td><strong>Exponential Form</strong></td>
<td>$8^\frac{2}{3}$ is in exponential form.</td>
<td></td>
</tr>
<tr>
<td><strong>Zero Exponent</strong></td>
<td>Any expression to the power of 0 will equal 1.</td>
<td>$(2xyz)^0 = 1$</td>
</tr>
<tr>
<td><strong>Negative Exponent</strong></td>
<td>Reciprocate the base and perform repeated multiplication OR use repeated division.</td>
<td>$5^{-3} = \left(\frac{1}{5}\right)^3 = \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = \frac{1}{125}$</td>
</tr>
<tr>
<td><strong>Multiply Powers with the Same base</strong></td>
<td>Add the exponents.</td>
<td>$m^5 \times m^2 = m^7$</td>
</tr>
<tr>
<td><strong>Dividing Powers with the same base.</strong></td>
<td>Subtract the exponents.</td>
<td>$q^6 \div q^4 = q^2$</td>
</tr>
<tr>
<td><strong>Power of a Power</strong></td>
<td>Multiply the exponents.</td>
<td>$(x^2)^4 = x^8$</td>
</tr>
<tr>
<td><strong>Power of a Product</strong></td>
<td>Apply the exponent to all factors.</td>
<td>$(3x^2)^3 = 27x^6$</td>
</tr>
<tr>
<td><strong>Power of a Quotient</strong></td>
<td>Apply the exponent to both numerator AND denominator</td>
<td>$\left(\frac{a^3}{b^3}\right)^\frac{2}{3} = \frac{a^2}{b^2}$</td>
</tr>
</tbody>
</table>
**Vocabulary:**

**Exponent Laws:**

From Math 9, you should have learned how to simplify the following monomial expressions using the following exponent laws:

**Note:** DO NOT use exponent laws when bases aren’t equal

<table>
<thead>
<tr>
<th>Exponent Laws</th>
<th>Examples (simplify &amp; evaluate where possible)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Product of Powers</strong></td>
<td>$a^m \times a^n = $</td>
</tr>
<tr>
<td></td>
<td>a) $0.8^2 \times 0.8^7 =$</td>
</tr>
<tr>
<td></td>
<td>b) $3^4 \times 3 =$</td>
</tr>
<tr>
<td></td>
<td>c) $10^{10} \times 10^{-6} =$</td>
</tr>
<tr>
<td><strong>Quotient of Powers</strong></td>
<td>$a^m \div a^n =$</td>
</tr>
<tr>
<td></td>
<td>a) $5^5 \div 5^3 =$</td>
</tr>
<tr>
<td></td>
<td>b) $\left(\frac{-4}{5}\right)^{-6} \div \left(\frac{-4}{5}\right)^{-20} =$</td>
</tr>
<tr>
<td></td>
<td>c) $40m^8 \div 5m =$</td>
</tr>
<tr>
<td><strong>Negative Exponent</strong></td>
<td>$a^{-m} =$</td>
</tr>
<tr>
<td></td>
<td>a) $25^{-3} =$</td>
</tr>
<tr>
<td></td>
<td>b) $6^3 \div 6^5 =$</td>
</tr>
<tr>
<td><strong>Zero Exponent</strong></td>
<td>$a^0 =$</td>
</tr>
<tr>
<td></td>
<td>a) $(-7x^5y^{-6})^0 =$</td>
</tr>
<tr>
<td></td>
<td>b) $\left(\frac{5}{2}\right)^4 \div \left(\frac{5}{2}\right)^4 =$</td>
</tr>
</tbody>
</table>
Example: Evaluate or simplify the following expressions.

1. $3^2 =$

2. $(-3)^2 =$

3. $-3^2 =$

4. $-5^0 =$

5. $6^{-2} =$

6. $-2^{-4} =$

7. $(-2)^{-4} =$

8. $x^3 \cdot x^4 =$

9. $x^3 \cdot \frac{1}{x^4} =$

10. $6m^4 \cdot 2m \div 3m^{-2} =$
Introduction to Exponents

Challenge #1: Solve each riddle using any strategy that works.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$3^2 \times 3^2$</td>
<td>2</td>
<td>$2^2 \times 2^2 \div 2^3$</td>
<td>3</td>
<td>$x^3 \times x^5$</td>
<td>4</td>
<td>$8x^4 \div 4x^3$</td>
</tr>
</tbody>
</table>

Rate the riddle: Easy, Medium, Hard

5. Find a strategy that is different from the one you used in Question 1 and solve the question again.

6. Find a strategy that is different from the one you used in Question 4 and solve the question again.
What is an Exponent?
Exponents are symbols that indicate an operation to be performed on the base.

- positive exponents → Repeated Multiplication
- negative exponents → Repeated Division

\[ b^e \]
\[ b \] is the base, and \[ e \] is the exponent. Together, we call them a power.

Some examples...

\[ 2^1, 2^2, 2^3, 2^4, 2^5 \] are the first five powers of 2.
\[ x^1, x^2, x^3, x^4, x^5 \] are the first five powers of \( x \).

All organisms begin as one cell and then through a process called mitosis the single cell splits into two, then each of those split into two, etc. Eventually, these cells together form a multi-celled organism with trillions of cells.

\[ \begin{align*}
1 \text{ cell} & \rightarrow 2^0 \\
2 \text{ cells} & \rightarrow 2^1 \\
4 \text{ cells} & \rightarrow 2^2 \\
8 \text{ cells} & \rightarrow 2^3
\end{align*} \]

** Guess the next few numbers _______ , _______ , _______

When numbers are written in a form such as \( 2^3 \) it is called a ____________, the “2” is the ____________ and the “3” is the ____________. The exponent represents the number of times the base is multiplied by itself.

\[
\begin{array}{|c|c|}
\hline
a^x & a \text{ is the base, } x \text{ is the exponent and } a^x \text{ is the power.} \\
\hline
5^2 & \text{Is read 5 to the exponent 2 and equals } 5 \times 5 \text{ as a repeated multiplication and evaluates to 25.} \\
2^5 & \text{Is read 2 to the exponent 5 and equals } 2 \times 2 \times 2 \times 2 \times 2 \text{ as a repeated multiplication and evaluates to 32.} \\
\hline
\end{array}
\]
Positive Integral Exponent (multiplication)  
\( a^n = 1 \times a \times a \times \ldots \times a \)  
(n factors)

Eg. \( 3^4 = 1 \times 3 \times 3 \times 3 \times 3 = 81 \)

Zero Exponent  
\( a^0 = 1, \quad (a \neq 0) \)

Eg. \( 5^0 = 1, \quad \left(\frac{3}{2}\right)^0 = 1 \)

Negative Integral Exponent (repeated division)  
\( a^{-n} = 1 \div a^n \)

\[ a^{-n} = \frac{1}{a^n} \]

Eg. \( 5^{-2} = \frac{1}{5^2} = \frac{1}{25} \)

**Challenge #2**

7. Evaluate each of the following and examine the pattern:

\[
\begin{align*}
2^4 &= \\
2^3 &= \\
2^2 &= \\
2^1 &= \\
2^0 &= \\
2^{-1} &= \\
2^{-2} &= \\
2^{-3} &= \\
2^{-4} &= 
\end{align*}
\]

8. What patterns do you notice in the list you created to the left?

9. Does the value of \(2^0\) make sense when put into this list?

10. Do negative exponents make sense in this list?

11. Why might people say negative exponents mean "repeated division?"
12. Identify the base in the following equation. 
\[ 4^3 = 64 \]

13. Identify the power in the following equation. 
\[ 2^5 = 32 \]

14. Identify the exponent in the following equation. 
\[ -3^2 = -9 \]

15. Which of the following is equivalent to \(-16\)?
- \(-4^2\)
- \((-4)^2\)
- \(4^{-2}\)
- \(-4^{-2}\)

16. Which of the following is equivalent to \(-81\)?
- \(-9^2\)
- \((-3)^4\)
- \(9^{-2}\)
- \(-3^{-4}\)

17. Which of the following are equivalent to 1.
- \(-3^0\)
- \(\frac{2x^3}{2x^3}\)
- \((5x)^0\)

18. Which of the following is equivalent to 9?
- \(-3^2\)
- \((-3)^2\)
- \(3^{-2}\)
- \((-3)^{-2}\)

19. Evaluate. 
\[ -2^6 = -1 \times 2 \times 2 \times 2 \times 2 \times 2 = -64 \]

20. Evaluate. 
\[ (-3)^3 \]

21. \(-4^2\)

22. \((-4)^{-2}\)

23. \(-4^{-2}\)

24. \(3^{-4}\)
\[
= \frac{1}{3^4} = \frac{1}{3 \times 3 \times 3 \times 3} = \frac{1}{81}
\]

25. \((-3)^{-4}\)

26. \(-3^{-4}\)

27. \(4^2\)

28. \((-4)^2\)

29. \(-4^2\)
| 30. $5^0$ | 31. $-5^0$ | 32. $\left(\frac{3a^4}{2x^3}\right)^0$ |

The Exponent Laws:

**Challenge #3**

33. **Multiply.**  

\[ a^3 \times a^6 \]  

Explain your steps.

**Challenge #4**

34. **Divide.**  

\[ g^7 \div g^3 \]  

Explain your steps.

**Challenge #5**

35. **Multiply.**  

\[ 5m^4 \times 3m^2 \]  

Explain your steps.
Simplify the following, write your answers using exponents.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>36. $a^3 \times a^6$</td>
<td>$a^{3+6}$</td>
<td>$a^9$</td>
</tr>
<tr>
<td>37. $a^2 \times a^{-4}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>38. $f^2 \times f^x$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>39. $\frac{1}{x^3} \times \frac{2}{x^2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40. $2^3 \times 2^{-5}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>41. $g^7 \div g^3$</td>
<td>$g^{7-3}$</td>
<td>$g^4$</td>
</tr>
<tr>
<td>42. $m^4 \div m^6$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>43. $t^3 \div t^{-5}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>44. $\frac{x^{13}}{x^3}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>45. $5m^4 \times 3m^2$</td>
<td>$5 \times 3 \times m^{4+2}$</td>
<td>$15m^6$</td>
</tr>
<tr>
<td>46. $-10x^3 \div -2x^{-2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>47. $\frac{4a^4}{-8a^2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>48. $\frac{2}{3}x^3 \times \frac{6}{5}x^4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>49. $\frac{2}{a^3} \div \frac{6}{a^5}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50. Evaluate.</td>
<td>$(\frac{2}{3})^3 \left(\frac{-6}{4}\right)^2$</td>
<td></td>
</tr>
</tbody>
</table>

Multiplying Powers with the same Base:
Add the exponents.

Eg. $x^5 \times x^2 = x^{5+2} = x^7$

$\frac{2}{a^5} \times \frac{1}{a^3} = a^{3-5} = a^{-2}$

$3x^2 \times 2x^5 = 3 \times 2 \times x^2 \times x^5 = 6x^7$

Dividing Powers with the same Base:
Subtract the exponents.

Eg. $d^4 \div d^3 = d^{4-3} = d^1 = d$

$\frac{y^6}{y^2} = y^{6-(-2)} = y^8$
Warm-Up:
8. $100x^4 \div 50x^8$
9. $a^9 \div a^{12}$
10. $6x^4 + 6x^5$
11. $(\frac{2}{5})^{-3}$
12. $6m^{12} \div 12m^{12}$
13. $30m^8 \div -10m$
14. $\frac{3m^{-2}p^4}{4a^{-1}}$
Exponent Laws:

From Math 9, you should have learned how to simplify the following monomial expressions using the following exponent laws:

Note: DO NOT use exponent laws when bases aren’t equal

<table>
<thead>
<tr>
<th>Exponent Laws</th>
<th>Examples (simplify &amp; evaluate where possible)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Power of a Power</strong></td>
<td></td>
</tr>
<tr>
<td>((a^m)^n = )</td>
<td></td>
</tr>
<tr>
<td>a) ((0.25^{-3})^{-5})</td>
<td></td>
</tr>
<tr>
<td>b) ((8^2)^4)</td>
<td></td>
</tr>
<tr>
<td>c) ((m^5)^3)</td>
<td></td>
</tr>
<tr>
<td>d) ((2m^{10})^3)</td>
<td></td>
</tr>
<tr>
<td><strong>Power of a Product</strong></td>
<td></td>
</tr>
<tr>
<td>((ab)^m = )</td>
<td></td>
</tr>
<tr>
<td>a) ((-6my^7)^3)</td>
<td></td>
</tr>
<tr>
<td>b) ((x^4y^{-2})^5)</td>
<td></td>
</tr>
<tr>
<td>c) ((8x^{-4})^2)</td>
<td></td>
</tr>
<tr>
<td>d) ((3m^{-2}y^5)^{-3})</td>
<td></td>
</tr>
<tr>
<td>e) ((3t^0)^4)</td>
<td></td>
</tr>
</tbody>
</table>

(More Complicated) Examples ☹: Evaluate or simplify the following expressions.

1. \(\left(\frac{2x^4y^{-3}}{3x^{-2}y}\right)^{-2}\)

2. \((-10xy^4)^2 \cdot (5x^2y^3)^{-2}\)
### Challenge #6

51. **Evaluate.**

\[(5^2)^3\]

Explain your steps.

\[
\begin{align*}
(5^2)^3 &= 5^{2 \cdot 3} \\
&= 5^6 \\
&= 15625
\end{align*}
\]

### Challenge #7

52. **Simplify.**

\[(m^3)^2\]

Explain your steps.

\[
\begin{align*}
(m^3)^2 &= m^{3 \cdot 2} \\
&= m^6
\end{align*}
\]

### Challenge #8

53. **Simplify.**

\[(2m^4)^3\]

Explain your steps.

\[
\begin{align*}
(2m^4)^3 &= 2^3 \cdot (m^4)^3 \\
&= 8m^{12}
\end{align*}
\]
Simplify the following.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>54. $(m^3)^2$</td>
<td>$m^6$</td>
</tr>
<tr>
<td>55. $(r^4)^0$</td>
<td>$1$</td>
</tr>
<tr>
<td>56. $(x^2y^3)^{-3}$</td>
<td>$\frac{1}{x^6y^9}$</td>
</tr>
<tr>
<td>57. $(2m^4)^3$</td>
<td>$8m^{12}$</td>
</tr>
<tr>
<td>58. $(2c^4d^3)^{-3}$</td>
<td>$\frac{1}{8c^{12}d^{9}}$</td>
</tr>
<tr>
<td>59. $(-3x^{-5}y^3)^{-4}$</td>
<td>$\frac{1}{81x^{20}y^{-12}}$</td>
</tr>
<tr>
<td>60. $(3x^{-2}y^{-3})^{-3}$</td>
<td>$\frac{1}{27x^6y^9}$</td>
</tr>
<tr>
<td>61. $(-2xy^2)(-3x^2y^3)^2$</td>
<td>$18x^5y^8$</td>
</tr>
<tr>
<td>62. $(2a^3(4a^2b)^2$</td>
<td>$16a^7b^4$</td>
</tr>
</tbody>
</table>

### Power of a Power:
Multiply the exponents.

**Example:**

\[
(5^2)^3 = (5 \times 5)^3 = (5 \times 5 \times 5 \times 5) = 5^6
\]

**The Rule:**

\[
(a^m)^n = a^{m \times n}
\]

If you have a power of a power ... multiply exponents.

**Example:**

\[
(x^2)^5 = x^{2 \times 5} = x^{10}
\]

### Power of a Product:
Apply the exponent to all factors.

**Example:**

\[
(5 \times 2)^3 = (5 \times 5 \times 5 \times 5 \times 5) = 5^6
\]

**The Rule:**

\[
(ab)^m = a^m b^m
\]

If you have a power of a product ... apply the exponent to EVERY factor in the product.

**Example:**

\[
(a^2b^3)^{-3} = a^{2 \times -3}b^{3 \times -3} = a^{-6}b^{-9}
\]
Challenge #9
63. Evaluate.
\[
\left( \frac{2}{5} \right)^3
\]
Explain your steps.

\[
\frac{8}{125}
\]

Challenge #10
64. Evaluate.
\[
\left( \frac{2}{5} \right)^{-3}
\]
Explain your steps.

\[
\frac{125}{8}
\]

Challenge #11
65. Simplify.
\[
\left( \frac{x}{2} \right)^3
\]
Explain your steps.

Challenge #12
66. Simplify.
\[
\left( \frac{6x^5y^3}{8y^4} \right)^{-2}
\]
Explain your steps.
Warm-Up: Simplify or evaluate as far as possible. Express answers with positive exponents.

1. $7^{-3}$
2. $2^6 \times 2^4$
3. $x^9 \div x^3$
4. $7m^4 \times 2m$
5. $(-8xy^5)^2$
6. $50p^9 \div 10p^{-2}$
7. $(3m^0)(9m)^0$
8. $(5m)^{-2}$
9. $(2^{-3})^{-2}$
10. $(10y^{-3})(6y^4)^2$
11. $(4x^2y^3)^{-3}$
12. $\frac{6m^8y^2z^{-4}}{12my^8z^{-8}}$
13. $\frac{-10ab^{-1}c^{-4}}{4a^{-2}c^7}$
14. $x^{-3} \cdot x^{-\frac{4}{3}} \cdot x^\frac{1}{3}$
Exponent Laws:

From Math 9, you should have learned how to simplify the following monomial expressions using the following exponent laws:

<table>
<thead>
<tr>
<th>Exponent Laws</th>
<th>Examples (simplify &amp; evaluate where possible)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power of a Quotient</td>
<td>[(\frac{a}{b})^m = \frac{a^m}{b^m} ]</td>
</tr>
<tr>
<td>Power of a Quotient</td>
<td>[(\frac{a}{b})^{-m} = \frac{b^m}{a^m} ]</td>
</tr>
</tbody>
</table>

(More Complicated) Examples 🤓: Evaluate or simplify the following expressions.

1. \[\left(\frac{x^4y^m}{x^7y^2m^5}\right)^{-6} \]

2. \[\frac{(5m^{-1}y^3)^2}{m^y} \]

3. \[\left(\frac{2x^{-1}y^6}{x^{-y}y^4}\right)^{-2} \]

4. \[\left(\frac{1}{2a^6b^5}\right)^{-4} \]

5. \[\left(\frac{8xb^{-7}}{-12x^2b^{-3}}\right)^{-3} \]
**Power of a Quotient:**

Apply the exponent to numerator AND denominator.

\[
\left( \frac{2}{3} \right)^3 = \left( \frac{2^3}{3^3} \right)
\]

\[
= \frac{8}{27}
\]

If asked to write using exponents

\[
= \frac{8}{27}
\]

If asked to simplify.

\[
\left( \frac{2}{3} \right)^{-3}
\]

The negative exponent means "flip the base".

\[
= \frac{2^{-3}}{3^{-3}}
\]

\[
= \frac{8}{27}
\]

\[
= \frac{8}{125}
\]

\[
\frac{a}{b}^m = \frac{a^m}{b^m}
\]

\[
\left( \frac{a}{b} \right)^{-m} = \frac{b^m}{a^m}
\]

**Simplify the following.**

67. \( \left( \frac{x^3}{2} \right)^3 = \frac{x^9}{8} \)

68. \( \left( \frac{a^4}{b} \right)^4 \)

69. \( \left( \frac{x^5}{y^3} \right)^5 \)

70. \( \left( \frac{-2a^2}{3y^3} \right)^3 \)

71. \( \left( \frac{a^{-3}}{b^{-2}} \right)^2 \)

72. \( \left( \frac{4x^2}{3y} \right)^2 \)

73. \( \left( \frac{6x^5y^3}{8y^4} \right)^{-2} \)

\[
= \frac{(8)^2(y^4)^2}{(6)^2(x^5)^2(y^3)^2}
\]

\[
= \frac{64y^8}{36x^{10}y^6}
\]

\[
= \frac{16y^2}{9x^{10}}
\]

74. \( \left( \frac{5ab^2c^3}{2a^{-2}c^{-3}} \right)^2 \)

75. \( \left( \frac{2m^2n^2}{m^3n^3} \right)^{-1} \)

\[
= \left( \frac{2m^2n^2}{m^3n^3} \right)^{-1} \]

\[
= \frac{m^3n^3}{2m^2n^2}
\]
Simplify the following.

76. \( \left( \frac{6a^3 b^2}{2ab} \right)^3 \)

77. \( (4x^{-3}y^4)^{-2} \)

78. Show why \( \frac{2a^2}{b^3} \) is the same as \( 2a^2 \times b^{-3} \).

79. Show why \( \frac{12x^3}{y} \) is the same as \( 12x^3 \times y^{-1} \).

Challenge #13

80. Write the following without using any negative exponents.

\[ 3a^2 b^{-5} \]

81. Write the following without using any negative exponents.

\[ \frac{3}{a^{-2} b^5} \]

Challenge #14

82. Simplify using positive exponents.

\( \left( \frac{2x^{-2}y^4}{x^{-2}y^3} \right)^3 \)

Explain your steps.
Writing Expressions with Positive Exponents. (Why? Because it is standard practice.)

An expression with powers is simplified if there are no brackets and no negative exponents.

Sometimes you will use the laws above and end up with an answer with negative exponents. The quick way to convert a negative exponent into a positive exponent is to move it across the division line. The solution in question 83 shows why this works.

Simplify the following. (No brackets, no negative exponents)

<table>
<thead>
<tr>
<th>83. $3a^2b^{-5}$</th>
<th>84. $a^2b^{-3}$</th>
<th>85. $\frac{2xy^5}{x^{-3}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$= 3a^2 \times \frac{1}{b^5}$</td>
<td>$= \frac{3a^2}{b^5}$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>86. $3a^2b^{-3}c^{-5}$</th>
<th>87. $(x^4y^{-3}z^{-1})^{-2}$</th>
<th>88. $\frac{(3x^{-3}y^{-5})^2}{2xy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>89. $\left(\frac{2x^{-2}y^4}{x^{-3}y^3}\right)^{-3}$</th>
<th>90. $\left(\frac{2a^3b^2}{4a^{-2}b^{-7}}\right)^{-3}$</th>
<th>91. $\frac{(4m^n-n^3)(7m^{-3}n^2)}{14m^n}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$= \left(\frac{x^{-3}y^3}{2x^{-2}y^4}\right)^3$</td>
<td>$= \left(\frac{2a^3b^2}{4a^{-2}b^{-7}}\right)^{-3}$</td>
<td>$= \frac{(4m^n-n^3)(7m^{-3}n^2)}{14m^n}$</td>
</tr>
<tr>
<td>$= \frac{x^{-9}y^9}{8x^{-6}y^{12}}$</td>
<td>$= \frac{8}{2a^3b^2}$</td>
<td></td>
</tr>
<tr>
<td>$= \frac{x^{-3}y^{-3}}{8}$</td>
<td>$= \frac{1}{8x^3y^3}$</td>
<td></td>
</tr>
</tbody>
</table>

92. Why does moving a power across the division line in a fraction change the sign on the exponent?
Simplify the following. (No brackets, no negative exponents)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>93. $\left( \frac{12x^2y^{-1}}{-8x^{-1}y^5} \right)^{-2}$</td>
<td>94. $\left( \frac{4a^3b^{-2}}{-6a^2b^{-1}} \right)^{-3}$</td>
</tr>
<tr>
<td>95. $\left( \frac{8x^2y^{-3}}{4x^{-1}y^{-5}} \right)^{-3}$</td>
<td>96. $\left( \frac{12x^{-3}y^5}{16x^3y^{-2}} \right)^{-1}$</td>
</tr>
</tbody>
</table>
**Warm-Up #1:** Simplify or evaluate as far as possible. Express answers with positive exponents.

1. \( \frac{3x^2y^4}{4x^3y^3} \)

2. \( \frac{2ab^3}{-2a^3b^2} \)

3. \( \frac{-2x^2y^{-5}}{-3x^{-4}y^3} \)

4. \( \frac{(n^2)^4(-n^0)^3}{-n^2} \)

5. \( \frac{2a^2bc^{-4}}{5^{-1}a^{-3}b^3c^2} \)

6. \( \left( \frac{x^2y}{mp^3} \right)^5 \)

7. \( \left( \frac{3e}{5d} \right)^{-2} \)

8. \( \left( \frac{15m^6y}{3my^{-5}} \right)^{-3} \)

9. \( \left( \frac{27m^2n}{9mn} \right)^{-2} \)

10. \( \left( \frac{2a^2b^{-2}}{8a^3b} \right)^{-4} \)
**Warm-Up #2:** Use your calculator to complete the following tables:

1. Explain the effect the exponent $\frac{1}{2}$ has on the value of $x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$x^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td></td>
</tr>
</tbody>
</table>

Write a rule to describe this relationship:

$$\frac{1}{x^2} =$$

2. Explain what effect the exponent $\frac{1}{3}$ has on the value of $y$.

<table>
<thead>
<tr>
<th>$y$</th>
<th>$y^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td></td>
</tr>
<tr>
<td>64</td>
<td></td>
</tr>
<tr>
<td>125</td>
<td></td>
</tr>
<tr>
<td>216</td>
<td></td>
</tr>
</tbody>
</table>

Write a rule to describe this relationship:

$$\frac{1}{y^3} =$$

3. What do you think $x^{\frac{1}{2}}$ means? Test your prediction on your calculator, letting $x = 16$.

4. What would $x^{\frac{1}{n}}$ mean (as a radical)?
### Exponent Law:

<table>
<thead>
<tr>
<th>Exponent Laws</th>
<th>Example #1 (simplify &amp; evaluate where possible)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^{\frac{1}{n}} = )</td>
<td>a) (100^{\frac{1}{2}})</td>
</tr>
<tr>
<td></td>
<td>b) ((-8)^{\frac{1}{3}})</td>
</tr>
<tr>
<td></td>
<td>c) (1024^{\frac{1}{5}})</td>
</tr>
<tr>
<td></td>
<td>d) ((625m^4)^{\frac{1}{4}})</td>
</tr>
<tr>
<td></td>
<td>e) ((81m)^{\frac{1}{3}})</td>
</tr>
<tr>
<td></td>
<td>f) (-343^{\frac{1}{3}})</td>
</tr>
<tr>
<td></td>
<td>g) ((-49)^{\frac{1}{2}})</td>
</tr>
<tr>
<td></td>
<td>h) (16^{-\frac{1}{4}})</td>
</tr>
<tr>
<td></td>
<td>i) (1000^{\frac{1}{3}})</td>
</tr>
</tbody>
</table>

### Rational Exponents (with numerator = 1)

**Example #2:** Simplify the following in radical form.

1. \(\sqrt{121}\)

2. \(\sqrt[5]{-32}\)

3. \(\frac{1}{\sqrt[5]{125}}\)

4. \(10\sqrt{3xy}\)
97. **Challenge #15**

If $\sqrt{9} \times \sqrt{9} = 9$,

and $9^a \times 9^a = 9$

Then what is the value of 'a'?

98. **Challenge #16**

If $\sqrt[3]{2} \times \sqrt[3]{2} \times \sqrt[3]{2} = 2$,

and $2^a \times 2^a \times 2^a = 2$

Then what is the value of 'a'?

99. Write a “rule” that relates a rational (fraction) exponent to an equivalent radical expression.
Rational Exponents in the form: \( x^{\frac{1}{n}} \)

Remember, rational often refers to fractions.

What does a rational exponent mean?
Recall: \( \sqrt{9} \times \sqrt{9} = 9 \).
But \( 9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 9 \).
And \( 3 \times 3 = 9 \).
So, \( \sqrt{9} = 9^{\frac{1}{2}} = 3 \).

If \( \sqrt{2} \times \sqrt{2} \times \sqrt{2} = 2 \).
But \( 2^{\frac{1}{3}} \times 2^{\frac{1}{3}} \times 2^{\frac{1}{3}} = 2 \).
So, \( \sqrt[3]{2} = 2^{\frac{1}{3}} \).

The Rule...

\[ a^{\frac{1}{n}} = \sqrt[n]{a} \quad \text{and} \quad a^{\frac{1}{n}} = \frac{1}{\sqrt[n]{a}} \]

Evaluate or simplify the following.

101. \( 49^{\frac{1}{2}} \)
102. \( -16^{\frac{1}{2}} \)
103. \( (-16)^{\frac{1}{2}} \)

104. \( 64^{\frac{1}{3}} \)
105. \( 27^{-\frac{1}{3}} \)
106. \( 32^{-\frac{1}{3}} \)

107. \( 10000^{\frac{1}{5}} \)
108. \( (4x^2)^{\frac{1}{3}} \)
109. \( (27x^6)^{-\frac{1}{3}} \)
Write in radical form.

110. $7^{\frac{1}{2}}$

111. $(3x)^{\frac{1}{3}}$

112. $4^{\frac{1}{3}}$

113. $4^{-\frac{1}{5}}$

114. $-6^{\frac{1}{3}}$

115. $64^{-\frac{1}{3}}$

Write in exponential form.

116. $\sqrt{13}$

117. $-3\sqrt{x}$

118. $\sqrt[3]{2y}$

119. $\sqrt[4]{4}$

120. $\sqrt[4]{4}$

121. $\frac{1}{\sqrt[3]{x}}$

Consider the following...

Step 1: $32^{\frac{3}{2}} = (32^{\frac{1}{2}})^3$

Step 2: $32^{\frac{3}{2}} = (\sqrt[3]{32})^3$

Step 3: $32^{\frac{3}{2}} = (2)^3$

Step 4: $32^{\frac{3}{2}} = 8$

122. Challenge #17. Complete the following as shown above.

    Step 1: $27^{\frac{2}{3}} = $ Explain: _____________________________

    ______________

    Step 2: $27^{\frac{2}{3}} = $

    ______________

    Step 3: $27^{\frac{2}{3}} = $

    ______________

    Step 4: $27^{\frac{2}{3}} = $
Warm-Up #1: Simplify or evaluate as far as possible (#1-6), or re-write radicals as exponents (#7-10). Express answers with positive exponents.

1. \(16^{\frac{1}{4}}\)
2. \(27^{-\frac{1}{3}}\)
3. \(-25^{\frac{1}{2}}\)
4. \((-25)^{\frac{1}{2}}\)
5. \(1024^{0.5}\)
6. \((-2)^{-2})^{\frac{1}{2}}\)
7. \(8^{\frac{1}{3}}\)
8. \(\sqrt{16y^8}\)
9. \(\frac{50}{\sqrt[3]{x^y}}\)
10. \((\sqrt[3]{7/2})^6\)
Warm-Up #2:

1. Re-write the exponents below as a product of two fractions, remembering that \( \frac{a}{b} = \frac{a}{1} \times \frac{1}{b} \). Then, evaluate. The first one is done as an example 😊😊

   a. \( 9^\frac{3}{2} = \left(9^\frac{1}{2}\right)^1 = (729)^\frac{1}{2} = \sqrt{729} = 27 \)

   b. \( 100^\frac{5}{2} \)

   c. \( 216^\frac{2}{3} \)

This works, but there’s an easier way!

**Exponent Law:**

<table>
<thead>
<tr>
<th>Exponent Laws</th>
<th>Example #1 (simplify &amp; evaluate where possible)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^{\frac{m}{n}} = )</td>
<td>a) ( 32^{\frac{3}{5}} )</td>
</tr>
<tr>
<td></td>
<td>b) ((-32)^{\frac{3}{5}})</td>
</tr>
<tr>
<td></td>
<td>c) (16^{\frac{7}{2}})</td>
</tr>
<tr>
<td></td>
<td>d) ((-27)^{\frac{2}{3}})</td>
</tr>
<tr>
<td></td>
<td>e) ((-25)^{\frac{5}{2}})</td>
</tr>
<tr>
<td></td>
<td>f) (-25^{\frac{3}{2}})</td>
</tr>
<tr>
<td></td>
<td>g) (-25^{-\frac{5}{2}})</td>
</tr>
<tr>
<td></td>
<td>h) (16^{1.5})</td>
</tr>
<tr>
<td></td>
<td>i) (1000^{-\frac{2}{3}})</td>
</tr>
</tbody>
</table>
Example #2: Write the following with exponents. Then, use exponent laws and evaluate.

1. $\sqrt{8} \times \sqrt{8}^3$

2. $\sqrt{g^5} \times \sqrt{g^7}$

3. $\sqrt[3]{16^3}$

4. $\sqrt[3]{x^2} \cdot \sqrt[6]{x}$

5. $(\sqrt[5]{18})^2 \cdot \sqrt[5]{18^3}$

6. $\sqrt[3]{64} \cdot \sqrt[4]{16^3}$

Example #3: Find the area of a triangle that has a base of $8\frac{4}{5}$ cm and a height of $8\frac{11}{2}$ cm. (Hint: $A = \frac{b \times h}{2}$)
Rational Exponents in the form: \( x^{\frac{m}{n}} \) where \( m \) is not 1.

Consider the power \( 27^{\frac{2}{3}} \). To understand the meaning of the rational exponent we can use the exponent law:

\((a^m)^n = a^{mn}\).

If we take \( 27^{\frac{2}{3}} \) and split the exponent into two parts we get the following...

\[
27^{\frac{2}{3}} = \left(27^{\frac{1}{3}}\right)^2
\]

This can then be written as...

\[
\left(\sqrt[3]{27}\right)^2
\]

The power can be evaluated from this point...

\[
\left(\sqrt[3]{27}\right)^2 = (3)^2 = 9
\]

The Rule...

\[
a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m \quad \text{and} \quad a^{-\frac{m}{n}} = \frac{1}{\sqrt[n]{a^m}} = \frac{1}{\left(\sqrt[n]{a}\right)^m}
\]

Two more examples:

Eg.1 Evaluate \( 8^{\frac{2}{3}} \) without using a calculator.

\[
8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4
\]

Means square of the cube root of 8.

Eg.2 Evaluate \( 9^{-\frac{3}{2}} \) without using a calculator.

\[
9^{-\frac{3}{2}} = \left(\frac{1}{9}\right)^{\frac{3}{2}} = \left(\frac{1}{\sqrt{9}}\right)^3 = \frac{1}{(\sqrt{9})^3} = \frac{1}{(3)^3} = \frac{1}{27}
\]

Means “the reciprocal” of the cube of the square root of 9.
Write each of the following using radicals. (Do not evaluate)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>123. $\sqrt[5]{4}$</td>
<td>124. $\sqrt[5]{4}$</td>
<td>125. $\sqrt[5]{4}$</td>
</tr>
<tr>
<td>126. $\sqrt[5]{4}$</td>
<td>127. $\sqrt[5]{4}$</td>
<td>128. $\sqrt[5]{4}$</td>
</tr>
</tbody>
</table>

Evaluate each of the following.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>129. $\sqrt[2]{4}$</td>
<td>130. $\sqrt[3]{125}$</td>
<td>131. $\sqrt[3]{8}$</td>
</tr>
<tr>
<td>132. $\sqrt[3]{81}$</td>
<td>133. $\sqrt[2]{4}$</td>
<td>134. $\sqrt[4]{16}$</td>
</tr>
<tr>
<td>135. $(-27)^{\frac{2}{3}}$</td>
<td>136. $(-8)^{\frac{2}{3}}$</td>
<td>137. $9^{\frac{2}{3}}$</td>
</tr>
<tr>
<td>138. $(-1)^{\frac{2}{5}}$</td>
<td>139. $(\frac{100}{9})^{\frac{2}{3}}$</td>
<td>140. $(\frac{27}{8})^{\frac{2}{3}}$</td>
</tr>
</tbody>
</table>
### Write each of the following using exponents. (Do not evaluate)

Eg. \( \sqrt[2]{12} = 12^{\frac{1}{2}} \)

<table>
<thead>
<tr>
<th>141. ( \sqrt[7]{7} )</th>
<th>142. ( \sqrt[3]{34} )</th>
<th>143. ( \sqrt[11]{-11} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>144. ( \sqrt[2]{a^2} )</td>
<td>145. ( \sqrt[4]{6^4} )</td>
<td>146. ( (\sqrt[3]{x})^2 )</td>
</tr>
<tr>
<td>147. ( (\sqrt[6]{6})^3 )</td>
<td>148. ( (\sqrt[2]{2x})^5 )</td>
<td>149. ( \frac{1}{\sqrt[8]{8}} )</td>
</tr>
<tr>
<td>150. ( \frac{1}{(\sqrt[2]{x})^3} )</td>
<td>151. ( \frac{1}{\sqrt[3]{x^3}} )</td>
<td>152. ( \sqrt[11]{2b^3} )</td>
</tr>
</tbody>
</table>

### Evaluate if possible.

<table>
<thead>
<tr>
<th>153. ( (-9)^{\frac{1}{2}} )</th>
<th>154. ( 100000^{\frac{3}{10}} )</th>
<th>155. ( \left(\frac{2^{2/3}}{8}\right)^{\frac{2}{3}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>156. ( 3^{\frac{2}{2}} \times 3^{\frac{2}{2}} )</td>
<td>157. ( -9^{\frac{1}{2}} )</td>
<td>158. ( (2^{5/4})^{0.4} )</td>
</tr>
</tbody>
</table>
### Evaluate if possible.

<table>
<thead>
<tr>
<th>159.</th>
<th>160. $4^2 + 16^2$</th>
<th>161. $(-1)^{-2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. $-8^{\frac{1}{4}}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. $(-8)^{\frac{1}{3}}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**What important rule is explored above?**

<table>
<thead>
<tr>
<th>162. $(\sqrt{5^2})(\sqrt{5})$</th>
<th>163. $(\sqrt{16})(\sqrt{32})$</th>
<th>164. $\sqrt{729}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Evaluate to two decimal places using a calculator.

<table>
<thead>
<tr>
<th>165. Evaluate to two decimal places using a calculator</th>
<th>166. Evaluate to two decimal places using a calculator</th>
<th>167. Evaluate to two decimal places using a calculator</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{\sqrt{300}}$</td>
<td>$\frac{5}{\sqrt{256}}$</td>
<td>$\frac{1}{\sqrt{2500}}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Challenge

Write the following radicals as a single power.

$$\left(\sqrt[n]{x^3}\right) \left(\sqrt[n]{x}\right)$$
Write each of the following radicals as a single power.

169. \( \sqrt[11]{x^3} (\sqrt[7]{x}) \)

\[
\left( x^{\frac{3}{11}} \right) \left( x^{\frac{1}{7}} \right) \]

Write as powers (both base-\(x\)).

\[
\left( x^{\frac{3}{11}} \right) \left( x^{\frac{1}{7}} \right) 
\]

Create common denominators.

\[
\left( x^{\frac{3}{11}} \right) \left( x^{\frac{1}{7}} \right) 
\]

Add numerators.

\[
\left( x^{\frac{11}{77}} \right) 
\]

More rational exponents...

172. The height and the base of a triangle each measure \(2 \frac{3}{5}\) cm. Without using a calculator, what is the area of the triangle?

173. Find the area of a rectangle if the length is \(5 \frac{2}{5}\) and the width is \(5 \frac{2}{5}\). Write your answer in exponential form, then approximate to two decimal places.

174. Inscribed a square inside another square such that the corners of the internal square contact the midpoint of sides of the larger square. If the side length of the larger square is \(\sqrt{7}\), what is the area of the inscribed square? Answer in exact form.

175. Simplify (write as a single power.)

\[
\left( \left( \sqrt[4]{x^4} \right) \left( \sqrt[4]{x} \right) \right)^{-\frac{2}{3}} 
\]

176. Simplify (write as a single power.)

\[
\left( \left( \sqrt[4]{x^9} \right) \left( \sqrt[3]{x^6} \right) \right)^{-\frac{2}{3}} 
\]
177. Ei-Q evaluated $64^\frac{3}{4}$ using the following steps. In which step did she make her first error?

**Step 1:** $64^\frac{3}{4} = (\sqrt[4]{64})^3$

**Step 2:** $64^\frac{3}{4} = (8)^3$

**Step 3:** $64^\frac{3}{4} = 24$

a) In step 1.
b) In step 2.
c) In step 3.
d) She made no error.

178. Flinflan started to evaluate $81^{-\frac{3}{4}}$ in two different ways shown below. Which of the following statements is correct?

**Method 1:** $81^{-\frac{3}{4}} = (\sqrt[4]{81})^{-3}$

**Method 2:** $81^{-\frac{3}{4}} = \frac{1}{\sqrt[4]{81^3}}$

a) Method 1 will produce the correct answer but method 2 will not.
b) Method 2 will produce the correct answer but method 1 will not.
c) Both methods will produce the correct answer.
d) Neither method will produce the correct answer.

179. Simplify: \[\left[\sqrt[4]{x^3}\right]\left[\sqrt[3]{x^2}\right]^{-1}\]

180. Simplify: \[\left[\sqrt[3]{a^2}\right]\left[\sqrt[4]{a^3}\right]^2\]

181. Simplify: \[\sqrt[3]{\left(\frac{2}{a^\frac{1}{3}}\right)^3}\]

182. Simplify: \[\sqrt[4]{\left(\frac{1}{x^\frac{1}{3}}\right)^4}\]
Match each item in column 1 with an equivalent item in column 2

<table>
<thead>
<tr>
<th>Column 1</th>
<th>Column 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>183. ((\frac{1}{x})^\frac{2}{3})</td>
<td>A. (\sqrt[3]{\frac{x}{3}})</td>
</tr>
<tr>
<td>184. ((\frac{1}{x})^\frac{3}{2})</td>
<td>B. (-\left(\frac{1}{x}\right)^\frac{3}{2})</td>
</tr>
<tr>
<td>185. ((\frac{1}{y})^\frac{2}{3})</td>
<td>C. (\sqrt[3]{\frac{y}{3}})</td>
</tr>
<tr>
<td>186. ((\frac{1}{y})^\frac{3}{2})</td>
<td>D. (-\left(\frac{1}{y}\right)^\frac{3}{2})</td>
</tr>
<tr>
<td>187. ((\frac{1}{y})^\frac{3}{2})</td>
<td>E. (\sqrt[3]{\frac{1}{y}})</td>
</tr>
<tr>
<td>188. Which of the following is equivalent to (\frac{3 \alpha^\frac{1}{2} \times (5\alpha)^\frac{1}{2}}{3\alpha^\frac{1}{2} \times (5\alpha)^\frac{1}{2}})</td>
<td>(2x^\frac{1}{2} \times (3x)^\frac{1}{2})</td>
</tr>
<tr>
<td>a. 15 (\alpha)</td>
<td>a. 6(x)</td>
</tr>
<tr>
<td>b. (\alpha\sqrt{15})</td>
<td>b. (x\sqrt{6})</td>
</tr>
<tr>
<td>c. 3(\sqrt{5}\alpha)</td>
<td>c. 2(\sqrt{3}\alpha)</td>
</tr>
<tr>
<td>d. 3(\alpha\sqrt{5})</td>
<td>d. 2(x\sqrt{3})</td>
</tr>
</tbody>
</table>
190. Which of the following is not equivalent to $x^2$?

a. $\frac{1}{\sqrt{x^2}}$  

b. $\sqrt{\sqrt{x}}$  

c. $(x^2)^{\frac{1}{2}}$  

d. $\sqrt[3]{x^3}$

191. Which of the following is not equivalent to $a^3$?

a. $\frac{1}{\sqrt[4]{a^4}}$  

b. $\frac{1}{\sqrt[3]{a^3}}$  

c. $\sqrt[3]{a^3}$  

d. $\sqrt{\sqrt[6]{a^6}}$


$$\frac{3^0 + 2^{-1}}{3^2 + 2^2}$$


$$\frac{3^{-2} + 3^2}{3^{-2} + 2^6}$$
Answers:
1. 55. 1
6. 56. \( x^{-6}y^{-9} = \frac{1}{x^6y^9} \)
7. 57. 8m^12
8. 58. \( 2^{-3}c^{-12}d^{-9} = \frac{1}{8c^{12}d^{9}} \)
9. 59. \( (-3)^{-4}x^6y^{-12} = \frac{x^6}{81y^{12}} \)
60. \( 3^{-3}x^6y^9 = \frac{1}{27}x^6y^9 \) or \( x^6y^9 = 27 \)
61. \( -18x^5y^9 \)
62. \( 128a^2b^2 \)
63. 8
64. \( \frac{a^3}{b^3} \)
65. \( \frac{16x^2}{9y^2} \)
66. \( \frac{9x^2}{10} \)
67. \( \frac{1}{8} \)
68. \( a^{5n} \)
69. \( x^5y^3 \)
70. \( \frac{1}{27}a^6 \)
71. \( \frac{a^6}{x^3} \)
72. \( \frac{16x^2}{9y^2} \)
73. \( \frac{16x^2}{25a^6b^4} \)
74. \( \frac{1}{8} \)
75. \( 27b^6 \)
76. \( 4x^{10} \)
77. \( \frac{y^{12}}{2x^4} \)
78. \( \frac{b^5}{a^3} = \frac{2a^2}{x^3} \) and \( \frac{1}{m^3} = \frac{b^3}{a^3} \)
79. \( \frac{12x^3}{y^2} = \frac{12x^3}{y^2} \) and \( \frac{1}{y^2} = \frac{y^{1-2}}{y} \)
80. \( \frac{3x^4}{2y^3} \)
81. \( \frac{1}{x^n} \)
82. \( \frac{a^3}{b^3} \)
83. \( \frac{3x^4}{2y^3} \)
84. \( \frac{d}{x^2} \)
85. \( 2x^2y^5 \)
86. \( \frac{3a^4}{y^5} \)
87. \( \frac{x^6}{y^4} \)
88. \( \frac{2x^5}{y^4} \)
90. \( \frac{1}{2x^3} \)
91. \( \frac{m^2n}{m^2n} \)
92. \( \frac{1}{2}a^3 = -\frac{a^3}{2} \)
47. \( \frac{1}{4}a^2 = \frac{a^2}{4} \)
48. \( \frac{1}{2} \)
49. \( \frac{3}{2} \)
50. \( \frac{1}{3} \)
51. \( 15625 \)
52. \( m^6 \)
53. \( 8m^{12} \)
54. \( m^6 \)
93. \( \sqrt{4} = 2 \)
94. \( \frac{1}{(\sqrt{4})^3} = \frac{1}{8} \)
95. \( \frac{1}{(\sqrt{9})^5} = \frac{1}{9} \)
96. \( \frac{1}{(\sqrt{9})^3} = \frac{1}{27} \)
97. \( \frac{1}{3} \)
98. \( \frac{1}{3} \)
99. \( x^2 = \sqrt{x} \)
100. Possible answer:
\( \sqrt[3]{3} \times \sqrt[3]{3} \times \sqrt[3]{3} = 3 \)
\( \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = 3 \)
101. \( 7 \)
102. \(-4 \)
103. no real number
104. \( 4 \)
105. \( \frac{1}{2} \)
106. \( \frac{1}{2} \)
107. \( 10 \)
108. \( 2 \)
109. \( \frac{1}{3} \)
110. \( \sqrt[3]{3} \)
111. \( \sqrt[3]{x} \)
112. \( \sqrt[3]{y} \)
113. \( \frac{1}{\sqrt[3]{3}} \)
114. \( -\sqrt[3]{64} \)
115. \( \frac{1}{\sqrt[3]{27}} \)
116. \( \frac{3}{2} \)
117. \(-3 \)
118. \( (2y)^3 \)
119. \( 2 \)
120. \( \frac{1}{\sqrt{3}} \)
121. \( (3x)^{-1} \)
122. \( 27 \frac{2}{3} = (27)^\frac{2}{3} \)
123. \( \sqrt[3]{4} \) or \( (\sqrt[3]{4})^2 \)
124. \( \sqrt[3]{4} \) or \( (\sqrt[3]{4})^3 \)
125. \( \sqrt[3]{4} \) or \( (\sqrt[3]{4})^4 \)
126. \( \frac{1}{\sqrt[3]{4}} \) or \( (\sqrt[3]{4})^{-1} \)
127. \( \frac{1}{\sqrt[3]{4}} \) or \( (\sqrt[3]{4})^{-1} \)
128. \( \frac{1}{\sqrt[3]{4}} \) or \( (\sqrt[3]{4})^{-1} \)
129. \( \sqrt[3]{4} = 2 \)
130. \( \sqrt[3]{27} = 3 \)
131. \( (\sqrt[3]{4})^2 = 4 \)
132. \( (\sqrt[3]{8})^3 = 27 \)
133. \( (\sqrt[3]{2})^3 = 8 \)
134. \( \frac{1}{(\sqrt{9})^3} = \frac{1}{9} \)
135. \( \frac{1}{(\sqrt{9})^5} = \frac{1}{27} \)
136. \( \frac{1}{(\sqrt{9})^3} = -\frac{1}{3} \)
137. \( \sqrt[3]{9} = (\sqrt[3]{9})^5 = 243 \)
138. \( 1 \)
139. \( \frac{1}{1000} \)
140. \( \frac{1}{4} \)
141. \( \frac{1}{7} \)
142. \( 347 \)
143. \( (11)^2 \)
144. \( a^2 \)
145. \( 6^2 \)
146. \( x^2 \)
147. \( 67 \)
148. \( (2x)^2 \)
149. \( a^2 \)
150. \( x^4 \)
151. \( x^2 \)
152. \( 3b \)
153. no real solution
154. \( 1000 \)
155. \( 2 \)
156. \( 3 \)
157. \( 3 \)
158. \( 1 \)
159. \( a^2 \) b) 16
160. \( 4 \)
161. no real solution
162. \( 5 \)
163. \( 4 \)
164. \( 3 \)
165. \( 0.32 \)
166. \( 0.198 \)
167. \( 0.55 \)
168. \( \frac{1}{10} \)
169. Answered on page.
170. \( x^2 \)
171. \( x^2 \)
172. \( 4 \) cm²
173. \( 5 \) cm²
174. \( 2.5 \) cm²
175. \( x^3 \) or \( \frac{4}{x^3} \)
176. \( x^3 \)
177. \( c \)
178. \( c \)
179. \( x^2 = 1 \)
180. \( a^{2\pi} = 1 \)
181. \( a^\pi \)
182. \( x = \frac{1}{x} \)
183. \( C \)
184. \( C \)
185. \( A \)
186. \( E \)
187. \( C \)
188. \( D \)
189. \( D \)
190. \( C, D \)
191. \( B \)
192. \( \frac{3}{2} \)
193. \( \frac{1}{2} \)