Foundations & Pre-Calculus 10
Homework & Notebook

Name:______________________________ **Block:**_________
Teacher: Miss Zukowski **Date Submitted:** ____/______/ 2018

Unit #____:___________________________________

Submission Checklist: *(make sure you have included all components for full marks)*

- ❏ Cover page & Assignment Log
- ❏ Class Notes
- ❏ Homework *(attached any extra pages to back)*
- ❏ Quizzes *(attached original quiz + corrections made on separate page)*
- ❏ Practice Test/ Review Assignment

Assignment Rubric: Marking Criteria			
Excellent (5) - Good (4) - Satisfactory (3) - Needs Improvement (2) - Incomplete (1) - NHI (0)		**Self Assessment**	**Teacher Assessment**
Notebook	• *All* teacher notes complete • Daily homework assignments have been recorded & completed *(front page)* • Booklet is neat, organized & well presented *(ie: name on, no rips/stains, all pages, no scribbles/doodles, etc)*	/5	/5
Homework	• *All* questions attempted/completed • *All* questions **marked** *(use answer key, correct if needed)*	/5	/5
Quiz *(1mark/dot point)*	• Corrections have been made **accurately** • Corrections made in a *different colour pen/pencil* *(+½ mark for each correction on the quiz)*	/2	/2
Practice Test *(1mark/dot point)*	• Student has completed all questions • Mathematical working out leading to an answer is shown • Questions are **marked** *(answer key online)*	/3	/3
Punctuality	• *All checklist items were submitted, and completed on the day of the unit test. (-1 each day late)*	/5	/5
Comments:		/20	/20

Homework Assignment Log

& Textbook Pages:________________________________

Date	Assignment/Worksheet	Due Date	Completed?

Quizzes & Tests:

What?	When?	Completed?
Quiz 1		
Quiz 2		
Unit/ Chapter test		

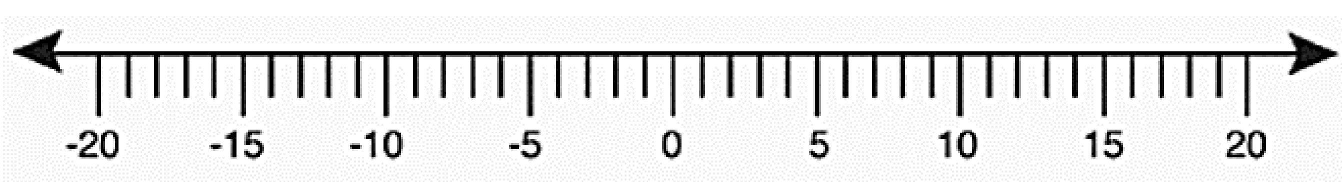
Math 10

Unit 1: Real Numbers and Radicals

Lesson 1: pages 1-7

Example: Place the following numbers on the number line below:

A	B	C	D	E	F	G	H
$\frac{6}{2}$	$-5.\overline{6}$	$\sqrt{20}$	$\frac{0}{12}$	$10.3\overline{25}$	$\sqrt[4]{4}$	$\sqrt[3]{4913}$	3π



Real Numbers & Radicals

Key Terms

Term	Definition	Example
Real Number (R)		
Rational Number (Q)		
Irrational Number ($\bar{Q}$)		
Integer (Z)		
Whole Number (W)		
Natural Number (N)		
Factor		
Factor Tree		
Prime Number		
Prime Factorization		
GCF		
Multiple		
LCM		
Radical		
Index		
Root • Square root • Cube root		
Power		
Entire Radical		
Mixed Radical		

The Real Number System

Real numbers are the set of numbers that we can place on the number line.

Real numbers may be positive, negative, decimals that repeat, decimals that stop, decimals that don't repeat or stop, fractions, square roots, cube roots, other roots. Most numbers you encounter in high school math will be real numbers.

The square root of a negative number is an example of a number that does not belong to the Real Numbers.

There are 5 subsets we will consider.

<table>
<tr><th colspan="4"><u>Real Numbers</u></th></tr>
<tr><td colspan="3"><u>Rational Numbers</u> (Q)

Numbers that can be written in the form $\frac{m}{n}$ where m and n are both integers and n is not 0.

Rational numbers will be terminating or repeating decimals.

Eg. 5, -2. 3, $\frac{4}{3}$, $2\frac{3}{8}$</td><td rowspan="2"><u>Irrational Numbers</u> ($\bar{Q}$)

Cannot be written as $\frac{m}{n}$.
Decimals will not repeat, will not terminate.

Eg. $\sqrt{3}$, $\sqrt{7}$, π, 53.123423656787659…</td></tr>
<tr><td><u>Natural</u> (N)

{1, 2, 3,…}</td><td><u>Whole</u> (W)

{0, 1, 2, 3,…}</td><td><u>Integers</u> (Z)

{…,-3,-2,-1, 0, 1, 2, 3,…}</td></tr>
</table>

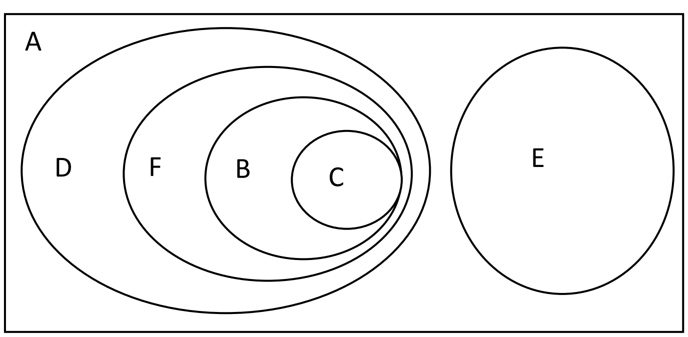
Name all of the sets to which each of the following belong?

1. 8	2. $\frac{4}{5}$	3. $\frac{15}{5}$
4. $\sqrt{7}$	5. $\sqrt{0.5}$	6. $12.3\overline{4}$
7. -17	8. $-\left(\frac{2}{3}\right)^3$	9. $2.7328769564923\ldots$

Write each of the following Real Numbers in decimal form. Round to the nearest thousandth if necessary. Label each as Rational or Irrational.

10. $\frac{2}{9}$	11. $-3\frac{3}{7}$	12. $\sqrt{8}$
13. $\sqrt[3]{9}$	14. $\sqrt[4]{256}$	15. $\sqrt[5]{25}$

16. Fill in the following diagram illustrating the relationship among the subsets of the real number system. (Use descriptions on previous page)



A____________________________

B____________________________

C____________________________

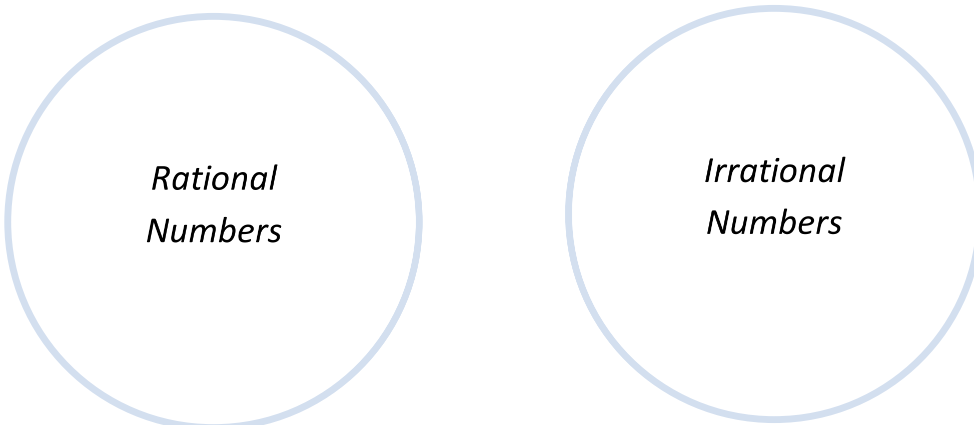
D____________________________

E____________________________

F____________________________

17. Place the following numbers into the appropriate set, rational or irrational.

$$5, \quad \sqrt{2}, \quad 2.\overline{13}, \quad \sqrt{16}, \quad \frac{1}{2}, \quad 5.1367845\ldots, \frac{\sqrt{7}}{2}, \quad \sqrt[3]{8}, \quad , \quad \sqrt[3]{25}$$



18. Which of the following is a rational number?

a. $\frac{\sqrt{3}}{2}$
b. $\sqrt[3]{16}$
c. $\frac{5}{7}$
d. $12.356528349875\ldots$

19. Which of the following is an irrational number?

a. $\sqrt{\frac{16}{9}}$
b. π
c. $\frac{3}{8}$
d. $\sqrt[3]{27}$

20. To what sets of numbers does -4 belong?

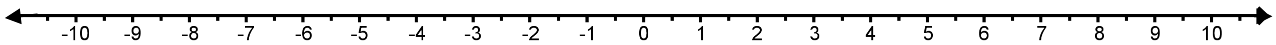
a. natural and whole
b. irrational and real
c. integer and whole
d. rational and integer

21. To what sets of numbers does $-\frac{4}{3}$ belong?

a. natural and whole
b. irrational and real
c. integer and whole
d. rational and real

Your notes here…

The Real Number Line



All real numbers can be placed on the number line. We could never list them all, but they all have a place.

Estimation:

It is important to be able to estimate the value of an irrational number. It is one tool that allows us to check the validity of our answers.

Without using a calculator, estimate the value of each of the following irrational numbers.
Show your steps!

22. $\sqrt{7}$

Find the perfect squares on either side of 7.
→4 and 9
Square root 4 = 2
Square root 9 = 3

Guess & Check:
2.6 x 2.6 =6.76
2.7 x 2.7 = 7.29
$\therefore \sqrt{7}$ is about 2.6

23. $\sqrt{14}$

24. $\sqrt{75}$

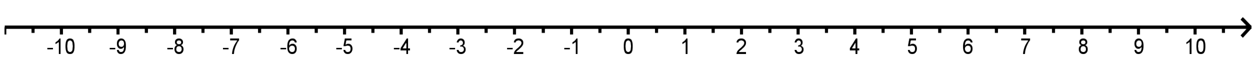
25. $\sqrt[3]{11}$

26. $\sqrt[3]{90}$

27. $\sqrt[3]{150}$

28. Place the corresponding letter of the following Real Numbers on the number line below.

A. -6 B. $\frac{2}{3}$ C. $-\frac{2}{3}$ D. $5\frac{1}{4}$ E. $\sqrt{2}$ F. $-\sqrt{7}$ G. $\frac{\sqrt{3}}{2}$ H. $-\frac{\sqrt{4}}{3}$



Math 10 **Unit 1: Real Numbers and Radicals** ____________________

Lesson 2: pages 8-11 _______________

A. **Factor (noun):** ____

Example: List the factors of 24.

B. **Factor (verb):** ____

Example: Factor 24.

C. **Greatest Common Factor (GCF)** [think: largest into all]

<u>TO FIND GCF: List the primes that are in both numbers and multiply them.</u>

Example #1: Find the GCF of 36 & 126.

Example #2: Find the GCF of 42, 90, & 84.

D. **Lowest Common Multiple (LCM)**

Example #1: List the first 6 multiples of 20:

24:

LCM of 20 & 24 is the lowest number that they both divide evenly into.

<u>TO FIND LCM: List the largest power of each prime number & multiply them.</u>

Example #2: Find the LCM of 45 & 60.

Example #3: Find the LCM of 84, 28, & 72.

Factors, Factoring, and the Greatest Common Factor

We often need to find factors and multiples of integers and whole numbers to perform other operations.

For example, we will need to find common multiples to add or subtract fractions.
For example, we will need to find common factors to reduce fractions.

<u>Factor</u>: (NOUN)

Factors of 20 are {1,2,4,5,10,20} because 20 can be evenly divided by each of these numbers.
Factors of 36 are {1,2,3,4,6,9,12,18,36}
Factors of 198 are { 1,2,3,6,9,11,18,22,33,66,99,198}

> Use division to find factors of a number. Guess and check is a valuable strategy for numbers you are unsure of.

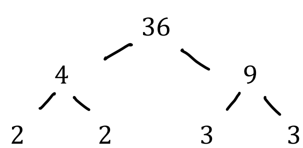
<u>To Factor</u>: (VERB) The act of writing a number (or an expression) as a product.

To factor the number 20 we could write 2×10 or 4×5 or 1×20 or $2 \times 2 \times 5$ or $2^2 \times 5$.
When asked to factor a number it is most commonly accepted to write as a product of prime factors.
<u>Use</u> <u>powers</u> where appropriate.

Eg. $20 = 2^2 \times 5$ Eg. $36 = 2^2 \times 3^2$ Eg. $198 = 2 \times 3^2 \times 11$

> Prime:
> When a number is only divisible by 1 and itself, it is considered a prime number.

A <u>factor tree</u> can help you "factor" a number.



$\therefore\ 36 = 2^2 \times 3^2$

Write each of the following numbers as a product of their prime factors.

29. 100	30. 120	31. 250

Write each of the following numbers as a product of their prime factors.

32. 324

33. 1200

34. 800

Greatest Common Factor

At times it is important to find the largest number that divides evenly into two or more numbers…the **Greatest Common Factor (GCF)**.

Challenge:

35. Find the GCF of 36 and 198.

Challenge:

36. Find the GCF of 80, 96 and 160.

Some Notes…

Find the GCF of each set of numbers.

37. 36, 198

$36 = 2^2 \times 3^2$
$198 = 2 \times 3^2 \times 11$

-Prime factors in common are 2 and 3^2.

*-GCF is **2×3^2**= 18*

-Alternate method:
List all factors…choose largest in both lists.

38. 98, 28

39. 80, 96, 160

$80 = 2^4 \times 5$
$96 = 2^5 \times 3$
$160 = 2^5 \times 5$

-Prime factors in common are 2^4.

-GCF is 2^4=16

-Alternate method:
List all factors…choose largest in both lists.

40. 24, 108

41. 126, 189, 735, 1470

42. 504, 1050, 1386

Multiples and Least Common Multiple

Challenge

43. Find the first seven multiples of 8.

Challenge

44. Find the least common multiple of 8 and 28.

Multiples of a number

Multiples of a number are found by multiplying that number by {1,2,3,4,5,…}.

Find the first five multiples of each of the following numbers.

45. 8

46. 28

47. 12

Find the least common multiple of each of the following sets of numbers.

48. 8,28

$8 = 2^3$
$28 = 2^2 \times 7$

-Look for largest power of each prime factor…

-In this case, 2^3 and 7.

-LCM = 2^3 x 7
LCM = 56

49. 72,90

50. 25, 220

51. 8, 12, 22

52. 4, 15, 25

53. 18, 20, 24, 36

54. Use the least common multiple of 2, 6, and 8 to add:
$$\frac{3}{8}+\frac{5}{6}+\frac{1}{2}$$

55. Use the least common multiple of 2, 5, and 7 to evaluate:
$$\frac{3}{5}-\frac{2}{7}+\frac{3}{2}$$

56. Use the least common multiple of 3, 8, and 9 to evaluate:
$$\frac{7}{9}-\frac{1}{3}-\frac{1}{8}$$

Math 10

Unit 1: Real Numbers and Radicals

Lesson 3: pages 12-17

____

____

1. $\sqrt{4+5} =$

2. $\sqrt{2+2\times 7} =$

3. $\sqrt{\frac{49}{81}} =$

4. $\sqrt{-576} =$

5. $\sqrt[3]{-512} =$

6. $\sqrt{2\cdot 2\cdot 3\cdot 3\cdot 11\cdot 11\cdot 11\cdot 11} =$

7. $\sqrt{25x^2} =$

8. $\sqrt{100x^6} =$

9. $\sqrt[3]{27x^6} =$

10. Use the prime factorization of 1728 to determine if it is a perfect cube. If so, determine $\sqrt[3]{1728}$.

Radicals:

Radicals are the name given to square roots, cube roots, quartic roots, etc.

$$\sqrt[n]{x}$$

<u>The parts of a radical</u>:

Radical sign	$\sqrt{}$	(Operations under the radical are evaluated as if inside brackets.)
Index	n	(tells us what type of root we are looking for, if blank…index is 2)
Radicand	x	(the number to be "rooted")

> A radical and its simplified form are equivalent expressions.

Square Roots

Square root of 81 looks like $\sqrt{81}$. It means to find what value must be multiplied by itself twice to obtain the number we began with.

$\sqrt{81}$ *we think* … $81 = 9 \times 9 \rightarrow \sqrt{81} = 9$ $\sqrt{a^4}$ we think … $a^4 = a^2 \times a^2 \rightarrow \sqrt{a^4} = a^2$

<u>PERFECT SQUARE NUMBER</u>: A number that can be written as a product of two equal factors.

$81 = 9 \times 9$ } 81 is a perfect square. Its square root is 9.

First 15 Perfect Square Numbers:
1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, …

Your notes here…

Operations inside a $\sqrt{}$ must be considered as if they were inside brackets...do them

Evaluate the following.

57. $\sqrt{49}$

58. $\sqrt{-25}$

59. $-\sqrt{36}$

60. Finish the statement:

    I know that $\sqrt{16} = 4$ because...

61. Finish the statement:

    I know that $\sqrt{\frac{64}{81}} = \frac{8}{9}$ because...

62. Finish the statement:

    I know that $\sqrt{-36} \neq -6$ because...

63. $\sqrt{121}$

64. $\sqrt{45 - 20}$

65. $2\sqrt{40 - (-9)}$

66. Simplify. $\sqrt{x^2}$

67. Simplify. $\sqrt{4x^2}$

68. Simplify. $\sqrt{16x^4}$

Cube Roots:

PERFECT CUBE NUMBER: A number that can be written as a product of three equal factors.

Cube root of 64 looks like $\sqrt[3]{64}$.

The index is 3. So we need to multiply our answer by itself 3 times to obtain 64. $4 \times 4 \times 4 = 64$

First 10 Perfect Cube Numbers: 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, …

Evaluate or simplify the following.

69. $\sqrt[3]{8}$
Explain what the small 3 in this problem means.

70. $\sqrt[3]{8}$

71. How could a factor tree be used to help find $\sqrt[3]{125}$?

72. Evaluate $\sqrt[3]{125}$.

73. $\sqrt[3]{-27}$

74. $\sqrt[3]{1000}$

75. $\sqrt[3]{-8}$

76. Show how prime factorization can be used to evaluate $\sqrt[3]{27}$.

77. $\sqrt[3]{343}$

78. $\sqrt[3]{-216}$

79. $\sqrt[3]{27} \times \sqrt{20 \times 5}$

80. $\sqrt[3]{64} \times \sqrt{45 - 20}$

81. $\sqrt[3]{-125}$

82. $\sqrt[4]{a^{12}}$

83. $\sqrt[3]{a^6}$

84. $\sqrt[3]{8x^3}$

Other Roots.

85. How does $\sqrt[6]{729}$ differ from $\sqrt[3]{729}$? Explain, do not simply evaluate.

86. Evaluate if possible. $\sqrt[4]{16}$

87. Evaluate if possible. $\sqrt[4]{-16}$.

88. Evaluate if possible. $\sqrt[5]{32}$.

89. Evaluate if possible. $\sqrt[4]{81}$.

90. Evaluate if possible. $\sqrt[6]{64}$.

91. Evaluate if possible. $\sqrt[3]{24-16}$.

92. Evaluate if possible. $\sqrt[4]{2(32-24)}$.

93. Evaluate if possible. $\sqrt[3]{4(5-3)}$.

Using a calculator, evaluate the following to two decimal places.

94. $\sqrt[3]{27}-\sqrt[5]{27}$

95. $2\sqrt{10}+\sqrt[4]{64}$

96. $\sqrt[5]{-32}-\sqrt[4]{16}$

97. $19-\sqrt[3]{18}$

98. $\frac{\sqrt{12}-\sqrt[3]{7}}{2}$

99. $\frac{\sqrt[3]{9}-\sqrt[3]{27}}{3}$

100. Describe the difference between radicals that are rational numbers and those that are irrational numbers.

Evaluate or simplify the following.

101. $\sqrt[3]{125}$	102. $\sqrt{2(15-(-3))}$	103. $\sqrt{\sqrt{16}}$
104. $\sqrt{0.16}$	105. $\sqrt{0.0001}$	106. $3\sqrt{25}-4\sqrt[3]{8}$
107. $\sqrt{\frac{1}{4}}$	108. $\sqrt{\frac{16}{49}}$	109. $\sqrt{\frac{100}{400}}$
110. $\sqrt{a^4}$	111. $\sqrt[3]{-x^6}$	112. $\sqrt[3]{8x^3}$

Evaluate or simplify the following.

113. $\sqrt{5^2}$

114. $\left(\sqrt{5}\right)^2$

115. $-\sqrt{(-5)^2}$

116. $\left(\sqrt{49}-\sqrt{64}\right)^3$

117. $\sqrt{\sqrt{16}+\sqrt{25}}$

118. What would be the side length of a square with an area of 1.44 cm²?

119. $\left(\sqrt[4]{16}\right)^3$

120. $\sqrt[5]{-32}$

121. $\sqrt[8]{256}$

122. Use the prime factors of 324 to determine if 324 is a perfect square. If so, find $\sqrt{324}$.

Answer:

$324 = 2^2 \times 3^4$ if fully factored

$\therefore \sqrt{324} = \sqrt{2 \times 2 \times 3^2 \times 3^2}$

$\therefore \sqrt{324} = \sqrt{(2 \times 3^2) \times (2 \times 3^2)}$

$\therefore \sqrt{324} = (2 \times 3^2)$

$\therefore \sqrt{324} = 18$

123. Use the prime factors of 576 to determine if 576 is a perfect square. If so, find $\sqrt{576}$.

124. Use the prime factors of 1728 to determine if it is a perfect cube. If so, find $\sqrt[3]{1728}$.

125. Use the prime factors of 5832 to determine if it is a perfect cube. If so, find $\sqrt[3]{5832}$.

Math 10 **Unit 1: Real Numbers and Radicals**

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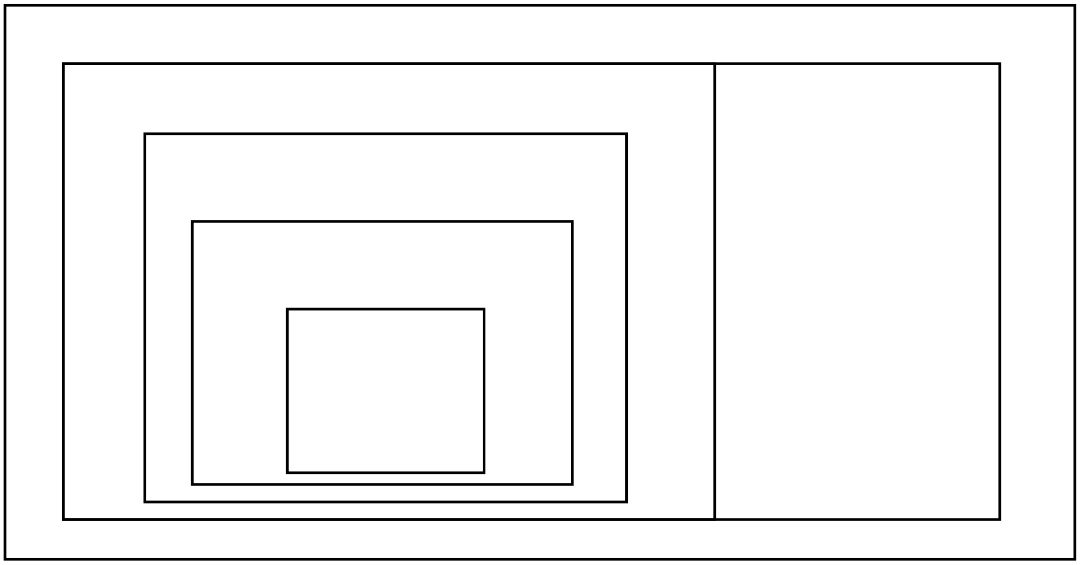
Lesson 4: pages 18-19 ________________

Part 1: Undefined Roots



What values of square roots are UNDEFINED? (ie: NO real solution)

What values of x make these roots undefined?

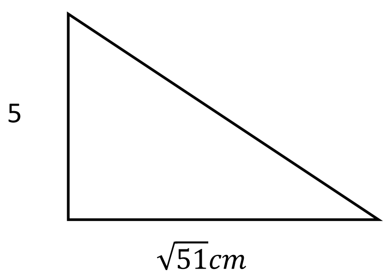
1. $\sqrt{x+4}$

2. $\sqrt{10-5x}$

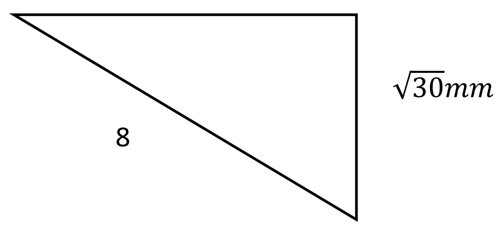
Part 2: Pythagoras ($a^2 + b^2 = c^2$) can only be used if a triangle has a ___________ angle!

Calculate the perimeter of the following triangles.

1.



2.



Part 3: Squares and Cubes

1. Is this a perfect square? $\sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 3 \cdot 5}$

2. Is this a perfect cube? $\sqrt[3]{3 \cdot 7 \cdot 3 \cdot 7 \cdot 3 \cdot 7}$

3. The volume of a cube is $729cm^3$. Find the surface area of the cube.

126. An engineering student developed a formula to represent the maximum load, in tons, that a bridge could hold. The student used 1.7 as an approximation for $\sqrt{3}$ in the formula for his calculations. When the bridge was built and tested in a computer simulation, it collapsed. The student had predicted the bridge would hold almost three times as much.

The formula was:
$5000(140 - 80\sqrt{3})$

What weight did the student think the bridge would hold?

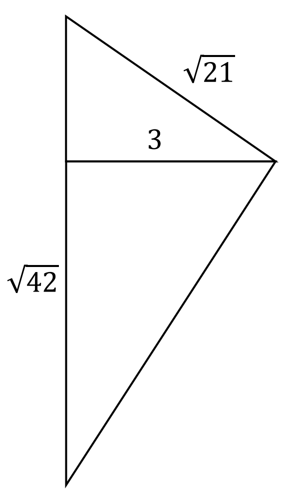
Calculate the weight the bridge would hold if he used $\sqrt{3}$ in his calculator instead.

127. For what values of x is $\sqrt{x-2}$ not defined?

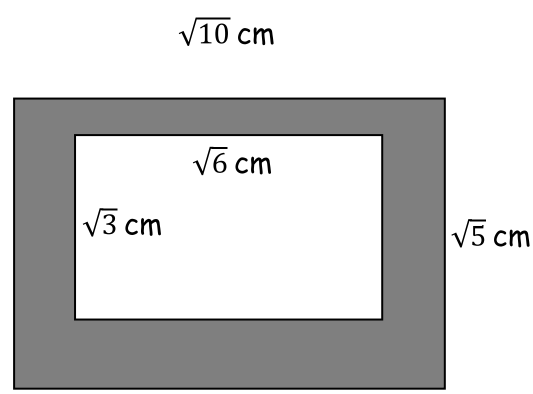
128. For what values of x is $\sqrt{x+3}$ not defined

129. For what values of x is $\sqrt{5-x}$ not defined

130. Calculate the perimeter to the nearest tenth. The two smaller triangles are right triangles.



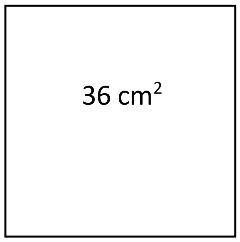
Calculate the area of the shaded region.



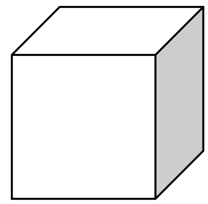
131. To the nearest tenth:

132. As an *expression* using radicals:
(you may need to come back to this one)

133. Consider the square below. Why might you think $\sqrt{}$ is called a square root?



134. Consider the diagram below. Why do you think $\sqrt[3]{}$ is called a cube root?



64 cm^3

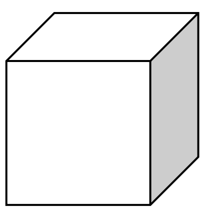
135. Find the side length of the square above.

136. Find the edge length of the cube above.

137. Why do you think 81 is called a "perfect square" number?

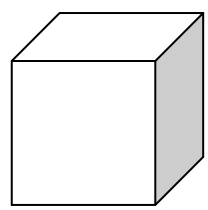
138. Why do you think 729 is called a "perfect cube" number?

139. Find the surface area of the following cube.



125 cm^3

140. Find the surface area of the following cube.



216 cm^3

141. A cube has a surface area of 294 m^2. Find its edge length in centimetres.

142. A cube has a surface area of 1093.5 m^2. Find its edge length in centimetres.

Answers:

The Real Number System Answer Key

1. Q, Z, W, N
2. Q
3. Q, Z, W, N
4. $\bar{Q}$
5. $\bar{Q}$
6. Q
7. Q, Z
8. Q
9. $\bar{Q}$
10. $0.\bar{2}, Q$
11. $-3.429, Q$ (rounded, actually $3.\overline{428571}$)
12. $2.828, \bar{Q}$ (rounded)
13. $2.080, \bar{Q}$ (rounded)
14. $4, Q$
15. $1.904, \bar{Q}$
16. A: real numbers
    B: whole numbes
    C: natural numbers
    D: rational numbers
    E: irrational numbers
    F: integers
17. Rational:
    $5, 2.\overline{13}, \sqrt{16}, \frac{1}{2}, \sqrt[3]{8}$
    Irrational:
    $\sqrt{2}, 5.1367845\ldots, \frac{\sqrt{7}}{2}, \sqrt[3]{25}$
18. c
19. b
20. d
21. d
22. answered on page
23. 3.7
24. 8.7
25. 2.2
26. 4.5
27. 5.3
28. From left to right:
    $A, F, C/H, B, G, E, D$
29. $5^2 \times 2^2$
30. $2^3 \times 3 \times 5$
31. $5^3 \times 2$
32. $2^2 \times 3^4$
33. $2^4 \times 3 \times 5^2$
34. $2^5 \times 5^2$
35. 18
36. 16
37. 18
38. 14
39. 16
40. 12
41. 21
42. 42
43. 8,16,24,32,40,48,56
44. 56
45. 8,16,24,32,40
46. 28,56,84,112, 140
47. 12,24,36,48,60
48. $2^3 \times 7 = 56$
49. $2^3 \times 3^2 \times 5 = 360$
50. 1100
51. 264
52. 300
53. 360
54. $\frac{41}{24}$
55. $\frac{127}{70}$
56. $\frac{23}{72}$
57. 7
58. $no\ real\ solution$
59. -6
60. $4 \times 4 = 16 \quad or \quad 4^2 = 16$
61. $\frac{8}{9} \times \frac{8}{9} = \frac{64}{81}$
62. There is no real number that can be multiplied by itself to produce a negative number.
63. 11
64. 5
65. 14
66. x
67. $2x$
68. $4x^2$
69. $Cube\ or\ Third\ root\ of\ 8.$ Which means find a number that if multiplied by itself 3 times would have a product of 8. You could also think: $?^3 = 8$
70. 2
71. $125 = 5 \times 5 \times 5 = 5^3$ A third power and a third (cube) root are inverse operations.
72. 5
73. -3
74. 10
75. -2
76. $27 = 3 \times 3 \times 3 = 3^3$
    $\therefore \sqrt[3]{27} = \sqrt[3]{3^3} = 3$
77. 7
78. -6
79. 30
80. 20
81. -5
82. a^3
83. a^2
84. $2x$
85. $\sqrt[6]{729}$ means sixth root
    $729 = 3^6$
    $\therefore \sqrt[6]{729} = 3$
    $\sqrt[3]{729}$ means third root
    $729 = 9^3$
    $\therefore \sqrt[3]{729} = 9$
86. 2
87. $no\ real\ solution$
88. 2
89. 3
90. 2
91. 2
92. 2
93. 2
94. 1.07
95. 9.15
96. -4.00
97. 16.38
98. 0.78
99. -0.31
100. Radicals that are rational numbers contain radicands that are perfect squares, cubes, etc. Radicals that are irrational numbers do not.
101. 5
102. 6
103. 2
104. 0.4
105. 0.01
106. 7
107. $\frac{1}{2}$
108. $\frac{4}{7}$
109. $\frac{1}{2}$
110. a^2
111. $-x^2$
112. $2x$
113. 5
114. 5
115. -5
116. -1
117. 3
118. 1.2 cm
119. 8
120. -2
121. 2
122. Yes, $\sqrt{324} = \sqrt{2^2 \times 3^4}$
     $= 2 \times 3^2 = 18$
123. Yes, $\sqrt{576} = \sqrt{2^6 \times 3^2}$
     $= 2^3 \times 3 = 24$
124. Yes,
     $\sqrt[3]{1728} = \sqrt[3]{2^6 \times 3^3}$
     $= 2^2 \times 3 = 12$

125. Yes,

$$\sqrt[3]{5832} = \sqrt[3]{2^3 \times 3^6}$$
$$\sqrt[3]{5832} = 2 \times 3^2 = 18$$

ρςφ Student calculated 20 000 tons. The student would hav e calculated 7179.7 tons if he did not round $\sqrt{3}$ to 1.7.

ρςχ $\sqrt{x} - 2$ is not defined for val ues of x less than 2. That is, if $x < 2$.

ρςψ $x < -3$

ρςω $x > 5$

ρσπ 21.7 units

ρσρ 2.8 cm²

ρσς $10\sqrt{5} \times \sqrt{5} - \sqrt{3} \times \sqrt{6} = 2\sqrt{2}$ cm²

ρσσ Perhaps because the side l ength of a square is the sq uare root of that square's ar ea.

ρστ Perhaps because the edge l ength of a cube is the cube root of that cube's volume.

ρσυ 6 cm

ρσφ 4 cm

ρσχ Its square root is an integer.

ρσψ Its cube root is an integer.

ρσω 150cm²

ρτπ 216 cm²

ρτρ 700 cm

ρτς 1350 cm

Exponents: Integral & Rational

Term	Definition	Example
Power	$2^1, 2^2, 2^3, 2^4, \ldots$ are powers of 2. A power is made up of a base and an exponent.	
Exponent	The smaller number written to the upper right of the base that tells you how many times to multiply the base by itself.	$2^4 = 2 \times 2 \times 2 \times 2$ 4 is the exponent.
Base	The "larger" number that the exponent is applied to. (The bottom number in a power)	$2^4 = 2 \times 2 \times 2 \times 2$ 2 is the base.
Rational number	Numbers that can be written as fractions.	
Rational Exponent	The exponent on a power is a rational number (fraction). $x^{\frac{2}{3}} = \left(\sqrt[3]{x}\right)^2$	$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = (3)^2 = 9$
Integral number	An integer {…-3,-2,-1,0,1,2,3,…}.	
Integral Exponent	The exponent on a power is an integer.	Such as x^2, x^{-3}.
Coefficient	The numbers in front of the letters in mathematical expressions.	In $3x^2$, 3 is the coefficient.
Variable	The letters in mathematical expressions.	In $3x^2$,'x' is the variable.
Undefined	If there is no good way to describe something, we say it is undefined.	$\frac{3}{0}$ is undefined because we cannot divide by zero.
Radical form	$\left(\sqrt[3]{8}\right)^2$ is in radical form.	
Exponential Form	$8^{\frac{2}{3}}$ is in exponential form.	
Zero Exponent	Any expression to the power of 0 will equal 1.	$(2xyz)^0 = 1$
Negative Exponent	Reciprocate the base and perform repeated multiplication OR use repeated division.	$5^{-3} = \left(\frac{1}{5}\right)^3 = \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = \frac{1}{125}$
Multiply Powers with the Same base	Add the exponents.	$m^5 \times m^2 = m^7$
Dividing Powers with the same base.	Subtract the exponents.	$q^6 \div q^4 = q^2$
Power of a Power	Multiply the exponents.	$(x^2)^4 = x^8$
Power of a Product	Apply the exponent to all factors.	$(3x^2)^3 = 27x^6$
Power of a Quotient	Apply the exponent to both numerator AND denominator	$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$

Math 10

Unit 2: Exponents

Lesson 1: pages 1-9

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Vocabulary:

Exponent Laws:

From Math 9, you should have learned how to simplify the following monomial expressions using the following exponent laws:

Note: DO NOT use exponent laws when bases aren't equal

Exponent Laws	Examples (simplify & evaluate where possible)
Product of Powers $a^m \times a^n =$	a) $0.8^2 \times 0.8^7 =$ b) $3^4 \times 3 =$ c) $10^{10} \times 10^{-6} =$
Quotient of Powers $a^m \div a^n =$	a) $5^5 \div 5^3 =$ b) $\left(-\frac{4}{5}\right)^{-6} \div \left(-\frac{4}{5}\right)^{-20} =$ c) $40m^8 \div 5m =$
Negative Exponent $a^{-m} =$	a) $25^{-3} =$ b) $6^3 \div 6^5 =$
Zero Exponent $a^0 =$	a) $(-7x^5y^{-6})^0 =$ b) $\left(\frac{5}{2}\right)^4 \div \left(\frac{5}{2}\right)^4 =$

Example: Evaluate or simplify the following expressions.

1. $3^2 =$

2. $(-3)^2 =$

3. $-3^2 =$

4. $-5^0 =$

5. $6^{-2} =$

6. $-2^{-4} =$

7. $(-2)^{-4} =$

8. $x^3 \cdot x^4 =$

9. $x^3 \cdot x^{\frac{1}{4}} =$

10. $6m^4 \cdot 2m \div 3m^{-2} =$

Introduction to Exponents

Challenge #1: Solve each riddle using any strategy that works.

1. Evaluate. $3^2 \times 3^2$	2. Evaluate. $2^2 \times 2^2 \div 2^3$	3. Evaluate. $x^3 \times x^5$	4. Evaluate. $8x^4 \div 4x^3$
Rate the riddle: Easy, Medium, Hard	Rate the riddle: Easy, Medium, Hard	Rate the riddle: Easy, Medium, Hard	Rate the riddle: Easy, Medium, Hard

5. Find a strategy that is different from the one you used in Question 1 and solve the question again.	6. Find a strategy that is different from the one you used in Question 4 and solve the question again.

What is an Exponent?

Exponents are symbols that indicate an operation to be performed on the base.

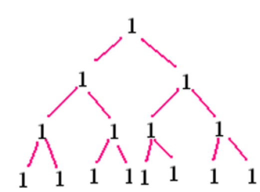
positive exponents → Repeated Multiplication
negative exponents → Repeated Division

b^e $\quad$ b is the base, and e is the exponent. $\quad$ Together, we call them a *power*.

Some examples…

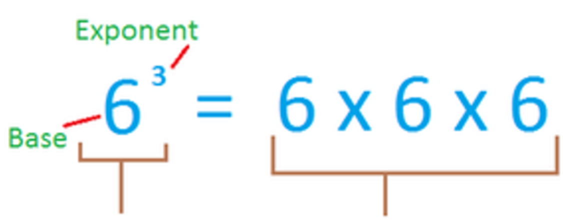
$2^1, 2^2, 2^3, 2^4, 2^5$ are the first five ***powers of 2***. $\quad$ x^1, x^2, x^3, x^4, x^5 are the first five ***powers of x***.

All organisms begin as one cell and then through a process called mitosis the single cell splits into two, then each of those split into two, etc. Eventually, these cells together form a multi-celled organism with trillions of cells.



1 cell - 2^0
2 cells - 2^1
4 cells - 2^2
8 cells - 2^3

** Guess the next few numbers _________, __________, __________



When numbers are written in a form such as 2^3 it is called a __________________, the "2" is the _________________ and the "3" is the __________________. The exponent represents the number of times the base is multiplied by itself.

a^x	a is the base, x is the exponent and a^x is the power.
5^2	Is read 5 to the exponent 2 and equals 5×5 as a repeated multiplication and evaluates to 25.
2^5	Is read 2 to the exponent 5 and equals $2 \times 2 \times 2 \times 2 \times 2$ as a repeated multiplication and evaluates to 32.

Positive Integral Exponent (multiplication)	**Zero Exponent**	**Negative Integral Exponent (repeated division)**
$a^n = 1 \times a \times a \times a \times \ldots \times a$ (n factors) Eg. $3^4 = 1 \times 3 \times 3 \times 3 \times 3 = 81$	$a^0 = 1, \quad (a \neq 0)$ Eg. $5^0 = 1, \quad \left(\frac{3}{2}\right)^0 = 1$	$a^{-n} = 1 \div a^n$ $= \frac{1}{a^n}$ Eg. $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

Challenge #2

7. Evaluate each of the following and examine the pattern:

$2^4 =$

$2^3 =$

$2^2 =$

$2^1 =$

$2^0 =$

$2^{-1} =$

$2^{-2} =$

$2^{-3} =$

$2^{-4} =$

8. What patterns do you notice in the list you created to the left?

9. Does the value of 2^0 make sense when put into this list?

10. Do negative exponents make sense in this list?

11. Why might people say negative exponents mean "repeated division?"

12. Identify the base in the following equation.

$4^3 = 64$

13. Identify the power in the following equation.

$2^5 = 32$

14. Identify the exponent in the following equation.

$-3^2 = -9$

15. Which of the following is equivalent to -16?

-4^2

$(-4)^2$

4^{-2}

-4^{-2}

16. Which of the following is equivalent to -81?

-9^2

$(-3)^4$

9^{-2}

-3^{-4}

17. Which of the following are equivalent to 1.

-3^0 $\quad \frac{2x^3}{2x^3}$ $\quad (5x)^0$

18. Which of the following is equivalent to 9?

-3^2

$(-3)^2$

3^{-2}

$(-3)^{-2}$

19. Evaluate.

-2^6

$$= \mathbf{-1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$$
$$= \mathbf{-64}$$

20. Evaluate.

$(-3)^3$

21. -4^2

22. $(-4)^{-2}$

23. -4^{-2}

24. 3^{-4}

$$= \frac{\mathbf{1}}{\mathbf{3^4}}$$

$$= \frac{\mathbf{1}}{\mathbf{81}}$$

$$= \mathbf{1 \div 3 \div 3 \div 3 \div 3}$$

$$= \mathbf{1 \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}}$$

$$= \frac{\mathbf{1}}{\mathbf{81}}$$

25. $(-3)^{-4}$

26. -3^{-4}

27. 4^2

28. $(-4)^2$

29. $-(4)^2$

30. 5^0	31. -5^0	32. $\left(\frac{34a^2}{2x}\right)^0$

The Exponent Laws:

Challenge #3

33. Multiply. $a^3 \times a^6$	Explain your steps.

Challenge #4

34. Divide. $g^7 \div g^3$	Explain your steps.

Challenge #5

35. Multiply. $5m^4 \times 3m^2$	Explain your steps.

Simplify the following, write your answers using exponents.

36. $a^3 \times a^6$
$= a^{3+6}$
$= a^9$

37. $a^2 \times a^{-4}$

38. $f^2 \times f^x$

39. $x^{\frac{1}{4}} \times x^{\frac{6}{8}}$

40. $2^3 \times 2^{-5}$

41. $g^7 \div g^3$
$= g^{7-3}$
$= g^4$

42. $m^4 \div m^0$

43. $t^0 \div t^{-5}$

44. $\frac{x^{13}}{x^3}$

45. $5m^4 \times 3m^2$
$= 5 \times 3 \times m^{4+2}$
$= 15m^6$

46. $-10x^4 \div -2x^{-2}$

47. $\frac{4a^4}{-8a^2}$

48. $\frac{2}{3}x^3 \times \frac{6}{5}x^4$

49. $\frac{2}{a^3} \div \frac{6}{a^6}$

50. Evaluate.
$\left(\frac{2}{3}\right)^3 \left(\frac{-6}{4}\right)^2$

Multiplying Powers with the same Base:
Add the exponents.

Eg.
$x^5 \times x^2 = x^{5+2} = x^7$

$a^{\frac{2}{3}} \times a^{\frac{1}{3}} = a^{\frac{3}{3}} = a^1 = a$

$3x^2 \times 2x^5 = 3 \times 2 \times x^2 \times x^5 = 6x^7$

Dividing Powers with the same Base:
Subtract the exponents.

Eg.
$d^4 \div d^3 = d^{4-3} = d^1 = d$

$\frac{y^6}{y^{-2}} = y^{6-(-2)} = y^8$

Math 10

Unit 2: Exponents

Lesson 2: pages 10-12

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Warm-Up:

1. 5^{-2}
2. 8^{-1}
3. 3^{-3}
4. $(-2)^4$
5. $\left(\frac{3}{10}\right)^{-2}$
6. $a^{\frac{8}{3}} \times a^{\frac{1}{3}}$
7. $a^{-8} \div a$
8. $100x^4 \div 50x^8$
9. $a^9 \div a^{12}$
10. $6x^4 \div 6x^5$
11. $\left(\frac{2}{5}\right)^{-3}$
12. $6m^{12} \div 12m^{12}$
13. $30m^8 \div -10m$
14. $\frac{3m^{-2}p^4}{4q^{-1}}$

Exponent Laws:

From Math 9, you should have learned how to simplify the following monomial expressions using the following exponent laws:

Note: DO NOT use exponent laws when bases aren't equal

Exponent Laws	**Examples (simplify & evaluate where possible)**
Power of a Power $(a^m)^n =$	a) $(0.25^{-3})^{-5}$ b) $(8^2)^4$ c) $(m^5)^3$ d) $(2m^{10})^3$
Power of a Product $(ab)^m =$	a) $(-6my^7)^3$ b) $(x^4y^{-2})^5$ c) $(8x^{-4})^2$ d) $(3m^{-2}y^5)^{-3}$ e) $(3t^0)^4$

(More Complicated) Examples ☺ : Evaluate or simplify the following expressions.

1. $\left(\frac{2x^4y^{-9}}{3x^{-2}y}\right)^{-2}$

2. $(-10xy^4)^2 \cdot (5x^2y^3)^{-2}$

Challenge #6

51. Evaluate. $(5^2)^3$	Explain your steps. 15625

Challenge #7

52. Simplify. $(m^3)^2$	Explain your steps.

Challenge #8

53. Simplify. $(2m^4)^3$	Explain your steps.

Simplify the following.

54. $(m^3)^2$

$= m^3 \times m^3$
$= m^6$

$= m^{3\times 2}$
$= m^6$

55. $(t^4)^0$

56. $(x^2y^3)^{-3}$

57. $(2m^4)^3$

$2m^4 \times 2m^4 \times 2m^4$
$= 2 \times 2 \times 2 \times m^4 \times m^4 \times m^4$
$= 8m^{12}$

OR

$= 2^3 m^{4\times 3}$
$= 8m^{12}$

58. $(2c^4d^3)^{-3}$

59. $(-3x^{-2}y^3)^{-4}$

60. $(3x^{-2}y^{-3})^{-3}$

61. $(-2xy^3)(-3x^2y^3)^2$

62. $(2a^2)^3(4a^3b)^2$

Power of a Power:
Multiply the exponents.

Eg$(5^2)^3 = (5\times 5)^3$.
$= (5\times 5)(5\times 5)(5\times 5)$
$= 5\times 5\times 5\times 5\times 5\times 5$
$= 5^6$

THE RULE:

$$(a^m)^n = a^{m\times n}$$

If you have a power of a power … multiply exponents.

Eg. $(x^2)^5 = x^{2\times 5} = x^{10}$

Power of a Product:
Apply the exponent to all factors.

Eg. $(5\times 2)^3$
$= (5\times 2)\times(5\times 2)\times(5\times 2)$
$= 5\times 5\times 5\times 2\times 2\times 2$
$= 5^3\times 2^3$

THE RULE:

$$(ab)^m = a^m b^m$$

If you have a power of a product … apply the exponent to EVERY factor in the product.

Eg. $(a^2b^3)^{-3} = a^{2\times -3}b^{3\times -3} = a^{-6}b^{-9}$

Challenge #9

63. Evaluate.

$$\left(\frac{2}{5}\right)^3$$

$\frac{8}{125}$

Explain your steps.

Challenge #10

64. Evaluate.

$$\left(\frac{2}{5}\right)^{-3}$$

$\frac{125}{8}$

Explain your steps.

Challenge #11

65. Simplify.

$$\left(\frac{x}{2}\right)^3$$

Explain your steps.

Challenge #12

66. Simplify.

$$\left(\frac{6x^5y^3}{8y^4}\right)^{-2}$$

Explain your steps.

Math 10

Unit 2: Exponents

Lesson 3: pages 13-16

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Warm-Up: Simplify or evaluate as far as possible. Express answers with positive exponents.

1. 7^{-3}
2. $2^6 \times 2^4$
3. $x^9 \div x^3$
4. $7m^4 \times 2m$
5. $(-8xy^5)^2$
6. $50p^9 \div 10p^{-2}$
7. $(3m^0)(9m)^0$
8. $(5m)^{-2}$
9. $(2^{-3})^{-2}$
10. $(10y^{-3})(6y^4)^2$
11. $(4x^2y^3)^{-3}$
12. $\frac{6m^8y^2z^{-4}}{12my^5z^{-8}}$
13. $\frac{-10ab^{-1}c^{-4}}{4a^{-2}c^7}$
14. $x^{-3} \cdot x^{-\frac{4}{3}} \cdot x^{\frac{1}{3}}$

Exponent Laws:

From Math 9, you should have learned how to simplify the following monomial expressions using the following exponent laws:

Exponent Laws	**Examples (simplify & evaluate where possible)**
Power of a Quotient $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{y^{-3}}{x^{\frac{3}{5}}}\right)^5$
Power of a Quotient $\left(\frac{a}{b}\right)^{-m} = \frac{b^m}{a^m}$	$\left(\frac{y^{-3}}{x^{\frac{3}{5}}}\right)^{-5}$

(More Complicated) Examples ☺ : Evaluate or simplify the following expressions.

1. $\left(\frac{x^4y^4m}{x^7y^2m^5}\right)^{-6}$

2. $\frac{(5m^{-1}y^3)^2}{my}$

3. $\left(\frac{7x^{-1}y^6}{x^{-4}y^4}\right)^{-2}$

4. $\frac{\left(\frac{1}{2}a^4b^5\right)^{-4}}{4}$

5. $\left(\frac{8xb^{-7}}{-12x^2b^{-9}}\right)^{-3}$

Power of a Quotient:
Apply the exponent to numerator AND denominator.

Eg. $\left(\frac{2}{5}\right)^3 = \left(\frac{2}{5}\right) \times \left(\frac{2}{5}\right) \times \left(\frac{2}{5}\right)$

$= \frac{2\times2\times2}{5\times5\times5}$

$= \frac{2^3}{5^3}$ If asked to write using exponents

$= \frac{8}{125}$ If asked to simplify.

$\left(\frac{2}{5}\right)^{-3}$ The negative exponent means "flip the base".

$= \frac{5\times5\times5}{2\times2\times2}$

$= \frac{5^3}{2^3}$

$= \frac{125}{8}$

THE RULE:

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$\left(\frac{a}{b}\right)^{-m} = \frac{b^m}{a^m}$$

Simplify the following.

67. $\left(\frac{x}{2}\right)^3$

$= \frac{x^3}{2^3}$

$= \frac{x^3}{8}$

68. $\left(\frac{a}{b}\right)^4$

69. $\left(\frac{x^2}{y^3}\right)^5$

70. $\left(\frac{-2a^2}{3y^3}\right)^3$

71. $\left(\frac{a^{-3}}{b^{-2}}\right)^{-2}$

72. $\left(\frac{4x}{3y}\right)^2$

73. $\left(\frac{6x^5y^3}{8y^4}\right)^{-2}$

$= \frac{(8)^2(y^4)^2}{(6)^2(x^5)^2(y^3)^2}$

$= \frac{64y^8}{36x^{10}y^6}$

$= \frac{16y^2}{9x^{10}}$

74. $\left(\frac{5ab^2c^3}{2a^{-2}c^{-3}}\right)^2$

75. $\left[\left(\frac{2m^2n^2}{mn^3}\right)^{-1}\right]^3$

Simplify the following.

76. $\left(\frac{6ab^3}{2ab}\right)^3$

77. $\left(\frac{4x^{-3}y^4}{8x^2y^{-2}}\right)^{-2}$

78. Show why $\frac{2a^2}{b^3}$ is the same as $2a^2 \times b^{-3.}$

79. Show why $\frac{12x^3}{y}$ is the same as $12x^3 \times y^{-1.}$

Challenge #13

80. Write the following without using <u>any</u> negative exponents.

$$3a^2b^{-5}$$

81. Write the following without using <u>any</u> negative exponents.

$$\frac{3}{a^{-2}b^5}$$

Challenge #14

82. Simplify using positive exponents.

$$\left(\frac{2x^{-2}y^4}{x^{-3}y^3}\right)^{-3}$$

Explain your steps

Writing Expressions with Positive Exponents. (Why? Because it is standard practice.)

An expression with powers is simplified if there are no brackets and no negative exponents.

Sometimes you will use the laws above and end up with an answer with negative exponents. The quick way to convert a negative exponent into a positive exponent is to ***move it across the division line***. The solution in question 83 shows why this works.

Simplify the following. (No brackets, no negative exponents)

83. $3a^2b^{-5}$

$$= 3a^2 \times \frac{1}{b^5}$$

$$= \frac{3a^2}{b^5}$$

84. a^2b^{-3}

85. $\frac{2xy^5}{x^{-4}}$

86. $3a^2b^{-3}c^{-5}$

87. $(x^4y^{-3}z^{-1})^{-2}$

88. $\frac{(3x^{-3}y^{-5})^2}{2xy}$

89. $\left(\frac{2x^{-2}y^4}{x^{-3}y^3}\right)^{-3}$

$$= \left(\frac{x^{-3}y^3}{2x^{-2}y^4}\right)^3$$

$$= \frac{x^{-9}y^9}{8x^{-6}y^{12}}$$

$$= \frac{x^{-3}y^{-3}}{8}$$

$$= \frac{1}{8x^3y^3}$$

90. $\frac{\left(\frac{2a^3b^2}{4a^{-2}b^{-1}}\right)^{-3}}{2}$

91. $\frac{(4m^2n^2)(7m^{-3}n^2)}{14mn^5}$

92. Why does moving a power across the division line in a fraction change the sign on the exponent?

Simplify the following. (No brackets, no negative exponents)

93. $\left(\frac{12x^3y^{-1}}{-8x^{-1}y^5}\right)^{-2}$

94. $\left(\frac{4a^3b^{-2}}{6a^2b^{-1}}\right)^{-3}$

95. $\left(\frac{8x^2y^{-3}}{4x^{-1}y^{-5}}\right)^{-3}$

96. $\left(\frac{12x^{-3}y^5}{16x^3y^{-2}}\right)^{-1}$

Math 10

Unit 2: Exponents

Lesson 4: pages 18-20

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Warm-Up #1: Simplify or evaluate as far as possible. Express answers with positive exponents.

1. $\frac{3x^2y^4}{4x^3y^3}$

2. $\frac{2ab^3}{-2a^3b^2}$

3. $\frac{-2x^2y^{-5}}{-3x^{-4}y^3}$

4. $\frac{(n^2)^4(-n^0)^3}{-n^2}$

5. $\frac{2a^2bc^{-4}}{5^{-1}a^{-3}b^3c^2}$

6. $\left(\frac{x^2y}{mp^8}\right)^5$

7. $\left(\frac{3c}{5d}\right)^{-2}$

8. $\left(\frac{15m^8y}{3my^{-5}}\right)^{-3}$

9. $\left(\frac{27m^2n}{9mn}\right)^{-2}$

10. $\left(\frac{2a^2b^{-2}}{8a^{-3}b}\right)^{-4}$

Warm-Up #2: Use your calculator to complete the following tables:

1.

x	$x^{\frac{1}{2}}$
1	
4	
9	
16	
25	
36	

Explain the effect the exponent $\frac{1}{2}$ has on the value of x.

Write a rule to describe this relationship:

$$x^{\frac{1}{2}} =$$

2.

y	$y^{\frac{1}{3}}$
1	
8	
27	
64	
125	
216	

Explain what effect the exponent $\frac{1}{3}$ has on the value of y.

Write a rule to describe this relationship:

$$y^{\frac{1}{3}} =$$

3. What do you think $x^{\frac{1}{4}}$ means? Test your prediction on your calculator, letting $x = 16$.

4. What would $x^{\frac{1}{n}}$ mean (as a radical)?

Exponent Law:

Exponent Laws	**Example #1 (simplify & evaluate where possible)**
Rational Exponents ***(with numerator = 1)*** $x^{\frac{1}{n}} =$	a) $100^{\frac{1}{2}}$ b) $(-8)^{\frac{1}{3}}$ c) $1024^{\frac{1}{5}}$ d) $(625m^4)^{\frac{1}{4}}$ e) $(81m)^{\frac{1}{4}}$ f) $-343^{\frac{1}{3}}$ g) $(-49)^{\frac{1}{2}}$ h) $16^{-\frac{1}{4}}$ i) $1000^{-\frac{1}{3}}$

Example #2: Simplify the following in radical form.

1. $\sqrt{121}$
2. $\sqrt[5]{-32}$
3. $\frac{1}{\sqrt[3]{125}}$
4. $10\sqrt{3xy}$

97. Challenge #15

If $\sqrt{9} \times \sqrt{9} = 9$,

and $9^a \times 9^a = 9$

Then what is the value of 'a'?

Explain:

98. Challenge #16

If $\sqrt[3]{2} \times \sqrt[3]{2} \times \sqrt[3]{2} = 2$,

and $2^a \times 2^a \times 2^a = 2$

Then what is the value of 'a'?

Explain:

99. Write a "rule" that relates a **rational** (fraction) **exponent** to an equivalent **radical expression**.

Rational Exponents in the form: $x^{\frac{1}{n}}$

Remember, *rational* often refers to fractions.

What does a rational exponent mean?

Recall: $\sqrt{9} \times \sqrt{9} = 9$.

But $9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 9$

And $3 \times 3 = 9$

$\text{So}, \sqrt{9} = 9^{\frac{1}{2}} = 3$

$\text{If } \sqrt[3]{2} \times \sqrt[3]{2} \times \sqrt[3]{2} = 2$

But $2^{\frac{1}{3}} \times 2^{\frac{1}{3}} \times 2^{\frac{1}{3}} = 2$

So, $\sqrt[3]{2} = 2^{\frac{1}{3}}$

100. Write another statement like the one to the left.

The Rule…

$$a^{\frac{1}{n}} = \sqrt[n]{a} \qquad and \qquad a^{-\frac{1}{n}} = \frac{1}{\sqrt[n]{a}}$$

Evaluate or simplify the following.

101. $49^{\frac{1}{2}}$	102. $-16^{\frac{1}{2}}$	103. $(-16)^{\frac{1}{2}}$
104. $64^{\frac{1}{3}}$	105. $27^{-\frac{1}{3}}$	106. $32^{-\frac{1}{5}}$
107. $10000^{\frac{1}{4}}$	108. $(4x^2)^{\frac{1}{2}}$	109. $(27x^6)^{-\frac{1}{3}}$

Write in radical form.

110. $7^{\frac{1}{2}}$	111. $(3x)^{\frac{1}{3}}$	112. $4^{\frac{1}{5}}$
113. $4^{-\frac{1}{5}}$	114. $-64^{\frac{1}{3}}$	115. $64^{-\frac{1}{3}}$

Write in exponential form.

116. $\sqrt{13}$	117. $-3\sqrt{x}$	118. $\sqrt{2y}$
119. $\sqrt[4]{4}$	120. $\sqrt[7]{4}$	121. $\frac{1}{\sqrt[5]{3x}}$

Consider the following…

Step 1: $32^{\frac{3}{5}} = \left(32^{\frac{1}{5}}\right)^3$

Step 2: $32^{\frac{3}{5}} = \left(\sqrt[5]{32}\right)^3$

Step 3: $32^{\frac{3}{5}} = (2)^3$

Step 4: $32^{\frac{3}{5}} = 8$

122. Challenge #17. Complete the following as shown above.

Step 1: $27^{\frac{2}{3}} =$ Explain:_________________________

Step 2: $27^{\frac{2}{3}} =$ _________________________

Step 3: $27^{\frac{2}{3}} =$ _________________________

Step 4: $27^{\frac{2}{3}} =$ _________________________

Math 10

Unit 2: Exponents

Lesson 5: pages 21-25

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Warm-Up #1: Simplify or evaluate as far as possible (#1-6), or re-write radicals as exponents (#7-10). Express answers with positive exponents.

1. $16^{\frac{1}{4}}$
2. $27^{-\frac{1}{3}}$
3. $-25^{\frac{1}{2}}$
4. $(-25)^{\frac{1}{2}}$
5. $1024^{0.5}$
6. $((-2)^{-2})^{\frac{1}{2}}$
7. $8\sqrt[3]{a}$
8. $\sqrt{16y^8}$
9. $\frac{50}{\sqrt[3]{xy}}$
10. $\left(\sqrt[3]{\sqrt[7]{z}}\right)^6$

Warm-Up #2:

1. Re-write the exponents below as a product of two fractions, remembering that $\frac{a}{b} = \frac{a}{1} \times \frac{1}{b}$. Then, evaluate. The first one is done as an example 😊

   a. $9^{\frac{3}{2}} = \left(9^{\frac{3}{1}}\right)^{\frac{1}{2}} = (729)^{\frac{1}{2}} = \sqrt{729} = 27$

   b. $100^{\frac{5}{2}}$

   c. $216^{\frac{2}{3}}$

This works, but there's an easier way!

Exponent Law:

Exponent Laws	**Example #1 (simplify & evaluate where possible)**
Rational Exponents ***(with numerator ≠ 1)*** $x^{\frac{m}{n}} =$	a) $32^{\frac{3}{5}}$ b) $(-32)^{\frac{3}{5}}$ c) $16^{\frac{7}{4}}$ d) $(-27)^{\frac{2}{3}}$ e) $(-25)^{\frac{5}{2}}$ f) $-25^{\frac{5}{2}}$ g) $-25^{-\frac{5}{2}}$ h) $16^{1.5}$ i) $1000^{-\frac{2}{3}}$

Exam

ple #2: Write the following with exponents. Then, use exponent laws and evaluate.

1. $\sqrt{8} \times \sqrt{8^3}$

2. $\sqrt{g^5} \times \sqrt{g^7}$

3. $\sqrt{\sqrt{16^3}}$

4. $\sqrt[3]{x^2} \cdot \sqrt[4]{x}$

5. $\left(\sqrt[5]{18}\right)^2 \cdot \sqrt[5]{18^3}$

6. $\sqrt[3]{64} \cdot \sqrt[4]{16^3}$

Example #3: Find the area of a triangle that has a base of $82^{\frac{4}{5}}cm$ and a height of $82^{\frac{11}{5}}cm$. (Hint: $A = \frac{b \times h}{2}$)

Rational Exponents in the form: $x^{\frac{m}{n}}$ where m is not 1.

Consider the power $27^{\frac{2}{3}}$. To understand the meaning of the rational exponent we can use the exponent law:
$(a^m)^n = a^{m\times n}$.

If we take $27^{\frac{2}{3}}$ and split the exponent into two parts we get the following…

$27^{\frac{2}{3}} = \left(27^{\frac{1}{3}}\right)^2$

This can then be written as…

$\left(\sqrt[3]{27}\right)^2$

The power can be evaluated from this point…

$\left(\sqrt[3]{27}\right)^2 = (3)^2 = 9$

The Rule…

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m \qquad and \qquad a^{-\frac{m}{n}} = \frac{1}{\sqrt[n]{a^m}} = \frac{1}{\left(\sqrt[n]{a}\right)^m}$$

Two more examples:

Eg.1 Evaluate $8^{\frac{2}{3}}$ without using a calculator.

$8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$

Means square of the cube root of 8.

Eg.2 Evaluate $9^{-\frac{3}{2}}$ without using a calculator.

$9^{-\frac{3}{2}} = \left(\frac{1}{9}\right)^{\frac{3}{2}} = \frac{\left(1^{\frac{1}{2}}\right)^3}{\left(9^{\frac{1}{2}}\right)^3} = \frac{1}{\left(\sqrt{9}\right)^3} = \frac{1}{(3)^3} = \frac{1}{27}$

Means "the reciprocal" of the cube of the square root of 9.

Write each of the following using radicals. (Do not evaluate)

123. $4^{\frac{2}{5}}$

124. $4^{\frac{3}{5}}$

125. $4^{\frac{4}{5}}$

126. $4^{-\frac{2}{5}}$

127. $4^{-\frac{3}{5}}$

128. $4^{-\frac{4}{5}}$

Evaluate each of the following.

129. $4^{\frac{1}{2}}$

130. $125^{\frac{1}{3}}$

131. $8^{\frac{2}{3}}$

132. $81^{\frac{3}{4}}$

133. $4^{\frac{3}{2}}$

134. $16^{-\frac{3}{4}}$

135. $(-27)^{-\frac{2}{3}}$

136. $(-8)^{-\frac{5}{3}}$

137. $9^{2.5}$

138. $(-1)^{-\frac{8}{5}}$

139. $\left(\frac{100}{9}\right)^{\frac{3}{2}}$

140. $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

Write each of the following using exponents. (Do not evaluate)

Eg. $\sqrt{12} = 12^{\frac{1}{2}}$ Eg. $\left(\sqrt[3]{7}\right)^4 = 7^{\frac{4}{3}}$ Eg. $\frac{1}{\left(\sqrt[3]{7}\right)^2} = 7^{-\frac{2}{3}}$

141. $\sqrt{7}$

142. $\sqrt[3]{34}$

143. $\sqrt[3]{-11}$

144. $\sqrt[5]{a^2}$

145. $\sqrt[3]{6^4}$

146. $\left(\sqrt[3]{x}\right)^2$

147. $\left(\sqrt[5]{6}\right)^3$

148. $\left(\sqrt[4]{2x}\right)^5$

149. $\frac{1}{\sqrt[3]{a}}$

150. $\frac{1}{\left(\sqrt[5]{x}\right)^4}$

151. $\frac{1}{\sqrt[4]{x^3}}$

152. $\sqrt[3]{2b^3}$

Evaluate if possible.

153. $(-9)^{\frac{1}{2}}$

154. $100000^{\frac{3}{5}}$

155. $\left(\frac{27}{8}\right)^{\frac{2}{3}}$

156. $3^{\frac{1}{2}} \times 3^{\frac{1}{2}}$

157. $-9^{\frac{1}{2}}$

158. $(2^5)^{0.4}$

Evaluate if possible.

159.

a. $-8^{\frac{4}{3}}$

b. $(-8)^{\frac{4}{3}}$

What important rule is explored above?

160. $4^{\frac{3}{2}} \div 16^{\frac{1}{4}}$

161. $(-1)^{-\frac{3}{2}}$

162. $\left(\sqrt[3]{5^2}\right)\left(\sqrt[3]{5}\right)$

163. $\left(\sqrt[4]{16}\right)\left(\sqrt[5]{32}\right)$

164. $\sqrt{\sqrt[3]{729}}$

165. Evaluate to two decimal places using a calculator

$$\frac{1}{\sqrt[5]{300}}$$

166. Evaluate to two decimal places using a calculator

$$\frac{5}{\sqrt[6]{256}}$$

167. Evaluate to two decimal places using a calculator

$$\frac{1}{\sqrt[13]{2500}}$$

168. Challenge

Write the following radicals as a single power.

$$\left(\sqrt{x^3}\right)\left(\sqrt[3]{x}\right)$$

Write each of the following radicals as a single power.

169. $\left(\sqrt{x^3}\right)\left(\sqrt[3]{x}\right)$

$\left(x^{\frac{3}{2}}\right)\left(x^{\frac{1}{3}}\right)$ Write as powers (both base-x).

$\left(x^{\frac{9}{6}}\right)\left(x^{\frac{2}{6}}\right)$ Create common denominators.

$\left(x^{\frac{9+2}{6}}\right)$ Add numerators.

$\left(x^{\frac{11}{6}}\right)$

170. $\left(\sqrt[3]{x^2}\right)\left(\sqrt[4]{x^3}\right)$

171. $\left(\sqrt[5]{x^3}\right)\left(\sqrt[3]{x^2}\right)$

More rational exponents…

172. The height and the base of a triangle each measure $2^{\frac{3}{2}}$ cm. Without using a calculator, what is the area of the triangle?

173. Find the area of a rectangle if the length is $5^{\frac{2}{3}}$ and the width is $5^{\frac{2}{5}}$. Write your answer in exponential form, then approximate to two decimal places.

174. Inscribe a square inside another square such that the corners of the internal square contact the midpoint of sides of the larger square. If the side length of the larger square is $\sqrt{7}$, what is the area of the inscribed square? Answer in exact form.

175. Simplify(write as a single power.)

$$\left[\left(\sqrt[3]{x^4}\right)\left(\sqrt[5]{x}\right)\right]^{-2}$$

176. Simplify (write as a single power.)

$$\left[\left(\sqrt[4]{x^9}\right)\left(\sqrt[3]{x^6}\right)\right]^{\frac{2}{3}}$$

177. Ei-Q evaluated $64^{\frac{3}{2}}$ using the following steps. In which step did she make her first error?

Step 1: $64^{\frac{3}{2}} = \left(\sqrt{64}\right)^3$

Step 2: $64^{\frac{3}{2}} = (8)^3$

Step 3: $64^{\frac{3}{2}} = 24$

a) In step 1.
b) In step 2.
c) In step 3.
d) She made no error.

---

178. Flinflan started to evaluate $81^{-\frac{3}{4}}$ in two different ways shown below. Which of the following statements is correct?

Method 1: $81^{-\frac{3}{4}} = \left(\sqrt[4]{81}\right)^{-3}$ Method 2: $81^{-\frac{3}{4}} = \frac{1}{\sqrt[4]{81^3}}$

a) Method 1 will produce the correct answer but method 2 will not.
b) Method 2 will produce the correct answer but method 1 will not.
c) Both methods will produce the correct answer.
d) Neither method will produce the correct answer.

---

179. Simplify: $\left[\left(\sqrt[3]{x^4}\right)\left(\sqrt[5]{x^2}\right)\right]^{-1}$

180. Simplify: $\left[\left(\sqrt[3]{a^5}\right)\left(\sqrt[4]{a^3}\right)\right]^{-2}$

181. Simplify:

$$\sqrt[3]{\left(a^{\frac{2}{3}}\right)^{\frac{1}{4}}}$$

182. Simplify:

$$\sqrt[4]{\left(x^{\frac{1}{3}}\right)^{\frac{1}{5}}}$$

Match each item in column 1 with an equivalent item in column 2

Column 1	Column 2
183. $\left(\frac{t}{j}\right)^{\frac{2}{3}}$	A. $\sqrt[3]{\frac{j^2}{t^2}}$
184. $\left(\frac{j}{t}\right)^{\frac{3}{2}}$	B. $-\left(\frac{j}{t}\right)^{\frac{3}{2}}$
185. $\left(\frac{t}{j}\right)^{-\frac{2}{3}}$	C. $\sqrt{\frac{j^3}{t^3}}$
186. $\left(\frac{j}{t}\right)^{-\frac{3}{2}}$	D. $-\left(\frac{t}{j}\right)^{\frac{2}{3}}$
187. $\left(\frac{t}{j}\right)^{-\frac{3}{2}}$	E. $\sqrt{\frac{t^3}{j^3}}$
	F. $\sqrt[3]{\frac{t^2}{j^2}}$
	G. $-\left(\frac{t}{j}\right)^{\frac{3}{2}}$

188. Which of the following is equivalent to $3a^{\frac{1}{2}} \times (5a)^{\frac{1}{2}}$

a. $15a$
b. $a\sqrt{15}$
c. $3\sqrt{5a}$
d. $3a\sqrt{5}$

189. Which of the following is equivalent to $2x^{\frac{1}{2}} \times (3x)^{\frac{1}{2}}$

a. $6x$
b. $x\sqrt{6}$
c. $2\sqrt{3x}$
d. $2x\sqrt{3}$

190. Which of the following is not equivalent to $x^{\frac{2}{3}}$?

a. $\sqrt[3]{x^2}$
b. $\left(\sqrt[6]{x}\right)^4$
c. $(x^2)\left(\sqrt[3]{x}\right)$
d. $\sqrt{x^3}$

191. Which of the following is not equivalent to $a^{\frac{3}{2}}$?

a. $\sqrt[4]{a^6}$
b. $\sqrt[3]{a^2}$
c. $a\sqrt{a}$
d. $\sqrt{\sqrt{a^6}}$

192. Evaluate. Answer in simplest fraction form.

$$\frac{3^0 + 2^{-1}}{3^2 + 2^2}$$

193. Evaluate. Answer in simplest fraction form.

$$\frac{3^{-2} + 3^2}{3^{-2} + 2^0}$$

Answers:

1. 81
2. 2
3. x^8
4. $2x$
5. $9 \times 9 = 81$ or $3 \times 3 \times 3 \times 3 = 81$ or $3^4 = 81$
6. Answers vary. Similar to above.
7. $16, 8, 4, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$
8. Divide by 2 as you go down the list
9. Fits the pattern above.
10. Yes follows the division pattern.
11. Decreasing exponent value is like dividing by two in this case.
12. 4
13. 2^5
14. 2
15. -4^2
16. -9^2
17. $\frac{2x^3}{2x^3}, (5x)^0$
18. $(-3)^2$
19. -64
20. -27
21. -16
22. $\frac{1}{16}$
23. $-\frac{1}{16}$
24. $\frac{1}{81}$
25. $\frac{1}{81}$
26. $-\frac{1}{81}$
27. 16
28. 16
29. -16
30. 1
31. -1
32. 1
33. a^9
34. g^4
35. $15m^6$
36. a^9
37. a^{-2}
38. f^{2+x}
39. x^1
40. 2^{-2}
41. g^4
42. m^4
43. t^5
44. x^{10}
45. $15m^6$
46. $5x^6$
47. $-\frac{1}{2}a^2 = -\frac{a^2}{2}$
48. $\frac{4x^7}{5}$
49. $\frac{a^3}{3}$
50. $\frac{2}{3}$
51. 15625
52. m^6
53. $8m^{12}$
54. m^6
55. 1
56. $x^{-6}y^{-9} = \frac{1}{x^6y^9}$
57. $8m^{12}$
58. $2^{-3}c^{-12}d^{-9} = \frac{1}{8c^{12}d^9}$
59. $(-3)^{-4}x^8y^{-12} = \frac{x^8}{81y^{12}}$
60. $3^{-3}x^6y^9 = \frac{1}{27}x^6y^9$ or $\frac{x^6y^9}{27}$
61. $-18x^5y^9$
62. $128a^{12}b^2$
63. $\frac{8}{125}$
64. $\frac{125}{8}$
65. $\frac{x^3}{8}$
66. $\frac{16y^2}{9x^{10}}$
67. $\frac{x^3}{8}$
68. $\frac{a^4}{b^4}$
69. $\frac{x^{10}}{y^{15}}$
70. $\frac{-8a^6}{27y^9}$
71. $\frac{a^6}{b^4}$
72. $\frac{16x^2}{9y^2}$
73. $\frac{16y^2}{9x^{10}}$
74. $\frac{25a^6b^4c^{12}}{4}$
75. $\frac{n^3}{8m^3}$
76. $27b^6$
77. $\frac{4x^{10}}{y^{12}}$
78. $\frac{2a^2}{b^3} = \frac{2a^2}{1} \times \frac{1}{b^3}$ and $\frac{1}{b^3} = b^{-3}$
79. $\frac{12x^3}{y} = \frac{12x^3}{1} \times \frac{1}{y}$ and $\frac{1}{y} = y^{-1}$
80. $\frac{3a^2}{b^5}$
81. $\frac{3a^2}{b^5}$
82. $\frac{1}{8x^3y^3}$
83. $\frac{3a^2}{b^5}$
84. $\frac{a^2}{b^3}$
85. $2x^5y^5$
86. $\frac{3a^2}{b^3c^5}$
87. $\frac{y^6z^2}{x^8}$
88. $\frac{9}{2x^7y^{11}}$
89. $\frac{1}{8x^3y^3}$
90. $\frac{4}{a^{15}b^9}$
91. $\frac{2}{m^2n}$
92. Remember that a negative exponent can be evaluated by reciprocating the base, therefore expressions like a^{-3} become $\frac{1}{a^3}$. Notice the exponent became positive.
93. $\frac{4y^{12}}{9x^8}$
94. $\frac{27b^3}{8a^3}$
95. $\frac{1}{8x^9y^6}$
96. $\frac{4x^6}{3y^7}$
97. $\frac{1}{2}$
98. $\frac{1}{3}$
99. $x^{\frac{1}{n}} = \sqrt[n]{x}$
100. Possible answer:
    $\sqrt[4]{3} \times \sqrt[4]{3} \times \sqrt[4]{3} \times \sqrt[4]{3} = 3$
    $3^{\frac{1}{4}} \times 3^{\frac{1}{4}} \times 3^{\frac{1}{4}} \times 3^{\frac{1}{4}} = 3$
    $\therefore \sqrt[4]{3} = 3^{\frac{1}{4}}$
101. 7
102. -4
103. $no\ real\ number$
104. 4
105. $\frac{1}{3}$
106. $\frac{1}{2}$
107. 10
108. $2x$
109. $\frac{1}{3x^2}$
110. $\sqrt{7}$
111. $\sqrt[3]{3x}$
112. $\sqrt[5]{4}$
113. $\frac{1}{\sqrt[5]{4}}$
114. $-\sqrt[3]{64}$
115. $\frac{1}{\sqrt[3]{64}}$
116. $13^{\frac{1}{2}}$
117. $-3x^{\frac{1}{2}}$
118. $(2y)^{\frac{1}{2}}$
119. $4^{\frac{1}{4}}$
120. $4^{\frac{1}{7}}$
121. $(3x)^{-\frac{1}{5}}$
122. $27^{\frac{2}{3}} = \left(27^{\frac{1}{3}}\right)^2$
    $27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$
    $27^{\frac{2}{3}} = (3)^2$
    $27^{\frac{2}{3}} = 9$
123. $\sqrt[5]{4^2}$ or $\left(\sqrt[5]{4}\right)^2$
124. $\sqrt[5]{4^3}$ or $\left(\sqrt[5]{4}\right)^3$
125. $\sqrt[5]{4^4}$ or $\left(\sqrt[5]{4}\right)^4$
126. $\frac{1}{\sqrt[5]{4^2}}$ or $\frac{1}{\left(\sqrt[5]{4}\right)^2}$
127. $\frac{1}{\sqrt[5]{4^3}}$ or $\frac{1}{\left(\sqrt[5]{4}\right)^3}$
128. $\frac{1}{\sqrt[5]{4^4}}$ or $\frac{1}{\left(\sqrt[5]{4}\right)^4}$
129. $\sqrt{4} = 2$
130. $\sqrt[3]{125} = 5$
131. $\left(\sqrt[3]{8}\right)^2 = 4$
132. $\left(\sqrt[4]{81}\right)^3 = 27$
133. $\left(\sqrt{4}\right)^3 = 8$
134. $\frac{1}{\left(\sqrt[4]{16}\right)^3} = \frac{1}{8}$
135. $\frac{1}{\left(\sqrt[3]{-27}\right)^2} = \frac{1}{9}$
136. $\frac{1}{\left(\sqrt[3]{-8}\right)^5} = -\frac{1}{32}$
137. $9^{\frac{5}{2}} = \left(\sqrt{9}\right)^5 = 243$
138. 1
139. $\frac{1000}{27}$
140. $\frac{4}{9}$
141. $7^{\frac{1}{2}}$
142. $34^{\frac{1}{3}}$
143. $(-11)^{\frac{1}{3}}$
144. $a^{\frac{2}{5}}$
145. $6^{\frac{4}{3}}$
146. $x^{\frac{2}{3}}$
147. $6^{\frac{3}{5}}$
148. $(2x)^{\frac{5}{4}}$
149. $a^{-\frac{1}{3}}$
150. $x^{-\frac{4}{5}}$
151. $x^{-\frac{3}{4}}$
152. $2^{\frac{1}{3}}b$
153. $no\ real\ solution$
154. 1000
155. $\frac{9}{4}$
156. 3
157. -3
158. 4
159. a)-16 b) 16
160. 4
161. $no\ real\ solution$
162. 5
163. 4
164. 3
165. 0.32
166. 1.98
167. 0.55
168. $x^{\frac{11}{6}}$
169. $Answered\ on\ page.$
170. $x^{\frac{17}{12}}$
171. $x^{\frac{19}{15}}$
172. $4\ cm^2$
173. $5^{\frac{16}{15}}$ cm² $\cong$ 5.57 cm²
174. $\frac{7}{2}\ or\ 3.5\ cm^2$
175. $x^{-\frac{46}{15}}$ or $\frac{1}{x^{\frac{46}{15}}}$
176. $x^{\frac{17}{6}}$
177. c
178. c
179. $x^{-\frac{26}{15}} = \frac{1}{x^{\frac{26}{15}}}$
180. $a^{-\frac{29}{6}} = \frac{1}{a^{\frac{29}{6}}}$
181. $a^{\frac{1}{18}}$
182. $x^{\frac{1}{60}}$
183. F
184. C
185. A
186. E
187. C
188. D
189. D
190. C,D
191. B
192. $\frac{3}{26}$
193. $\frac{41}{5}$