Lesson 1 (pages 1-7)

September 10, 2017 1:39 PM

Math 10

Unit 1: Real Numbers and Radicals

Lesson 1: pages 1-7

Ms. A

Sept 12/17

REAL NUMBERS ($\mathbb{R}$)

(can be placed on a number line)

RATIONAL NUMBERS ($\mathbb{Q}$)

(CAN BE WRITTEN as fraction)

- Decimals DO terminate or repeat

ex: 7, $3.\overline{6}$, 5, $\frac{1}{2}$

Three Subsets:

⊕⊖0 ① INTEGERS ($\mathbb{Z}$) $\{\ldots, -3, -2, -1, 0, 1, 2, \ldots\}$

⊕ 0 ② WHOLE #S (W) $\{0, 1, 2, 3, \ldots\}$

⊕ ③ NATURAL #S (N) $\{1, 2, 3, \ldots\}$

IRRATIONAL NUMBERS ($\overline{\mathbb{Q}}$) "not"

(CANNOT BE WRITTEN as fraction)

- Decimals DO NOT terminate or repeat

ex: π, $\sqrt{2}$, $3.62489\ldots$

$-\frac{5}{8}$ Q, R

8 N, W, Z, Q, R

Example: Place the following numbers on the number line below:

A	B	C	D	E	F	G	H
$\frac{6}{2}$ Q	$-5.\overline{6}$	$\sqrt{20}$ $\overline{\mathbb{Q}}$	$\frac{0}{12}$	$10.3\overline{25}$	$\sqrt[4]{4}$ $\overline{\mathbb{Q}}$	$\sqrt[3]{4913}$ Q	3π $\overline{\mathbb{Q}}$ "π" button
3	Q	4.47...	Q	Q	1.414...	17	9.424...

Practice Work: pages 4-7 (including 7)

Key Terms

Term	Definition	Example
Real Number (R)	All numbers that can be placed on a number line	$1, 2.\overline{5}, \sqrt{2}$
Rational Number (Q)	Numbers that can be written in a fraction, w/ a decimal that stops or repeats	$5, 2.\overline{13}, \frac{1}{2}$
Irrational Number ($\bar{Q}$)	# cannot be written as fraction, decimal does NOT stop or repeat.	$\sqrt{2}, \pi, \sqrt{3}$
Integer (Z)	All positive & negative #s and zero	-2, -1, 0, 1, 2
Whole Number (W)	All positive numbers and zero (no decimal)	0, 1, 2, 3
Natural Number (N)	All positive numbers but NOT zero (no decimal)	1, 2, 3
Factor	Numbers you can multiply tog. to get another #	a factor of 6 = 2
Factor Tree	A method to obtain the prime factors of a number using a tree shape/form.	$4 = 2 \times 2$ 2 2 $4 = 2^2$
Prime Number	A # only divisible by 1 and itself.	27, 3, 11
Prime Factorization	The act of writing a number (or an expression) as a product of PRIME #s	$24 = 3 \times 2^3$ 6 4 3 2 2 2 ☆ $\boxed{24 = 3 \times 2^3}$ ☆
GCF	"Greatest common factor" = the largest # that divides evenly into 2 or more #s	GCF of 20 and 16 = 4
Multiple	The result of multiplying a # by 1, 2, 3, 4, ...	First 3 multiples of 8: 8, 16, 24
LCM	"Least Common Multiple" the smallest multiple shared between 2 or more #s	LCM of 4 and 5 = 20
Radical	Name given to square roots, cube roots, quadratic roots, etc.	$\sqrt{36}, \sqrt[3]{49}$
Index	Represents what root the radical is.	index $\sqrt[n]{x}$
Root Square root Cube root	☆ Finding the root, square root, cube roots means what number will multiply itself 2 or 3 times to get/equal the designated #	S.R. = $\sqrt[2]{25} = 5 \times 5 \rightarrow \boxed{5}$ C.R. = $\sqrt[3]{343} = 7 \times 7 \times 7 \rightarrow \boxed{7}$
Power	An expression made up of an exponent & base	base 3^4 - exponent } power
Entire Radical	A radical where all number(s) are underneath the radical sign	$\sqrt{5}$
Mixed Radical	A radical with an integer outside of the radical sign (left).	$2\sqrt{15}$

☆ Tells you what kind of root you are looking for.

☆ A number made up of a rational number and an irrational #.

The Real Number System

Real numbers are the set of numbers that we can place on the number line.

Real numbers may be positive, negative, decimals that repeat, decimals that stop, decimals that don't repeat or stop, fractions, square roots, cube roots, other roots. Most numbers you encounter in high school math will be real numbers.

The square root of a negative number is an example of a number that does not belong to the Real Numbers.

There are 5 subsets we will consider.

Real Numbers

Rational Numbers (Q)

Numbers that can be written in the form $\frac{m}{n}$ where m and n are both integers and n is not 0.

Rational numbers will be terminating or repeating decimals.

Eg. 5, -2. 3, $\frac{4}{3}$, $2\frac{3}{8}$

Natural (N)	Whole (W)	Integers (Z)
{1, 2, 3,...}	{0, 1, 2, 3,...}	{...,-3,-2,-1, 0, 1, 2, 3,...}

Irrational Numbers ($\bar{Q}$)

Cannot be written as $\frac{m}{n}$.
Decimals will not repeat, will not terminate.

Eg. $\sqrt{3}$, $\sqrt{7}$, π, 53.123423656787659...

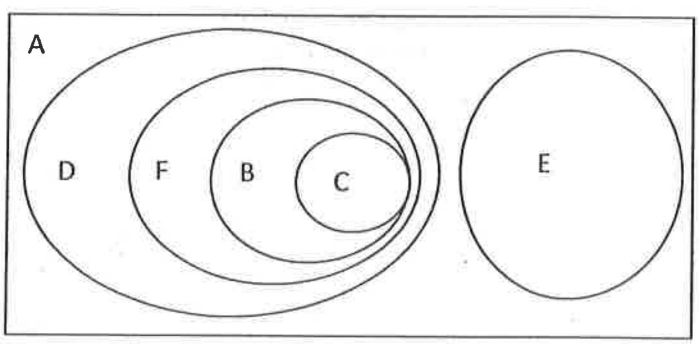
Name all of the sets to which each of the following belong?

1. 8 Q, Z, W, N	2. $\frac{4}{5}$ Q	3. $\frac{15}{5} = 3$ Q, Z, W, N
4. $\sqrt{7}$ $\bar{Q}$	5. $\sqrt{0.5}$ $\bar{Q}$	6. $12.3\bar{4}$ Q
7. -17 Q, Z	8. $-\left(\frac{2}{3}\right)^3 = -\frac{8}{27}$ Q	9. $2.7328769564923\ldots$ $\bar{Q}$

Write each of the following Real Numbers in decimal form. Round to the nearest thousandth if necessary. Label each as Rational or Irrational.

10. $\frac{2}{9}$ Q $0.22\bar{2}$	11. $-3\frac{3}{7}$ Q -3.429	12. $\sqrt{8}$ $\bar{Q}$ 2.828
13. $\sqrt[3]{9}$ $\bar{Q}$ 2.080	14. $\sqrt[4]{256}$ Q 4	15. $\sqrt[5]{25}$ $\bar{Q}$ 1.904

16. Fill in the following diagram illustrating the relationship among the subsets of the real number system. (Use descriptions on previous page)



A Real Numbers
B Whole Numbers
C Natural Numbers
D Rational Numbers
E Irrational Numbers
F Integers

17. Place the following numbers into the appropriate set, rational or irrational.

$5, \quad \sqrt{2}, \quad 2.\overline{13}, \quad \sqrt{16}, \quad \frac{1}{2}, \quad 5.1367845\ldots, \quad \frac{\sqrt{7}}{2}, \quad \sqrt[3]{8}, \quad , \quad \sqrt[3]{25}$

Rational Numbers

5 $\quad 2.\overline{13}$ $\quad \sqrt{16}$ $\quad \sqrt[3]{8}$ $\quad \frac{1}{2}$

Irrational Numbers

$\sqrt{2}$ $\quad \frac{\sqrt{7}}{2}$ $\quad 5.1367845\ldots$ $\quad \sqrt[3]{25}$

★ 18. Which of the following is a rational number?

a. $\frac{\sqrt{3}}{2}$
b. $\sqrt[3]{16}$
c. $\frac{5}{7}$
d. 12.356528349875 ...

19. Which of the following is an irrational number?

a. $\sqrt{\frac{16}{9}}$
b. π
c. $\frac{3}{8}$
d. $\sqrt[3]{27}$

20. To what sets of numbers does -4 belong?

a. natural and whole
b. irrational and real
c. integer and whole
d. rational and integer

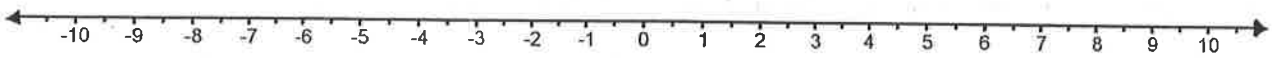
21. To what sets of numbers does $-\frac{4}{3}$ belong?

a. natural and whole
b. irrational and real
c. integer and whole
d. rational and real

Your notes here...

✓

The Real Number Line



All real numbers can be placed on the number line. We could never list them all, but they all have a place.

Estimation:

It is important to be able to estimate the value of an irrational number. It is one tool that allows us to check the validity of our answers.

Without using a calculator, estimate the value of each of the following irrational numbers.
Show your steps!

22. $\sqrt{7}$

Find the perfect squares on either side of 7.
→4 and 9
Square root 4 = 2
Square root 9 = 3

Guess & Check:
2.6 x 2.6 =6.76
2.7 x 2.7 = 7.29
∴ $\sqrt{7}$ is about 2.6

23. $\sqrt{14}$

square root 3 = 9
square root 4 = 16

$\boxed{\sqrt{14} \approx 3.7}$

3.7 × 3.7 = 13.69

24. $\sqrt{75}$

square root 8 = 64
square root 9 = 81

$\boxed{\sqrt{75} \approx 8.7}$

8.6 × 8.6 = 73.96
8.7 × 8.7 = 75.69

25. $\sqrt[3]{11}$

cube root 2 = 8
cube root 3 = 27

$\boxed{\sqrt[3]{11} \approx 2.2}$

2.2 × 2.2 × 2.2 = 10.648

26. $\sqrt[3]{90}$

cube root 64 = 4
cube root 125 = 5

$\sqrt[3]{90} \approx 4.5$

4.5 × 4.5 × 4.5 = 91.125

27. $\sqrt[3]{150}$

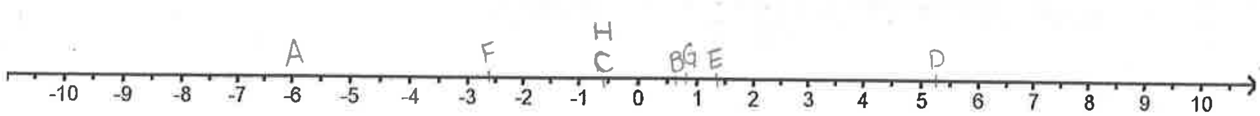
cube root 125 = 5
cube root 216 = 6

$\sqrt[3]{150} \approx 5.3$

5.3 × 5.3 × 5.3 = 148.877

28. Place the corresponding letter of the following Real Numbers on the number line below.

A. -6 B. $\frac{2}{3}$ C. $-\frac{2}{3}$ D. $5\frac{1}{4}$ E. $\sqrt{2}$ F. $-\sqrt{7}$ G. $\frac{\sqrt{3}}{2}$ H. $-\frac{\sqrt{4}}{3}$



Lesson 2 (pages 8-11)

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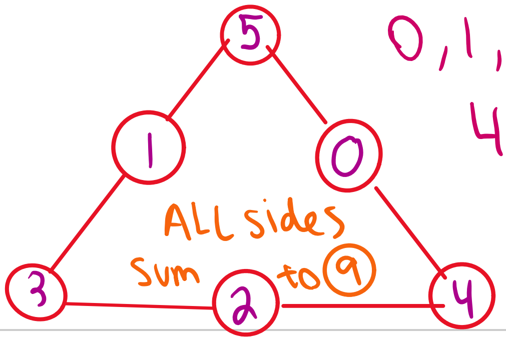


0, 1, 2, 3, 4, 5

① pgs 4-7
② M9 Review

Math 10 **Unit 1: Real Numbers and Radicals** Ms. A

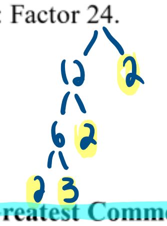
Lesson 2: pages 8-11 Sept. 14/17

A. **Factor (noun):** divides evenly

Example: List the factors of 24. 1, 2, 3, 4, 6, 8, 12, 24

B. **Factor (verb):** write as a product (of prime #s)

Example: Factor 24.



$24 = 2 \times 3 \times 2 \times 2$

$= 2^3 \times 3$

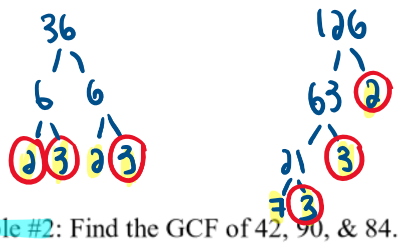
three 2s

C. **Greatest Common Factor (GCF)** [think: largest into all]

TO FIND GCF: List the primes that are in both numbers and multiply them.

① Draw tree diagrams
② Circle primes common to ALL trees
③ Multiply circled #s together

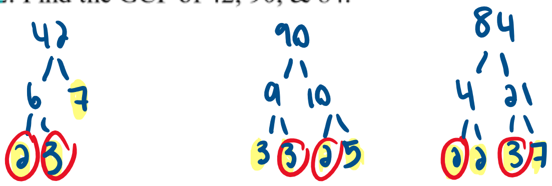
Example #1: Find the GCF of 36 & 126.



$GCF = 2 \times 3 \times 3$

$= \boxed{18}$

Example #2: Find the GCF of 42, 90, & 84.



$GCF = 2 \times 3$

$= \boxed{6}$

D. **Lowest Common Multiple (LCM)**

Example #1: List the first 6 multiples of 20: 20, 40, 60, 80, 100, 120

24: 24, 48, 72, 96, 120

LCM of 20 & 24 is the lowest number that they both divide evenly into.

TO FIND LCM: List the largest power of each prime number & multiply them.

Example #2: Find the LCM of 45 & 60.

$45 = 3 \times 3 \times 5 = 2^0 \times 3^2 \times 5^1$

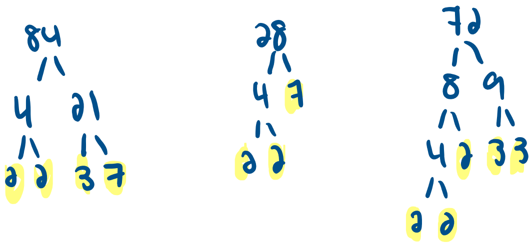
$60 = 2 \times 3 \times 2 \times 5 = 2^2 \times 3^1 \times 5^1$

same exponent so doesn't matter

$LCM = 2^2 \times 3^2 \times 5^1$

$= \boxed{180}$

Example #3: Find the LCM of 84, 28, & 72.



$84 = 2^2 \times 3 \times 7$

$28 = 7 \times 2^2$

$72 = 2^3 \times 3^2$

$LCM = 2^3 \times 3^2 \times 7$

$= \boxed{504}$

PW: pgs. 8-11

Factors, Factoring, and the Greatest Common Factor

We often need to find factors and multiples of integers and whole numbers to perform other operations.

For example, we will need to find common multiples to add or subtract fractions.
For example, we will need to find common factors to reduce fractions.

Factor: (NOUN)

Factors of 20 are {1,2,4,5,10,20} because 20 can be evenly divided by each of these numbers.
Factors of 36 are {1,2,3,4,6,9,12,18,36}
Factors of 198 are { 1,2,3,6,9,11,18,22,33,66,99,198}

> Use division to find factors of a number. Guess and check is a valuable strategy for numbers you are unsure of.

To Factor: (VERB) The act of writing a number (or an expression) as a product.

To factor the number 20 we could write 2×10 or 4×5 or 1×20 or $2 \times 2 \times 5$ or $2^2 \times 5$.
When asked to factor a number it is most commonly accepted to write as a product of prime factors.
Use powers where appropriate.

Eg. $20 = 2^2 \times 5$ Eg. $36 = 2^2 \times 3^2$ Eg. $198 = 2 \times 3^2 \times 11$

> Prime:
> When a number is only divisible by 1 and itself, it is considered a prime number.

A factor tree can help you "factor" a number.

```
        36
      /    \
    4        9
   / \      / \
  2   2    3   3
```

$\therefore\ 36 = 2^2 \times 3^2$

Write each of the following numbers as a product of their prime factors.

29. 100

```
  100
  / \
 50  2
 / \
25  2
/ \
5  5
```

$100 = 5^2 \times 2^2$

30. 120

```
  120
  / \
 60  2
 / \
30  2
/ \
15  2
/ \
3  5
```

$120 = 5 \times 3 \times 2^3$

31. 250

```
  250
  / \
125  2
/ | \
5 5  5
```

$250 = 5^3 \times 2$

Write each of the following numbers as a product of their prime factors.

32. 324

324 → 162, 2; 162 → 81, 2; 81 → 27, 3; 27 → 3, 9; 9 → 3, 3

$324 = 3^4 \times 2^2$

33. 1200

1200 → 400, 3; 400 → 200, 2; 200 → 100, 2; 100 → 50, 2; 50 → 25, 2; 25 → 5, 5

$1200 = 5^2 \times 3 \times 2^4$

34. 800

800 → 400, 2; 400 → 200, 2; 200 → 100, 2; 100 → 50, 2; 50 → 25, 2; 25 → 5, 5

$800 = 5^2 \times 2^5$

Greatest Common Factor

At times it is important to find the largest number that divides evenly into two or more numbers…the **Greatest Common Factor (GCF).**

Challenge:

35. Find the GCF of 36 and 198.

GCF = 18

$36 \div 2 = 18$

$198 \div 2 = 18$

36 → 2, 18; 18 → 9, 2; 9 → 3, 3

198 → 2, 99; 99 → 9, 11; 9 → 3, 3

$36 = 3^2 \times 2^2$

$198 = 11 \times 3^2 \times 2$

$3^2 \times 2 = \boxed{18}$

Challenge:

36. Find the GCF of 80, 96 and 160.

80 = 40, 20, 16

GCF = 16

$\boxed{2^4 = 16}$

$80 = 5 \times 2^4$

80 → 8, 10; 8 → 4, 2; 4 → 2, 2; 10 → 2, 5

$96 = 3 \times 2^5$

96 → 2, 48; 48 → 2, 24; 24 → 2, 12; 12 → 6, 2; 6 → 2, 3

$160 = 5 \times 2^5$

160 → 8, 20; 8 → 4, 2; 4 → 2, 2; 20 → 10, 2; 10 → 5, 2

Some Notes…

Find the GCF of each set of numbers.

37. 36, 198

$36 = 2^2 \times 3^2$

$198 = 2 \times 3^2 \times 11$

Prime factors in common are 2 and 3^2.

GCF is $2 \times 3^2 = 18$

(Alternate method: List all factors...choose largest in both lists.)

38. 98, 28

98 → 2, 49 → 7, 7

28 → 2, 14 → 2, 7

$98 = 7^2 \times 2$

$28 = 7 \times 2^2$

$7 \times 2 = 14$

GCF = 14

39. 80, 96, 160

$80 = 2^4 \times 5$

$96 = 2^5 \times 3$

$160 = 2^5 \times 5$

Prime factors in common are 2^4.

GCF is $2^4 = 16$

(Alternate method: List all factors...choose largest in both lists.)

40. 24, 108

$108 = 3^3 \times 2^2$

108 → 54, 2 → 27, 2 → 3, 9 → 3, 3

$24 = 3 \times 2^3$

24 → 12, 2 → 2, 6 → 3, 2

GCF $= 3 \times 2^2 = \boxed{12}$

41. 126, 189, 735, 1470

$126 = 7 \times 3^2 \times 2$

126 → 2, 63 → 7, 9 → 3, 3

$189 = 7 \times 3^3$

189 → 3, 63 → 7, 9 → 3, 3

$735 = 7^2 \times 5 \times 3$

735 → 49, 15 → 7, 7, 3, 5

$1470 = 7^2 \times 5 \times 3 \times 2$

1470 → 2, 735 → 5, 147 → 3, 49 → 7, 7

$7 \times 3 = 21$

$\boxed{GCF = 21}$

42. 504, 1050, 1386

$504 = 7 \times 3^2 \times 2^3$

504 → 2, 252 → 126, 2 → 2, 63 → 9, 7 → 3, 3

$1050 = 7 \times 5^2 \times 3 \times 2$

1050 → 105, 10 → 5, 21 → 7, 3; 10 → 2, 5

$1386 = 11 \times 7 \times 3^2 \times 2$

1386 → 3, 462 → 231, 2 → 3, 77 → 7, 11

$7 \times 3 \times 2 = 42$

$\boxed{GCF = 42}$

Multiples and Least Common Multiple

Challenge

43. Find the first seven multiples of 8.

8, 16, 24, 32, 40, 48, 56

Challenge

44. Find the least common multiple of 8 and 28.

28, 56

LCM = $\boxed{56}$

$8 = 2^3$

8 → 2, 4 → 2, 2

$28 = 2^2 \times 7$

28 → 2, 14 → 2, 7

LCM $= 2^3 \times 7 = \boxed{56}$

Multiples of a number

Multiples of a number are found by multiplying that number by {1,2,3,4,5,...}.

Find the first five multiples of each of the following numbers.

45. 8

8, 16, 24, 32, 40, 48

46. 28

28, 56, 84, 112, 140

47. 12

12, 24, 36, 48, 60

Find the least common multiple of each of the following sets of numbers.

48. 8,28

$8 = 2^3$
$28 = 2^2 \times 7$

Look for largest power of each prime factor...

In this case, 2^3 and 7.
LCM $= 2^3 \times 7$
LCM $= 56$

49. 72,90

$\boxed{3^2 \times 2^3}$

72 → 36, 2 → 2, 18 → 2, 9 → 3, 3

$90 = \boxed{5 \times 3^2 \times 2}$

90 → 30, 3 → 10, 3 → 5, 2

$3^2 \times 2^3 \times 5 = 360$

$\boxed{LCM = 360}$

50. 25, 220

$\boxed{1110}$

220 → 110, 2 → 55, 2 → 11, 5

$11 \times 5 \times 2^2$

25 → 5, 5

5^2

$11 \times 5^2 \times 2^2 = 1100$

LCM = 1100

51. 8, 12, 22

$8 = 2^3$ | $12 = 3 \times 2^2$ | $22 = 11 \times 2$

$11 \times 3 \times 2^3 = \boxed{LCM = 264}$

52. 4, 15, 25

$4 = 2^2$ | $15 = 5 \times 3$ | $25 = 5^2$

$5^2 \times 3 \times 2^2 = \boxed{LCM = 300}$

53. 18, 20, 24, 36

$18 = 3^2 \times 2$ | $20 = 5 \times 2^2$ | $24 = 3 \times 2^3$ | $36 = 3^2 \times 2^2$

$5 \times 3^2 \times 2^3 = \boxed{LCM = 360}$

54. Use the least common multiple of 2, 6, and 8 to add:

$$\frac{3}{8}+\frac{5}{6}+\frac{1}{2}$$

$$\frac{9}{24}+\frac{20}{24}+\frac{12}{24}$$

$$\boxed{\frac{41}{24} \text{ or } 1\frac{17}{24}}$$

55. Use the least common multiple of 2, 5, and 7 to evaluate:

$$\frac{3}{5}-\frac{2}{7}+\frac{3}{2}$$

$$\frac{42}{70}-\frac{20}{70}+\frac{105}{70}$$

$$\boxed{\frac{127}{70} \text{ or } 1\frac{57}{70}}$$

56. Use the least common multiple of 3, 8, and 9 to evaluate:

$$\frac{7}{9}-\frac{1}{3}-\frac{1}{8}$$

$3 = 3$
$9 = 3^2$
$8 = 2^3$

$= 2^3 \times 3^2 = 72$

$$\frac{56}{72}-\frac{24}{72}-\frac{9}{72}$$

$$\boxed{\frac{23}{72}}$$

Math 10

Unit 1: Real Numbers and Radicals

Lesson 3: pages 12-17

MS. A
Sept 19/17

BEDMAS

1. $\sqrt{4+5} = \sqrt{9} = 3$

2. $\sqrt{2+2\times7} = \sqrt{2+14} = \sqrt{16} = 4$

3. $\sqrt{\frac{49}{81}} = \frac{\sqrt{49}}{\sqrt{81}} = \frac{7}{9}$

4. $\sqrt[2]{-576} =$ NO SOLUTION (Error)

if even #, can't root negative

$(24)^2 = 576$

$(-24)^2 = +576$

NO WAY TO GET NEGATIVE

5. $\sqrt[3]{-512} = -8$

if odd #, CAN root negative

$\sqrt[n]{x}$

radical sign

index

radicand

6. $\sqrt{2\cdot2\cdot3\cdot3\cdot11\cdot11\cdot11\cdot11} = \sqrt{527076}$

$= 726$

7. $\sqrt{25x^2} = \sqrt{25}\cdot\cancel{\sqrt[2]{x^2}}$

$= \boxed{5x}$

8. $\sqrt{100x^6} = \sqrt{100}\cdot\sqrt[2]{x^6}$

$= \sqrt{100}\cdot\cancel{\sqrt{x^2}}\cdot\cancel{\sqrt{x^2}}\cdot\cancel{\sqrt{x^2}}$

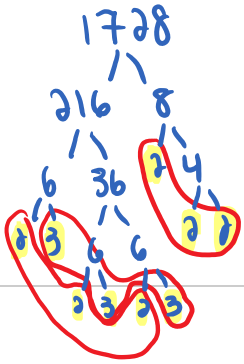
$= 10x^3$

9. $\sqrt[3]{27x^6} =$

$\sqrt[3]{27} \times \sqrt[3]{x^6}$

$= \boxed{3x^2}$

10. Use the prime factorization of 1728 to determine if it is a perfect cube. If so, determine $\sqrt[3]{1728}$.



perfect cube!

↳ ALL factors go in a group of 3

$\sqrt[3]{1728} = \boxed{12}$

1

pages 12-17

Radicals:

Radicals are the name given to square roots, cube roots, quartic roots, etc.

$$\sqrt[n]{x}$$

The parts of a radical:

Radical sign	$\sqrt{\ }$	(Operations under the radical are evaluated as if inside brackets.)
Index	n	(tells us what type of root we are looking for, if blank…index is 2)
Radicand	x	(the number to be "rooted")

A radical and its simplified form are equivalent expressions.

Square Roots

Square root of 81 looks like $\sqrt{81}$. It means to find what value must be multiplied by itself twice to obtain the number we began with.

$\sqrt{81}$ *we think* … $81 = 9 \times 9 \rightarrow \sqrt{81} = 9$ $\sqrt{a^4}$ we think …$a^4 = a^2 \times a^2 \rightarrow \sqrt{a^4} = a^2$

PERFECT SQUARE NUMBER: A number that can be written as a product of two equal factors.

$81 = 9 \times 9$ } 81 is a perfect square. Its square root is 9.

First 15 Perfect Square Numbers:
1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, …

Your notes here…

$i = \sqrt{-1}$

Operations inside a $\sqrt{}$ must be considered as if they were inside brackets...do them

Evaluate the following.

57. $\sqrt{49}$

7

58. $\sqrt{-25}$

$\sqrt{(25 \times -1)} = \sqrt{25} \times \sqrt{-1}$

$5 \times \sqrt{-1} = 5i$

59. $-(\sqrt{36})$

-6

60. Finish the statement:

I know that $\sqrt{16} = 4$ because...

$4 \times 4 = 16$

61. Finish the statement:

I know that $\sqrt{\frac{64}{81}} = \frac{8}{9}$ because...

$\frac{\sqrt{64}}{\sqrt{81}}$, $8 \times 8 = 64$, $9 \times 9 = 81$

62. Finish the statement:

I know that $\sqrt{-36} \neq -6$ because...

$-6 \times -6 = +36$

63. $\sqrt{121}$

11

64. $\sqrt{45 - 20}$

$\sqrt{25} = 5$

65. $2\sqrt{40 - (-9)}$

$2\sqrt{49}$

$2(7) = 14$

66. Simplify. $\sqrt{x^2}$

x

67. Simplify. $\sqrt{4x^2}$

$2x$

$\sqrt{4 \times x^2} =$

$\sqrt{4} \times \sqrt{x^2}$

$2x$

68. Simplify. $\sqrt{16x^4}$

$4x^2$

Cube Roots:

PERFECT CUBE NUMBER: A number that can be written as a product of three equal factors.

Cube root of 64 looks like $\sqrt[3]{64}$.

The index is 3. So we need to multiply our answer by itself 3 times to obtain 64. $4 \times 4 \times 4 = 64$

First 10 Perfect Cube Numbers: 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, …

EX. $125 = 5^3$
5 25
5 5

Evaluate or simplify the following.

69. $\sqrt[3]{8}$
Explain what the small 3 in this problem means.
It's asking for the cube root = the answer will multiply itself 3 times to obtain 8 → (2).

70. $\sqrt[3]{8} = 2$

71. How could a factor tree be used to help find $\sqrt[3]{125}$?
Do a factor tree for 125 and their should be $5 \times 5 \times 5$

72. Evaluate $\sqrt[3]{125}$. $= 5$

73. $\sqrt[3]{-27} = -3$

74. $\sqrt[3]{1000} = 10$

75. $\sqrt[3]{-8} = -2$

EX. $27 = 3^2$
3 9
3 3

76. Show how prime factorization can be used to evaluate $\sqrt[3]{27}$.
Find the prime factors of 27 and there should be $3 \times 3 \times 3$.

77. $\sqrt[3]{343} = 7$

78. $\sqrt[3]{-216} = -6$

79. $\sqrt[3]{27} \times \sqrt{20 \times 5}$
$3 \times 10 = \boxed{30}$

80. $\sqrt[3]{64} \times \sqrt{45 - 20}$
$4 \times 5 = \boxed{20}$

81. $\sqrt[3]{-125} = -5$

82. $\sqrt[4]{a^{12}} = a^3$

83. $\sqrt[3]{a^6} = a^2$

84. $\sqrt[3]{8x^3}$
$\sqrt[3]{8}$
$\sqrt[3]{8} \times \sqrt[3]{x^3}$
$2 \times x^1 = \boxed{2x}$

Other Roots.

85. How does $\sqrt[6]{729}$ differ from $\sqrt[3]{729}$? Explain, do not simply evaluate.

$\sqrt[6]{729}$ = is looking for a product of 6 equal numbers to equal 729
$\sqrt[3]{729}$ is looking for a product of 3 equal #s to equal 729

86. Evaluate if possible. $\sqrt[4]{16} = 2$

87. Evaluate if possible. $\sqrt[4]{-16}. =$

$\sqrt[4]{-1} \times \sqrt[4]{16}$
$i \times 2$
NOT POSSIBLE
$\boxed{2i}$

88. Evaluate if possible. $\sqrt[5]{32}$.

2

89. Evaluate if possible. $\sqrt[4]{81}$.

3

90. Evaluate if possible. $\sqrt[6]{64}$.

2

91. Evaluate if possible. $\sqrt[3]{24-16}$.

$\sqrt[3]{8} = \boxed{2}$

92. Evaluate if possible. $\sqrt[4]{2(32-24)}$.

$\to \sqrt[4]{2(8)}$
$\to \sqrt[4]{16}$
$= \boxed{2}$

93. Evaluate if possible. $\sqrt[3]{4(5-3)}$.

$\to \sqrt[3]{4(2)}$
$\to \sqrt[3]{8}$
$= 2$

Using a calculator, evaluate the following to two decimal places.

94. $\sqrt[3]{27} - \sqrt[5]{27}$

$3 - 1.93$
$= \boxed{1.07}$

95. $2\sqrt{10} + \sqrt[4]{64}$

$6.32 + 2.83$
$= \boxed{9.15}$

96. $\sqrt[5]{-32} - \sqrt[4]{16}$

$-2 - 2$
$= \boxed{-4.00}$

97. $19 - \sqrt[3]{18}$

$19 - 2.62$
$= \boxed{16.38}$

98. $\frac{\sqrt{12}-\sqrt[3]{7}}{2}$

$\to \frac{3.46 - 1.91}{2}$
$\to \frac{1.55}{2} = \boxed{0.78}$

99. $\frac{\sqrt[3]{9}-\sqrt[3]{27}}{3}$

$\to \frac{2.08 - 3}{3}$
$\frac{-0.92}{3} = \boxed{-0.31}$

100. Describe the difference between radicals that are rational numbers and those that are irrational numbers.

rational $\sqrt{16}$ irrational $\sqrt{13}$

All radicals that equal a rational # are perfect squares, cubes, etc. All radicals that equal irrational #s do not.

STEPS:
1. are last digits a perfect square?
2. count the number of decimal places
3. Divide # of decimals by index

Evaluate or simplify the following.

101. $\sqrt[3]{125}$

$125 = 5^3$

5 25

5 5

$\boxed{5}$

102. $\sqrt{2(15-(-3))}$

$\sqrt{2(18)}$

$\sqrt{36}$

$= \boxed{6}$

103. $\sqrt{\sqrt{16}}$

$\sqrt{\sqrt{16}}$

$\sqrt{4} = \boxed{2}$

104. $\sqrt{0.16}$

$\boxed{0.4}$

105. $\sqrt[3]{0.0001}$

$\boxed{0.01}$

106. $3\sqrt{25} - 4\sqrt[3]{8}$

$3(5) - 4(2)$

$15 - 8$

$= \boxed{7}$

107. $\sqrt{\frac{1}{4}}$

$\sqrt{0.25} =$

$\boxed{0.5} \rightarrow \frac{1}{2}$ ☆ ☆ ☆ ☆

108. $\sqrt{\frac{16}{49}}$

$\boxed{\frac{4}{7}}$

109. $\sqrt{\frac{100}{400}}$

$\frac{10}{20} = \boxed{\frac{1}{2}}$

110. $\sqrt[2]{a^4}$

$\cancel{\boxed{a^{0.5}}}$

$\sqrt[2]{a^4} = 4 \div 2 = 2$

$\boxed{a^2}$

111. $\sqrt[3]{-x^6}$

$\sqrt[3]{(-1)} \times \sqrt[3]{x^6}$

$-1 \times x^2$

$= \boxed{-x^2}$

112. $\sqrt[3]{8x^3}$

$\sqrt[3]{8} \times \sqrt[3]{x^3}$

$2 \times x$

$= \boxed{2x}$

square root and square cancel each other

Evaluate or simplify the following.

113. $\sqrt{5^2}$

$\sqrt{25} = \boxed{5}$

114. $(\sqrt{5})^2$

$\boxed{5}$

115. $-\sqrt{(-5)^2}$

$-\sqrt{25} = \boxed{-5}$

116. $(\sqrt{49} - \sqrt{64})^3$

$(7-8)^3$

$(-1)^3$

$= \boxed{-1}$

117. $\sqrt{\sqrt{16} + \sqrt{25}}$

$\sqrt{4+5} = \sqrt{9}$

$= \boxed{3}$

118. What would be the side length of a square with an area of 1.44 cm²?

$\boxed{1.44}$ $\sqrt{1.44} =$

$\boxed{l = 1.2\text{ cm}}$

119. $(\sqrt[4]{16})^3$

$(2)^3 = \boxed{8}$

120. $\sqrt[5]{-32}$

$\boxed{-2}$

121. $\sqrt[8]{256}$

$\boxed{2}$

122. Use the prime factors of 324 to determine if 324 is a perfect square. If so, find $\sqrt{324}$.

Answer:

$324 = 2^2 \times 3^4$ if fully factored

$\therefore \sqrt{324} = \sqrt{2 \times 2 \times 3^2 \times 3^2}$

$\therefore \sqrt{324} = \sqrt{(2 \times 3^2) \times (2 \times 3^2)}$

$\therefore \sqrt{324} = (2 \times 3^2)$

$\therefore \sqrt{324} = 18$

YES

123. Use the prime factors of 576 to determine if 576 is a perfect square. If so, find $\sqrt{576}$.

$576 = 3^2 \times 2^6$

576 → 3, 192 → 2, 96 → 2, 48 → 2, 24 → 3, 8 → 2, 4 → 2, 2

$\sqrt{576} = \sqrt{3^2 \times 2^6}$

$\sqrt{576} = \sqrt{(3 \times 2^3) \times (3 \times 2^3)}$

$= \sqrt{576} = (3 \times 2^3)$

$\boxed{= \sqrt{576} = 24}$ YES

124. Use the prime factors of 1728 to determine if it is a perfect cube. If so, find $\sqrt[3]{1728}$.

$1728 = 3^3 \times 2^6$

1728 → 4, 432; 4 → 2, 2; 432 → 4, 108; 4 → 2, 2; 108 → 2, 54; 54 → 2, 27; 27 → 3, 9; 9 → 3, 3

$\sqrt[3]{1728} = \sqrt[3]{3^3 \times 2^6}$

$\sqrt[3]{1728} = \sqrt[3]{(3 \times 2^2) \times (3 \times 2^2) \times (3 \times 2^2)}$

$\sqrt[3]{1728} = 3 \times 2^2$

$\sqrt[3]{1728} = \boxed{12}$

$\boxed{\text{YES}}$

125. Use the prime factors of 5832 to determine if it is a perfect cube. If so, find $\sqrt[3]{5832}$.

5832 → 2, 2916 → 2, 1458 → 2, 729 → 3, 243 → 3, 81 → 9, 9 → 3, 3, 3, 3

$\sqrt[3]{5832} = \sqrt[3]{3^6 \times 2^3}$

$\sqrt[3]{5832} = \sqrt[3]{(3^2 \times 2) \times (3^2 \times 2) \times (3^2 \times 2)}$

$\sqrt[3]{5832} = 3^2 \times 2$

$= \sqrt[3]{5832} = \boxed{18}$ $\boxed{\text{YES}}$

Lesson 4 (pages 18-22)

September 17, 2017 10:59 AM

Test yourself! How are you doing so far? **Remember-- Mid-Point Quiz NEXT class!!**

1. List 520 as a product of primes.

520
52 10
2 26 2 5
2 13

$= 2 \times 2 \times 13 \times 2 \times 5$

$= 2^3 \times 5 \times 13$

2. Find the GCF of 108 and 120.

108
3 36
6 6
2 3 2 3

120
12 10
2 6 2 5
2 3

$GCF = 3 \times 2 \times 2$

$= \boxed{12}$

3. To which sets of numbers does -13 belong?

R real #s

Q rational #s

Z integers

4. Which perfect squares would be used to estimate $\sqrt{53}$?

49 & 64

5. Evaluate the following to the nearest thousandth:

$$\frac{\sqrt{45} - \sqrt[3]{18}}{2}$$

$= \boxed{2.044}$

R Q / $\bar{Q}$ N Z W

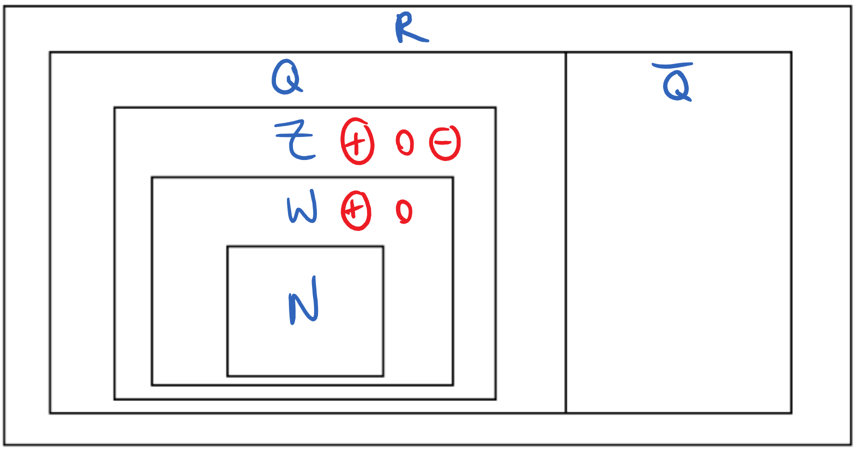
Math 10

Unit 1: Real Numbers and Radicals

Lesson 4: pages 18-22

Ms. A
Sept 21st

Part 1: Undefined Roots



What values of square roots are UNDEFINED? (ie: NO real solution)

NEGATIVE

What values of x make these roots undefined?

1. $\sqrt{x+4}$

$x+4 \geq 0$
$-4 \quad -4$
$\Rightarrow \boxed{x \geq -4}$

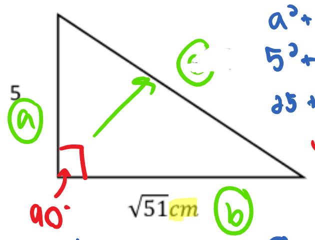
2. $\sqrt{10-5x}$

$10-\cancel{5x} \geq 0$
$+\cancel{5x} \quad +5x$
$\frac{10}{5} \geq \frac{5x}{5}$
$2 \geq x$ or $(x \leq 2)$

Part 2: Pythagoras ($a^2 + b^2 = c^2$) can only be used if a triangle has a 90° angle!

Calculate the perimeter of the following triangles.

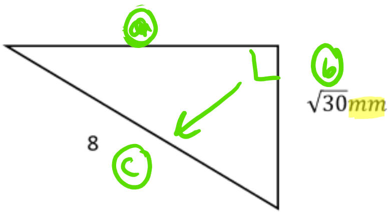
1.



$a^2+b^2=c^2$
$5^2+\cancel{\sqrt{51}}^2=c^2$
$25+51=c^2$
$\sqrt{76}=\sqrt{c^2}$
$c=\sqrt{76}cm$

Perimeter = ⓐ + ⓑ + ⓒ = $5+\sqrt{51}+\sqrt{76}$
$= \boxed{20.9 cm}$

2.



$a^2+b^2=c^2$
$a^2+\sqrt{30}^2=8^2$
$a^2+\cancel{30}=64$
$-\cancel{30} \quad -30$
$\sqrt{a^2}=\sqrt{34}$
$a=\sqrt{34}mm$

p = ⓐ + ⓑ + ⓒ
$=\sqrt{34}+\sqrt{30}+8$
$=\boxed{19.3 mm}$

Part 3: Squares and Cubes

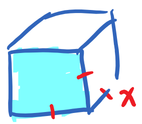
1. Is this a perfect square? $\sqrt[2]{2\cdot2\cdot3\cdot3\cdot5\cdot5\cdot3\cdot5}$

unpaired NOT a perfect square!

2. Is this a perfect cube? $\sqrt[3]{3\cdot7\cdot3\cdot7\cdot3\cdot7}$

YES!

3. The volume of a cube is $729cm^3$. Find the surface area of the cube.



$V = 729cm^3$

① $V = x\cdot x\cdot x$
$V = x^3$
$\sqrt[3]{729} = \sqrt[3]{x^3}$
$9 = x$

② SA: area of one face $= 9\cdot9 = 81$

SA of entire cube $= 6\cdot81$ (6 sides, area of one side)

$=\boxed{486 cm^2}$

126. An engineering student developed a formula to represent the maximum load, in tons, that a bridge could hold. The student used 1.7 as an approximation for $\sqrt{3}$ in the formula for his calculations. When the bridge was built and tested in a computer simulation, it collapsed. The student had predicted the bridge would hold almost three times as much.

The formula was:
$5000(140 - 80\sqrt{3})$

What weight did the student think the bridge would hold?

$5000(140-80(1.7))$
$= 5000(140-136) = 5000(4) = \boxed{20000 \text{ tons}}$

Calculate the weight the bridge would hold if he used $\sqrt{3}$ in his calculator instead.

$\boxed{7179.676973 \text{ tons}}$

127. For what values of x is $\sqrt{x-2}$ not defined?

$x \leq 1$ or $x < 2$

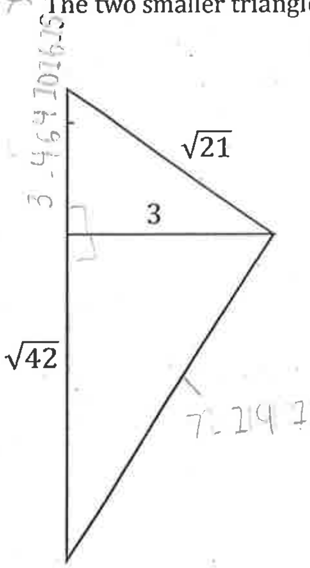
128. For what values of x is $\sqrt{x+3}$ not defined

$\boxed{x \leq -4}$ or $\boxed{x < -3}$

129. For what values of x is $\sqrt{5-x}$ not defined

$\boxed{x > 5}$

130. Calculate the perimeter to the nearest tenth. The two smaller triangles are right triangles.



3.46

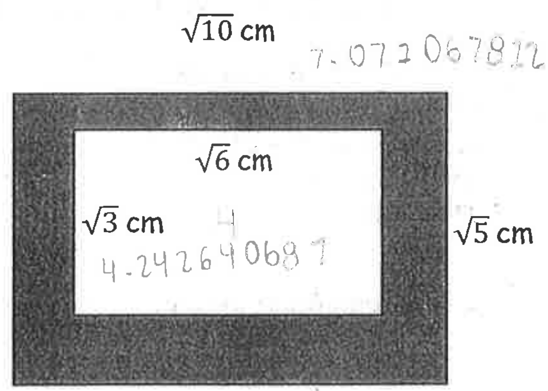
$3.464101615 + 6.480740698 + 7.1414284429 + 4.582575695$

~~P = 34.0 cm~~

P = 21.7 units

Calculate the area of the shaded region.

$\sqrt{50}$ $\sqrt{18}$



131. To the nearest tenth:

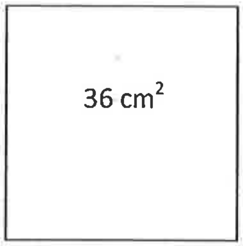
$7.071067812 - 4.242640687$ $\boxed{2.8 \text{ cm}^2}$

132. As an *expression* using radicals:
(you may need to come back to this one)

$\sqrt{10} \times \sqrt{5} - \sqrt{6} \times \sqrt{3}$
$\sqrt{50} - \sqrt{18}$
$\sqrt{25 \times 2} - \sqrt{2 \times 9}$
$5\sqrt{2} - 3\sqrt{2}$
$\boxed{2\sqrt{2} \text{ cm}^2}$

b/c square root is like finding the 2 sides lengths of a square (the perfect square = the area).

133. Consider the square below. Why might you think $\sqrt{\ }$ is called a square root?

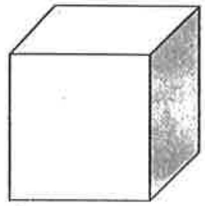


want to equal find 2 numbers that multiply together?

$\sqrt{\ }$ is called square root b/c it wants to know the 2 equals values of the sides of the square.

134. Consider the diagram below. Why do you think $\sqrt[3]{\ }$ is called a cube root?

b/c cube root is like finding 3 side lengths of a cube (perfect cube = the volume)



64 cm³

cube root is called cube root b/c the cube has 3 equal number that multiply and get that answer.

135. Find the side length of the square above.

$\sqrt{36} = \boxed{6\text{ cm}}$

136. Find the edge length of the cube above.

$\sqrt[3]{64} = \boxed{4\text{ cm}}$

137. Why do you think 81 is called a "perfect square" number?

Because 81 is the area of a square ($9 \times 9 \to$ no decimals)

138. Why do you think 729 is called a "perfect cube" number?

Because 729 is the volume of a cube ($l \times w \times h$) $\to$ cube = all equal side lengths/widths/heights ($9 \times 9 \times 9$)

139. Find the surface area of the following cube.

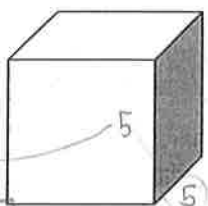
volume = s^3

$6s^2 = SA$

$6 \cdot (5)^2 = SA$

$6 \cdot 25 = SA$

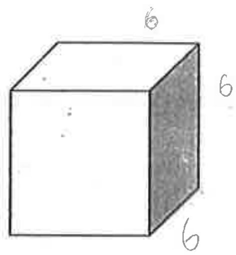
$\boxed{150\text{ cm}^2 = SA}$



125 cm³

$\boxed{150\text{ cm}^2}$

140. Find the surface area of the following cube.



216 cm³

$6 \times 6^2 = SA$

$6 \times 36 = SA$

$\boxed{216 = SA\ \text{cm}^2}$

36×6

$= \boxed{216\text{ cm}^2}$

141. A cube has a surface area of 294 m². Find its edge length in centimetres.

SA = 294 m²

$\frac{294}{6} = \frac{6s^2}{6}$

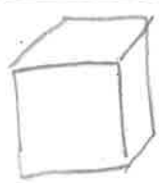
$\sqrt{49} = \sqrt{s^2}$

$\boxed{7\text{ m} = s}$

$L = 7\text{ m} \times 100 \to \boxed{L = 700\text{ cm}}$

142. A cube has a surface area of 1093.5 m². Find its edge length in centimetres.

SA = 1093.5 m²



$\frac{6s^2}{6} = \frac{1093.5}{6}$

$\sqrt{s^2} = \sqrt{182.25}$

$L = 13.5\text{ m} \times 100$

$L = 1350\text{ cm}$

Term	Definition	Example
Power	$2^1, 2^2, 2^3, 2^4, \ldots$ are powers of 2. A power is made up of a base and an exponent.	
Exponent	The smaller number written to the upper right of the base that tells you how many times to multiply the base by itself.	$2^4 = 2 \times 2 \times 2 \times 2$ 4 is the exponent.
Base	The "larger" number that the exponent is applied to. (The bottom number in a power)	$2^4 = 2 \times 2 \times 2 \times 2$ 2 is the base.
Rational number	Numbers that can be written as fractions.	
Rational Exponent	The exponent on a power is a rational number (fraction). $x^{\frac{2}{3}} = (\sqrt[3]{x})^2$	$27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = (3)^2 = 9$
Integral number	An integer {...-3,-2,-1,0,1,2,3,...}.	
Integral Exponent	The exponent on a power is an integer.	Such as x^2, x^{-3}.
Coefficient	The numbers in front of the letters in mathematical expressions.	In $3x^2$, 3 is the coefficient.
Variable	The letters in mathematical expressions.	In $3x^2$,'x' is the variable.
Undefined	If there is no good way to describe something, we say it is undefined.	$\frac{3}{0}$ is undefined because we cannot divide by zero.
Radical form	$(\sqrt[3]{8})^2$ is in radical form.	
Exponential Form	$8^{\frac{2}{3}}$ is in exponential form.	
Zero Exponent	Any expression to the power of 0 will equal 1.	$(2xyz)^0 = 1$
Negative Exponent	Reciprocate the base and perform repeated multiplication OR use repeated division.	$5^{-3} = \left(\frac{1}{5}\right)^3 = \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = \frac{1}{125}$
Multiply Powers with the Same base	Add the exponents.	$m^5 \times m^2 = m^7$
Dividing Powers with the same base.	Subtract the exponents.	$q^6 \div q^4 = q^2$
Power of a Power	Multiply the exponents.	$(x^2)^4 = x^8$
Power of a Product	Apply the exponent to all factors.	$(3x^2)^3 = 27x^6$
Power of a Quotient	Apply the exponent to both numerator AND denominator	$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$

Math 10

Unit 2: Exponents

Lesson 1: pages 1-9

Vocabulary:

power → x^m ← exponent

base → x

Exponent Laws:

From Math 9, you should have learned how to simplify the following monomial expressions using the following exponent laws:

Note: DO NOT use exponent laws when bases aren't equal

Exponent Laws	Examples (simplify & evaluate where possible)
Product of Powers $a^m \times a^n = a^{m+n}$	a) $0.8^2 \times 0.8^7 = 0.8^{2+7} = \boxed{0.8^9}$ b) $3^4 \times 3^1 = 3^{4+1} = \boxed{3^5}$ c) $10^{10} \times 10^{-6} = 10^{10+(-6)} = \boxed{10^4}$
Quotient of Powers $a^m \div a^n = a^{m-n}$	a) $5^5 \div 5^3 = 5^{5-3} = \boxed{5^2}$ b) $\left(-\frac{4}{5}\right)^{-6} \div \left(-\frac{4}{5}\right)^{-20} = \left(-\frac{4}{5}\right)^{-6-(-20)} = \boxed{\left(-\frac{4}{5}\right)^{14}}$ c) $40m^8 \div 5m = 40 \div 5 \; m^8 \div m^1 = 8m^7$
NEW ** **Negative Exponent** $a^{-m} = \frac{1}{a^m}$	a) $25^{-3} = \frac{1}{25^3}$ $\frac{25^{-3}}{1}$ TOP $\frac{1}{25^3}$ b) $6^3 \div 6^5 = 6^{3-5} = 6^{-2} = \boxed{\frac{1}{6^2}}$ $\frac{6 \cdot 6 \cdot 6}{6 \cdot 6 \cdot 6 \cdot 6 \cdot 6}$ c) $5^2 \div 5^5 = 5^{-3} = \boxed{\frac{1}{5^3}}$ $\cancel{\frac{5^2}{10^{-3}}}$ $\cancel{5^2 \cdot 10^3}$
Zero Exponent $a^0 = 1$	a) $(-7x^5y^{-6})^0 = \boxed{1}$ b) $\left(\frac{5}{2}\right)^4 \div \left(\frac{5}{2}\right)^4 = \left(\frac{5}{2}\right)^{4-4} = \left(\frac{5}{2}\right)^0 = \boxed{1}$

Example: Evaluate or simplify the following expressions.

1. $3^2 = 3 \cdot 3 = \boxed{9}$

2. $(-3)^2 = -3 \cdot -3 = \boxed{9}$

3. $-3^2 = - 3 \cdot 3 = \boxed{-9}$

4. $-5^0 = \boxed{-1}$

$\neq (-5)^0 = 1$

5. $6^{-2} = \frac{1}{6^2} = \boxed{\frac{1}{36}}$

Evaluate

6. $-\boxed{2^{-4}} = -\frac{1}{2^4} = \boxed{-\frac{1}{16}}$

simplify

7. $(-2)^{-4} = \frac{1}{(-2)^4} = \boxed{\frac{1}{16}}$

8. $x^3 \cdot x^4 = x^{3+4} = \boxed{x^7}$

9. $x^3 \cdot x^{\frac{1}{4}} = x^{\frac{3}{1} \cdot \frac{4}{4} + \frac{1}{4}} \rightarrow x^{\frac{12}{4} + \frac{1}{4}} = \boxed{x^{13/4}}$ or $\boxed{x^{3\,1/4}}$

10. $6m^4 \cdot 2m \div 3m^{-2} = \boxed{4m^7}$

$6 \cdot 2 \div 3 = 4$

$\underbrace{m^4 \cdot m^1}_{} \div m^{-2}$

$m^5 \div m^{-2} = m^{5-(-2)} = m^7$

ex: $x^2 \cdot x^{\frac{3}{4}}$

$= x^{2+\frac{3}{4}}$

$= x^{2\frac{3}{4}} = \boxed{x^{\frac{11}{4}}}$

PW: pgs 1-9 for Thursday

49

^1x a^6

(4a)

$$\frac{2}{a^3} \div \frac{6}{a^6} \rightarrow \frac{\cancel{2}^{1}}{a^3} \times \frac{a^6}{\cancel{6}_{3}} = \boxed{}$$

Introduction to Exponents

Challenge #1: Solve each riddle using any strategy that works.

1. Evaluate.
$3^2 \times 3^2$
$2+2=4$ $3^4 = \boxed{81}$
Rate the riddle:
Easy, Medium, Hard
Easy

2. Evaluate.
$2^2 \times 2^2 \div 2^3$
$= 2^4 \div 2^3$
$= 2^1 = \boxed{2}$
Rate the riddle:
Easy, Medium, Hard
Easy

3. Evaluate.
$x^3 \times x^5$
$\boxed{x^8}$
Rate the riddle:
Easy, Medium, Hard
Easy

4. Evaluate.
$8x^4 \div 4x^3$
$\frac{{}^{2}\cancel{8}x^4}{{}_{1}\cancel{4}x^3} = 2x^1 = \boxed{2x}$
Rate the riddle:
Easy, Medium, Hard
Easy

5. Find a strategy that is different from the one you used in Question 1 and solve the question again.
$3 \times 3 \times 3 \times 3 = 3^4 = \boxed{81}$
(Expand)

6. Find a strategy that is different from the one you used in Question 4 and solve the question again.
$8 \div 4 = 2$
$x^4 \div x^3 = xxxx \cdot xxx = x$
$= \boxed{2x}$

What is an Exponent?

Exponents are symbols that indicate an operation to be performed on the base.

positive exponents → Repeated Multiplication
negative exponents → Repeated Division

b^e b is the base, and e is the exponent. Together, we call them a *power.*

Some examples...

$2^1, 2^2, 2^3, 2^4, 2^5$ are the first five ***powers of 2.*** x^1, x^2, x^3, x^4, x^5 are the first five ***powers of x.***

Your Notes Here...

Positive Integral Exponent (multiplication)	Zero Exponent	Negative Integral Exponent (repeated division)
$a^n = 1 \times a \times a \times a \times \ldots \times a$ (*n* factors) Eg. $3^4 = 1 \times 3 \times 3 \times 3 \times 3 = 81$	$a^0 = 1, \quad (a \neq 0)$ Eg. $5^0 = 1, \quad \left(\frac{3}{2}\right)^0 = 1$	$a^{-n} = 1 \div a^n$ $= \frac{1}{a^n}$ Eg. $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

Challenge #2

7. Evaluate each of the following and examine the pattern:

$2^4 = 16$

$2^3 = 8$

$2^2 = 4$

$2^1 = 2$

$2^0 = 1$

$2^{-1} = \frac{1}{2}$

$2^{-2} = \frac{1}{4}$

$2^{-3} = \frac{1}{8}$

$2^{-4} = \frac{1}{16}$

8. What patterns do you notice in the list you created to the left?

If you divide each by 2 (when going down) or multiply each by 2 (when going up) you will get the answer

9. Does the value of 2^0 make sense when put into this list?

yes, b/c if you use the pattern I mentioned above it makes sense.

$2 \div 2 = \boxed{1}$

10. Do negative exponents make sense in this list?

yes you just have to change the negative to a positive and put it under 1 ($2^{-2} = \frac{1}{2^2} = \boxed{\frac{1}{4}}$)

11. Why might people say negative exponents mean "repeated division?"

Because going from a negative exponent to an even more negative exponent just means divide by 2 here. (you divide by 2 over and over = "repeatedly")

12. Identify the base in the following equation.

$4^3 = 64$

$\boxed{4}$

13. Identify the power in the following equation.

$2^5 = 32$

$\boxed{2^5}$

14. Identify the exponent in the following equation.

$-3^2 = -9$

$\boxed{2}$

15. Which of the following is equivalent to −16?

$-4^2 = -16$

$(-4)^2 = +16$

$4^{-2} = \frac{1}{16}$

$-4^{-2} = \frac{1}{-4^2} = \frac{1}{-16}$

16. Which of the following is equivalent to −81?

$-9^2 = -81$

$(-3)^4 = +81$

$9^{-2} = \frac{1}{81}$

$-3^{-4} = \frac{1}{-3^4} = \frac{1}{-81}$

17. Which of the following are equivalent to 1.

-3^0 $\frac{2x^3}{2x^3}$ $(5x)^0$

$-1 \times 1 = -1$ $1x^0 = 1 \times 1 = 1$ $+1$

correction

All

18. Which of the following is equivalent to 9?

$-3^2 = -9$

$(-3)^2 = +9$

$3^{-2} = \frac{1}{9}$

$(-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{9}$

19. Evaluate.

-2^6

$$= -1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$
$$= -64$$

$-2^6 = -1 \times 2^6$

$-1 \times 64 = \boxed{-64}$

20. Evaluate.

$(-3)^3$

$-3 \times -3 \times -3 = \boxed{-27}$

21. -4^2

$-1 \times 16 = \boxed{-16}$

22. $(-4)^{-2}$

$\frac{1}{(-4)^2} \rightarrow \boxed{\frac{1}{16}}$

23. -4^{-2}

$\frac{1}{-4^2} = \frac{1}{-1 \times 16} = \boxed{\frac{1}{-16}}$

☆BOOK SAYS $-\frac{1}{16}$ ☆

24. 3^{-4}

$= \frac{1}{3^4} = \boxed{\frac{1}{81}}$

$$= \frac{1}{3^4}$$

$$= 1 \div 3 \div 3 \div 3 \div 3$$

$$= 1 \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$$

$$= \frac{1}{81} \qquad = \frac{1}{81}$$

25. $(-3)^{-4}$

$\frac{1}{(-3)^4} = \frac{1}{-3 \times -3 \times -3 \times -3}$

$= \boxed{\frac{1}{81}}$

26. -3^{-4}

$\frac{1}{-3^4} = \frac{1}{-1 \times 3^4} = \boxed{\frac{1}{-81}}$

☆BOOKS SAYS $-\frac{1}{81}$ ☆

27. 4^2 $4 \times 4 = 16$

$\boxed{16}$

28. $(-4)^2$

$-4 \times -4 = \boxed{16}$

29. $-(4)^2$

$-1 \times 4 \times 4 = -1 \times 16$

$= \boxed{-16}$

30. $5^0 = \boxed{1}$

31. $-5^0 = -1 \times 1 = \boxed{-1}$

32. $\left(\frac{34a^2}{2x}\right)^0 = \frac{1}{1} = \boxed{1}$

The Exponent Laws:

Challenge #3

33. Multiply.

$$a^3 \times a^6$$

3 + 6 = 9

$\boxed{a^9}$

Explain your steps.

when bases are the same and powers are being multiplied, add exponents

Challenge #4

34. Divide.

$$g^7 \div g^3$$

7 - 3 = 4

$g^7 \div g^3$

$= \boxed{g^4}$

Explain your steps.

when bases are the same and powers are being divided, subtract exponents.

Challenge #5

35. Multiply.

$$5m^4 \times 3m^2$$

$5m^4 \times 3m^2$

$(5 \times 3) \times (m^4 \times m^2)$

$\boxed{15m^6}$

Explain your steps.

when powers are multiplied, and bases are the same, multiply the coefficients and add the exponents.

CORRECTION: $\frac{2^3}{3^3} \times \frac{(-6)^2}{4^2} = \frac{\cancel{8}^{1}}{\cancel{27}_{3}} \times \frac{\cancel{36}^{4}}{\cancel{16}_{2}} = \frac{4}{6} = \boxed{\frac{2}{3}}$

Simplify the following, write your answers using exponents.

36. $a^3 \times a^6$
$= a^{3+6}$
$= a^9$

37. $a^2 \times a^{-4}$
$2 + -4$
$\boxed{a^{-2}}$

38. $f^2 \times f^x$
$\cancel{\boxed{f^{2x}}} \rightarrow$ CORRECTION: $\boxed{f^{2+x}}$

39. $x^{\frac{1}{4}} \times x^{\frac{6}{8}}$ ☆USING EXPONENTS☆
$\frac{1}{4} = \frac{2}{8}, \frac{6}{8} = \frac{8}{8}$
$x^{\frac{8}{8}} = x^1 = \cancel{\boxed{x}} = \boxed{x^1}$

40. $2^3 \times 2^{-5}$
$3 + -5$
$2^3 \times 2^{-5} = \boxed{2^{-2}}$

41. $g^7 \div g^3$
$= g^{7-3}$
$= g^4$

42. $m^4 \div m^0$
$m^4 \div m^0 = \boxed{m^4}$

43. $t^0 \div t^{-5}$
$0 - (-5) = 0 + 5 = 5$
$t^0 \div t^{-5} = \boxed{t^5}$

44. $\frac{x^{13}}{x^3}$
$x^{13} \div x^3 = \boxed{x^{10}}$

45. $5m^4 \times 3m^2$
$= 5 \times 3 \times m^{4+2}$
$= 15m^6$

46. $-10x^4 \div -2x^{-2}$
$= (-10 \div -2) \times (x^4 \div x^{-2})$
$= 5 \times x^6$
$= \boxed{5x^6}$

47. $\frac{4a^4}{-8a^2}$
$\boxed{-\frac{1a^2}{2}} = \boxed{-\frac{a^2}{2}}$

48. $\frac{2}{3}x^3 \times \frac{6}{5}x^4$
$\left(\frac{2}{\cancel{3}} \times \frac{\cancel{6}^{2}}{5}\right) \times (x^3 \times x^4)$
$\frac{4}{5} \times x^7 = \boxed{\frac{4x^7}{5} \text{ or } \frac{4}{5}x^7}$
☆ ☆ ☆

49. $\frac{2}{a^3} \div \frac{6}{a^6}$
$\frac{2}{a^3} \div \frac{6}{a^6}$
$\frac{\cancel{2}^{1}}{\cancel{a^3}_{1}} \times \frac{\cancel{a^6}^{3}}{\cancel{6}_{3}} = \boxed{\frac{a^3}{3}}$

50. Evaluate.
$\left(\frac{2}{3}\right)^3 \left(\frac{-6}{4}\right)^2$?
$\frac{\cancel{8}^{1}}{\cancel{27}_{3}} \times \frac{-\cancel{36}^{-4}}{\cancel{16}_{2}} = \boxed{\frac{-4}{6}} = \frac{-2}{3}$ no negative?
$\rightarrow \boxed{\frac{2}{3}}$

Multiplying Powers with the same Base:	**Dividing Powers with the same Base:**
Add the exponents.	Subtract the exponents.
Eg.	Eg.
$x^5 \times x^2 = x^{5+2} = x^7$	$d^4 \div d^3 = d^{4-3} = d^1 = d$
$a^{\frac{2}{3}} \times a^{\frac{1}{3}} = a^{\frac{3}{3}} = a^1 = a$	$\frac{y^6}{y^{-2}} = y^{6-(-2)} = y^8$
$3x^2 \times 2x^5 = 3 \times 2 \times x^2 \times x^5 = 6x^7$	

✓

Lesson 2 (pgs 10-12)

October 11, 2017 4:38 PM

Math 10

$a^{-m} = \frac{1}{a^m}$

Unit 2: Exponents

Lesson 2: pages 10-12

Warm-Up:

1. $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

2. $8^{-1} = \frac{1}{8^1} = \frac{1}{8}$

3. $3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

4. $(-2)^4 = (-2)^4 = -2\cdot-2\cdot-2\cdot-2 = 16$

$(-2)^{-4}$

$\cancel{\frac{1}{-2^4} = \frac{1}{-2\cdot2\cdot2\cdot2} = -\frac{1}{16}}$

5. $\left(\frac{3}{10}\right)^{-2} = \frac{3^{-2}}{10^{-2}} = \frac{1}{3^2} \quad \frac{10^2}{3^2} = \frac{100}{9}$

6. $a^{\frac{8}{3}} \times a^{\frac{1}{3}} = a^{\frac{8}{3}+\frac{1}{3}} = a^{\frac{9}{3}} = a$

7. $a^{-8} \div a^1 = a^{-8-1} = a^{-9} \left(\text{or } \frac{1}{a^9}\right)$

8. $100x^4 \div 50x^8 = 2x^{4-8} = 2x^{-4} = \frac{2}{x^4}$

9. $a^9 \div a^{12} = a^{9-12} = a^{-3} = \frac{1}{a^3}$

$6^2 \div 6^3 = 6^{2-3} = 6^{-1}$

10. $6x^4 \div 6x^5 = 1x^{4-5} = 1x^{-1} = \frac{1}{x^1} = \frac{1}{x}$

11. $\left(\frac{2}{5}\right)^{-3} = \frac{2^{-3}}{5^{-3}} = \frac{5^3}{2^3} = \frac{125}{8}$

12. $6m^{12} \div 12m^{12} = \frac{6}{12} \rightarrow \frac{1}{2}m^{12-12}$

$\frac{1}{2}m^0 = 1$

13. $30m^8 \div -10m^1 = -3m^{8-1} = -3m^7 = \frac{1}{2}$

14. $\frac{3q^{-2}p^4}{4q^{-1}} = \frac{3q^1p^4}{4m^2}$

Exponent Laws:

From Math 9, you should have learned how to simplify the following monomial expressions using the following exponent laws:

Note: DO NOT use exponent laws when bases aren't equal

Exponent Laws	**Examples (simplify & evaluate where possible)**
$a^m \times a^n = a^{m+n}$ NOT SAME **Power of a Power** $(a^m)^n =$ $a^{m \times n}$	a) $(0.25^{-3})^{-5}$ $= 0.25^{15}$ b) $(8^2)^4$ $= 8^8$ $(2)^3$ $2(3)$ c) $(m^5)^3$ $= m^{15}$ NOT 2×3 $2\times2\times2$ d) $(2m^{10})^3$ $= 2^3 \times (m^{10})^3 = 8m^{30}$
Power of a Product $(ab)^m =$ $a^m b^m$	a) $(-6my^7)^3$ $= (-6)^3 m^3 y^{21} = -216m^3y^{21}$ 7×3 b) $(x^4y^{-2})^5$ $= x^{20} y^{-10}$ or $\frac{x^{20}}{y^{10}}$ c) $(8x^{-4})^2$ $= 8^2 x^{-8} = 64x^{-8} \rightarrow \frac{64}{x^8}$ d) $(3m^{-2}y^5)^{-3}$ $= 3^{-3} m^6 y^{-15} \rightarrow \frac{m^6}{3^3y^{15}}$ e) $(3t^0)^4$ $= 3^4 \cancel{t^0}{=1} = 3^4 = 81$

(More Complicated) Examples ☺ : Evaluate or simplify the following expressions.

1. $\left(\frac{2x^4y^{-9}}{3x^{-2}y}\right)^{-2}$ $\rightarrow \left(\frac{3x^{-2}y}{2x^4y^{-9}}\right)^2$

① Simplify in brackets: $\frac{2x^4y^{-9}}{3x^{-2}y} = \frac{2x^4x^2}{3y^9y^1} = \frac{2x^6}{3y^{10}}$

② $\left(\frac{2x^6}{3y^{10}}\right)^{-2} = \frac{2^{-2}x^{-12}}{3^{-2}y^{-20}} = \frac{3^2y^{20}}{2^2x^{12}} = \boxed{\frac{9y^{20}}{4x^{12}}}$

2. $(-10xy^4)^2 \cdot (5x^2y^3)^{-2}$

$= (-10)^2x^2y^8 \cdot 5^{-2}x^{-4}y^{-6}$

$= 100x^2y^8 \cdot 5^{-2}x^{-4}y^{-6}$

$= 100x^2y^8 \frac{1}{5^2}x^{-4}y^{-6}$

$\frac{100}{1} \times \frac{1}{25} = \frac{100}{25} = \boxed{4}$

$= 4x^{-2}y^2$

$= \frac{4y^2}{x^2}$

$a^m \times a^n = a^{m+n}$

$a^{-m} = \frac{1}{a^m}$ 5^{-3} $= 1$

x^2

$a^{-m} = \frac{1}{a^m}$ $\quad$ $5^{-3} = \cfrac{1}{\cfrac{5}{\cfrac{5}{5}}}$

PW: pgs 10-12 (including)

Challenge #6

51. Evaluate. = get # answer ★

$(5^2)^3$

$2 \times 3 = 6$

$\boxed{5^6} \rightarrow 5^6 = \boxed{15625}$

[Power of a Power]

Explain your steps.

When a power is raised to an exponent, multiply the exponents

15625

Challenge #7

52. Simplify.

$(m^3)^2$

$3 \times 2 = 6$

$\boxed{m^6}$

[Power of a Power]

Explain your steps.

When a power is raised to an exponent, multiply the exponents

Challenge #8

53. Simplify.

$(2m^4)^3$

$2^3 \times m^{4 \times 3}$

$8 \times m^{12}$

$\boxed{8m^{12}}$

[Power of a ~~Product~~]
Power?

Explain your steps.

When a power is raised to an exponent, put the exponent on the coefficient and evaluate and multiply the exponents.

√

★ A number multiplied by 0 is 0 ★

not negative? ★

$-3^{-4} \times x^{-2\times -4} \times y^{3\times -4}$

$\frac{1}{-3^4} \times x^8 \times y^{-12}$

$\frac{1}{-81} \times \frac{x^8}{1} \times \frac{1}{y^{12}} = \boxed{\frac{x^8}{-81y^{12}}}$

Simplify the following.

54. $(m^3)^2$

$= m^3 \times m^3$
$= m^6$

$= m^{3\times2}$
$= m^6$

55. $(t^4)^0$

$t^{4\times0}$

$t^0 = \boxed{1}$

56. $(x^2y^3)^{-3}$

$x^{2\times -3} \times y^{3\times -3}$

$x^{-6} \times y^{-9}$

★ $\boxed{x^{-6}y^{-9}}$ → ★ $\boxed{\frac{1}{x^6y^9}}$

NOT DONE!!

57. $(2m^4)^3$

$2m^4 \times 2m^4 \times 2m^4$
$= 2 \times 2 \times 2 \times m^4 \times m^4 \times m^4$
$= 8m^{12}$

OR

$= 2^3m^{4\times3}$
$= 8m^{12}$

58. $(2c^4d^3)^{-3}$

$2^{-3} \times c^{4\times -3} \times d^{3\times -3}$

$\frac{1}{8} \times c^{-12} \times d^{-9}$

$\frac{1}{8} c^{-12} d^{-9} = \frac{1}{8} \times \frac{1}{c^{12}d^9}$ $\boxed{\frac{1}{8c^{12}d^9}}$

★ do these go together?

59. $(-3x^{-2}y^3)^{-4}$

$-3^{-4} \times x^{-2\times -4} \times y^{3\times -4}$

$\frac{1}{-3^4} \times x^8 \times y^{-12}$

$\frac{1}{-81} \times x^8 \frac{1}{y^{12}}$

$\frac{1}{-81} \times \frac{1}{y^{12}} \times x^8$

$\boxed{\frac{1}{-81y^{12}} x^8}$ X

60. $(3x^{-2}y^{-3})^{-3}$

$3^{-3} \times x^{-2\times -3} \times y^{-3\times -3}$

$\frac{1}{27} \times x^6 \times y^9$

$\boxed{\frac{x^6y^9}{27}}$

61. $(-2xy^3)(-3x^2y^3)^2$

$= -2xy^3 \times (-3)^2 x^4y^6$

$= -2xy^3 \times 9x^4y^6$

$= -18x^{1+4}y^{3+6}$

$= \boxed{-18x^5y^9}$

62. $(2a^2)^3(4a^3b)^2$

$2^3a^6 \times 4^2a^6b^2$

$8a^6 \times 16a^6b^2$

$128a^{6+6}b^2$

$\boxed{128a^{12}b^2}$

$(-3x^{-2}y^3)^{-4}$

$(-3^{-4}x^{-2\times-4}y^{3\times-4}$

$\frac{1}{(-3)^4} \times x^8 y^{-12}$

$\frac{1}{(-3)^4} \times \frac{x^8}{1} \times \frac{1}{y^{12}}$

$\boxed{\frac{x^8}{81y^{12}}}$

Power of a Power:
Multiply the exponents.

Eg$(5^2)^3 = (5 \times 5)^3$.
$= (5 \times 5)(5 \times 5)(5 \times 5)$
$= 5 \times 5 \times 5 \times 5 \times 5 \times 5$
$= 5^6$

THE RULE:

$$(a^m)^n = a^{m\times n}$$

If you have a power of a power … multiply exponents.

Eg. $(x^2)^5 = x^{2\times5} = x^{10}$

Power of a Product:
Apply the exponent to all factors.

Eg. $(5 \times 2)^3$
$= (5 \times 2) \times (5 \times 2) \times (5 \times 2)$
$= 5 \times 5 \times 5 \times 2 \times 2 \times 2$
$= 5^3 \times 2^3$

THE RULE:

$$(ab)^m = a^mb^m$$

If you have a power of a product … apply the exponent to EVERY factor in the product.

Eg. $(a^2b^3)^{-3} = a^{2\times-3}b^{3\times-3} = a^{-6}b^{-9}$

$(2c^4d^3)^{-3} = 2^{-3}c^{4\times-3}d^{3\times-3}$

$= \frac{1}{8} c^{-12} d^{-9} = \frac{1}{8} \times \frac{1}{c^{12}} \times \frac{1}{d^9} = \boxed{\frac{1}{8c^{12}d^9}}$

✓

Challenge #9

63. Evaluate.

$$\left(\frac{2}{5}\right)^3$$

$\frac{2^3}{5^3} = \boxed{\frac{8}{125}}$

$\frac{8}{125}$

Explain your steps.

Apply exponent to numerator and denominator

Challenge #10

64. Evaluate.

$$\left(\frac{2}{5}\right)^{-3} = \frac{5^3}{2^3} = \frac{125}{8}$$

$\frac{2^{-3}}{5^{-3}} = \frac{1}{2^3} \div \frac{1}{5^3}$

$= \frac{1}{8} \div \frac{1}{125} = \frac{1}{8} \times \frac{125}{1}$

$= \boxed{\frac{125}{8}}$

$\frac{125}{8}$

Explain your steps.

1) Apply exponent to numerator and denominator

2) flip repricol

3) Simplify / multiply

Challenge #11

65. Simplify.

$$\left(\frac{x}{2}\right)^3$$

$\frac{x^3}{2^3} = \boxed{\frac{x^3}{8}}$

Explain your steps.

Apply exponent to numerator (variable) and denominator

Challenge #12

Can divide 8 and 6 first!

66. Simplify.

$$\left(\frac{6x^5y^3}{8y^4}\right)^{-2}$$

$\frac{8^2 y^{8^2}}{6^2 x^{10} y^6}$

$\frac{\overset{16}{\cancel{64}} y^2}{\underset{9}{\cancel{36}} x^{10}}$

$\boxed{\frac{16y^2}{9x^{10}}}$

$\frac{6^{-2} x^{-10} y^{-6}}{8^{-2} y^{-8}}$

$= \left(\frac{1}{36} \times \frac{1}{x^{10}} \times \frac{1}{y^6}\right) \div \left(\frac{1}{64} \times \frac{1}{y^8}\right)$

$\frac{1}{64y^8}$

$\frac{1}{\underset{9}{\cancel{36}} x^{10} \cancel{y^6}} \times \frac{\overset{16}{\cancel{64}} \cancel{y^8}^{2}}{1}$

$\boxed{\frac{16y^2}{9x^{10}}}$

Explain your steps.

Apply exponent to the numerator and denominator.

Lesson 3 (pgs. 13-16)

October 15, 2017 1:32 PM

Math 10

Unit 2: Exponents

Lesson 3: pages 13-16

Warm-Up: Simplify or evaluate as far as possible. Express answers with positive exponents.

1. $7^{-3} = \frac{1}{7^3} = \frac{1}{343}$

2. $2^6 \times 2^4 = 2^{6+4} = 2^{10} = 1024$

   $x^6 \cdot x^4 = x^{10}$

3. $x^9 \div x^3 = x^{9-3} = x^6$

4. $7m^4 \times 2m^1 = 14m^{4+1} = 14m^5$

5. $(-8xy^5)^2 = (-8)^2x^2y^{10} = 64x^2y^{10}$

6. $50p^9 \div 10p^{-2} = 5p^{9-(-2)} = 5p^{11}$

7. $(3m^0)(9m)^0 = (3\cdot 1)(1) = 3\cdot 1 = 3$

8. $(5m)^{-2} = \frac{1}{(5m)^2} = \frac{1}{5^2m^2} = \frac{1}{25m^2}$

9. $(2^{-3})^{-2} = 2^6 = 64$

10. $(10y^{-3})(6y^4)^2 = 10y^{-3}\cdot 6^2\cdot y^8 = 360y^{-3+8} = 360y^5$

11. $(4x^2y^3)^{-3} = 4^{-3}x^{-6}y^{-9} = \frac{1}{4^3x^6y^9} = \frac{1}{64x^6y^9}$

12. $\frac{6m^8y^2z^{-4}}{12my^5z^{-8}} = \frac{6m^8y^2z^8}{12my^5z^4}$ reduced by 6 $= \frac{1m^7y^{-3}z^4}{2} = \frac{1m^7z^4}{2y^3}$

13. $\frac{-10a^{-1}c^{-4}}{4a^{-2}c^7} = \frac{-5a^1c^{-11}}{2} = \frac{-5a^1}{2c^{11}}$

   $c^{-4-7} = c^{-11}$

   $a^{-1-(-2)}$

   $a^{-1+2} = a^1$

14. $x^{-3}\cdot x^{-\frac{4}{3}}\cdot x^{\frac{1}{3}}$

   $x^{\frac{-3\cdot 3}{1\cdot 3}+\left(-\frac{4}{3}\right)+\frac{1}{3}}$

   $= x^{-\frac{9}{3}-\frac{4}{3}+\frac{1}{3}}$

   $= x^{-\frac{12}{3}} = x^{-4} = \boxed{\frac{1}{x^4}}$

Exponent Laws:

From Math 9, you should have learned how to simplify the following monomial expressions using the following exponent laws:

Exponent Laws	Examples (simplify & evaluate where possible)
Power of a Quotient $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{y^{-3}}{x^{\frac{3}{5}}}\right)^5 = \frac{y^{-3\cdot5}}{x^{\frac{3}{5}\cdot5}} = \frac{y^{-15}}{x^3} = \boxed{\frac{1}{x^3y^{15}}}$; $\frac{3}{5}\times\frac{5}{1} = \frac{15}{5} = 3$
Power of a Quotient $\left(\frac{a}{b}\right)^{-m} = \frac{b^m}{a^m}$ Flip fraction $\left(\frac{b}{a}\right)^m$	$\left(\frac{y^{-3}}{x^{\frac{3}{5}}}\right)^{-5} = \left(\frac{x^{\frac{3}{5}}}{y^{-3}}\right)^5 = \frac{x^{\frac{3}{5}\cdot5}}{y^{-3\cdot5}} = \frac{x^3}{y^{-15}} = \boxed{x^3y^{15}}$

(More Complicated) Examples ☺ : Evaluate or simplify the following expressions.

1. $\left(\frac{x^4y^4m}{x^7y^2m^5}\right)^{-6} = \left(\frac{x^7y^2m^5}{x^4y^4m^1}\right)^6 = \left(x^3y^{-2}m^4\right)^6 = x^{18}y^{-12}m^{24} = \frac{x^{18}m^{24}}{y^{12}}$

   ① Flipped fraction
   ② Simplified brackets
   ③ Distributed "6" exponent

2. $\frac{(5m^{-1}y^3)^2}{my} = \frac{5^2m^{-2}y^6}{my} = \frac{25m^{-2}y^6}{m^1y^1} = 25m^{-3}y^5 = \frac{25y^5}{m^3}$

3. $\left(\frac{7x^{-1}y^6}{x^{-4}y^4}\right)^{-2} = \left(\frac{x^{-4}y^4}{7x^{-1}y^6}\right)^2 = \frac{x^{-8}y^8}{7^2x^{-2}y^{12}} = \frac{x^{-8-(-2)}y^{8-12}}{49} = \frac{x^{-6}y^{-4}}{49}$
   $= \frac{1}{49x^6y^4}$

4. $\frac{\left(\frac{1}{2}a^4b^5\right)^{-4}}{4} = \left(\frac{1}{2}a^4b^5\right)^{-4} \div 4$
   $= \left(\frac{1}{2}\right)^{-4}a^{-16}b^{-20} \div 4 \rightarrow \left(\frac{2}{1}\right)^4a^{-16}b^{-20} \div 4$
   $= 16a^{-16}b^{-20} \div 4$
   $= \frac{16}{a^{16}b^{20}} \div \frac{4}{1} \rightarrow \frac{16}{a^{16}b^{20}} \times \frac{1}{4}$
   $= \frac{\cancel{16}\,4}{1\,\cancel{4}a^{16}b^{20}}$
   $= \frac{4}{a^{16}b^{20}}$

5. $\left(\frac{8xb^{-7}}{-12x^2b^{-9}}\right)^{-3} = \left(\frac{-12x^2b^{-9}}{8xb^{-7}}\right)^3 = \frac{(-12)^3x^6b^{-27}}{8^3x^3b^{-21}} = \frac{-1728x^3b^{-6}}{512}$
   $= \frac{-\cancel{1728}\,27\,x^3}{\cancel{512}\,8\,b^6} = \frac{27x^3}{8b^6}$

PW: pgs 13 - 16 (including)

QUIZ on Thursday (pgs. 1-16)

After School MON room 227

Power of a Quotient:

Apply the exponent to numerator AND denominator.

Eg. $\left(\frac{2}{5}\right)^3 = \left(\frac{2}{5}\right) \times \left(\frac{2}{5}\right) \times \left(\frac{2}{5}\right)$

$= \frac{2\times2\times2}{5\times5\times5}$

$= \frac{2^3}{5^3}$ If asked to write using exponents

$= \frac{8}{125}$ If asked to simplify.

$\left(\frac{2}{5}\right)^{-3}$ The negative exponent means "flip the base".

$= \frac{5\times5\times5}{2\times2\times2}$

$= \frac{5^3}{2^3}$

$= \frac{125}{8}$

THE RULE:

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$\left(\frac{a}{b}\right)^{-m} = \frac{b^m}{a^m}$$

Simplify the following.

67. $\left(\frac{x}{2}\right)^3$

$= \frac{x^3}{2^3}$

$= \frac{x^3}{8}$

68. $\left(\frac{a}{b}\right)^4$

$\boxed{\frac{a^4}{b^4}}$

69. $\left(\frac{x^2}{y^3}\right)^5$

$\boxed{\frac{x^{10}}{y^{15}}}$

70. $\left(\frac{-2a^2}{3y^3}\right)^3$

$\frac{(-2)^3a^6}{3^3y^9}$

$\boxed{\frac{-8a^6}{27y^9}}$

71. $\left(\frac{a^{-3}}{b^{-2}}\right)^{-2}$

$\boxed{\frac{a^6}{b^4}}$

72. $\left(\frac{4x}{3y}\right)^2$

$\frac{4^2x^2}{3^2y^2} = \boxed{\frac{16x^2}{9y^2}}$

73. $\left(\frac{6x^5y^3}{8y^4}\right)^{-2}$

$= \frac{(8)^2(y^4)^2}{(6)^2(x^5)^2(y^3)^2}$

$= \frac{64y^8}{36x^{10}y^6}$

$= \frac{16y^2}{9x^{10}}$

74. $\left(\frac{5ab^2c^3}{2a^{-2}c^{-3}}\right)^2$

$\left(\frac{5ab^2c^3}{2a^{-2}c^{-3}}\right)^2 = \left(\frac{5a^3b^2c^6}{2}\right)^2$

$= \boxed{\frac{25a^6b^4c^{12}}{4}}$

75. $\left[\left(\frac{2m^2n^2}{mn^3}\right)^{-1}\right]^3$

$\left(\frac{mn^3}{2m^2n^2}\right)^3$

$\frac{m^3n^9}{8m^6n^6}$

$\boxed{\frac{n^3}{8m^3}}$

REMEMBER: $a = a^1$

Simplify the following.

76. $\left(\frac{6ab^3}{2ab}\right)^3$

$\frac{6^3a^3b^9}{2^3a^3b^3}$

$= \frac{\cancel{216}\cancel{a^3}b^{\cancel{9}6}}{\cancel{8}\cancel{a^3}\cancel{b^3}} \rightarrow \boxed{27b^6}$

77. $\left(\frac{4x^{-3}y^4}{8x^2y^{-2}}\right)^{-2}$

$\left(\frac{8x^2y^{-2}}{4x^{-3}y^4}\right)^2$

$\frac{\cancel{64}x^4x^{16}}{\cancel{16}x^{-6}y^{8}{}^{12}}$

$= \boxed{\frac{4x^{10}}{y^{12}}}$

78. Show why $\frac{2a^2}{b^3}$ is the same as $2a^2 \times b^{-3}$.

$\frac{2a^2}{b^3} = \frac{2a^2}{1} \times \frac{1}{b^3}$

$= \frac{2a^2}{b^3} = \frac{2a^2}{b^3}$

79. Show why $\frac{12x^3}{y}$ is the same as $12x^3 \times y^{-1}$.

$\frac{12x^3}{y} = \frac{12x^3}{1} \times \frac{1}{y^1}$

$\frac{12x^3}{y} = \frac{12x^3}{y}$

Challenge #13

80. Write the following without using <u>any</u> negative exponents.

$3a^2b^{-5}$

$\frac{3a^2}{1} \times \frac{1}{b^5}$

$= \boxed{\frac{3a^2}{b^5}}$

81. Write the following without using <u>any</u> negative exponents.

$\frac{3}{a^{-2}b^5}$

$\boxed{\frac{3a^2}{b^5}}$

Challenge #14

82. Simplify using positive exponents.

$\left(\frac{2x^{-2}y^4}{x^{-3}y^3}\right)^{-3}$

$\left(\frac{x^{-3}y^3}{2x^{-2}y^4}\right)^3$

$\frac{\cancel{x^{-9}}\cancel{y^9}}{8x^{\cancel{-6}3}y^{\cancel{12}3}}$

$= \boxed{\frac{1}{8x^3y^3}}$

Explain your steps

1. Flip/recipricol to make exponent (-3) positive.
2. Apply exponent to numerator and denominator
3. simplify

$3^2 = \frac{1}{3^{-2}}$ / $3^{-2} = \frac{1}{3^2}$ $\frac{3x}{yz^{-2}} = \frac{3xz^2}{y}$

Writing Expressions with Positive Exponents. (Why? Because it is standard practice.)

An expression with powers is simplified if there are no brackets and no negative exponents.

Sometimes you will use the laws above and end up with an answer with negative exponents. The quick way to convert a negative exponent into a positive exponent is to ***move it across the division line***. The solution in question 83 shows why this works.

Simplify the following. (No brackets, no negative exponents)

83. $3a^2b^{-5}$

$$= 3a^2 \times \frac{1}{b^5}$$

$$= \frac{3a^2}{b^5}$$

84. a^2b^{-3}

$\frac{a^2}{1} \times \frac{1}{b^3} = \boxed{\frac{a^2}{b^3}}$

85. $\frac{2xy^5}{x^{-4}}$

$\frac{2xy^5}{1} \div \frac{x^{-4}}{1}$

$\frac{2xy^5}{1} \times \frac{1}{xy^3} = \cancel{\boxed{\frac{2y^5}{x^3}}}$

$= \frac{2xy^5}{x^{-4}} = \frac{2xy^5x^4}{} \boxed{2x^5y^5}$

86. $3a^2b^{-3}c^{-5}$

$\boxed{\frac{3a^2}{b^3c^5}}$

87. $(x^4y^{-3}z^{-1})^{-2}$

$\left(\frac{1}{x^4y^{-3}z^{-1}}\right)^2$

$\frac{1}{x^8y^{-6}z^{-2}} = \boxed{\frac{y^6z^2}{x^8}}$

88. $\frac{(3x^{-3}y^{-5})^2}{2xy}$

$\frac{3^2x^{-6}y^{-10}}{2xy}$ $\frac{9}{2 \times x^1 \times y^1 \times x^6 \times y^{10}}$

$= \boxed{\frac{9}{2x^7y^{11}}}$

89. $\left(\frac{2x^{-2}y^4}{x^{-3}y^3}\right)^{-3}$

$$= \left(\frac{x^{-3}y^3}{2x^{-2}y^4}\right)^3$$

$$= \frac{x^{-9}y^9}{8x^{-6}y^{12}}$$

$$= \frac{x^{-3}y^{-3}}{8}$$

$$= \frac{1}{8x^3y^3}$$

90. $\left(\frac{2a^3b^2}{4a^{-2}b^{-1}}\right)^{-3}$

$\left(\frac{4a^{-2}b^{-1}}{8a^3b^2}\right)^3$

$\frac{(2a^{-5}b^{-3})^3}{2}$

$\frac{2^3a^{-15}b^{-9}}{2}$

$\frac{8a^{-15}b^{-9}}{2}$

$= \frac{4}{1} \times \frac{1}{a^{15}} \times \frac{1}{b^9} = \boxed{\frac{4}{a^{15}b^9}}$

91. $\frac{(4m^2n^2)(7m^{-3}n^2)}{14mn^5}$

$\frac{4 \times m^2 \times n^2 \times 7 \times m^{-3} \times n^2}{14 \times m \times n^5}$

$\frac{28m^{-1}n^4}{14mn^5}$

$\frac{28n^4}{14mn^5m^1}$

$\frac{28n^4}{14m^2n^5}$

$\boxed{\frac{2}{m^2n}}$

92. Why does moving a power across the division line in a fraction change the sign on the exponent?

B/c the base is reciprocated

✓

Simplify the following. (No brackets, no negative exponents)

93. $\left(\frac{12x^3y^{-1}}{-8x^{-1}y^5}\right)^{-2}$

$\left(\frac{-8x^{-1}y^5}{12x^3y^{-1}}\right)^2$

$\left(\frac{(-2)x^{-4}y^6}{3}\right)^2$

$\frac{4x^{-8}y^{12}}{3^2}$

$\boxed{\frac{4y^{12}}{9x^8}}$

94. $\left(\frac{4a^3b^{-2}}{6a^2b^{-1}}\right)^{-3}$

$\left(\frac{6a^2b^{-1}}{4a^3b^{-2}}\right)^3$

$\frac{216a^6b^{-3}}{64a^9b^{-6}}$

$\frac{27}{8}\;\frac{216b^3}{64a^3}$

$= \boxed{\frac{27b^3}{8a^3}}$

95. $\left(\frac{8x^2y^{-3}}{4x^{-1}y^{-5}}\right)^{-3}$

$\left(\frac{4x^{-1}y^{-5}}{8x^2y^{-3}}\right)^3$

$\left(\frac{1}{2x^3y^2}\right)^3$

$= \boxed{\frac{1}{8x^9y^6}}$

96. $\left(\frac{12x^{-3}y^5}{16x^3y^{-2}}\right)^{-1}$

$\left(\frac{16x^3y^{-2}}{12x^{-3}y^5}\right)^1$

$\left(\frac{4x^6}{3y^7}\right)^1$

$\boxed{\frac{4x^6}{3y^7}}$

Lesson 4 (pgs. 17-19)

October 15, 2017 1:46 PM

Math 10

Unit 2: Exponents

Lesson 4: pages 17-19

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Warm-Up #1: Simplify or evaluate as far as possible. Express answers with positive exponents.

1. $\frac{3x^2y^4}{4x^3y^3}$

$= \frac{3x^{2-3}y^{4-3}}{4}$

$= \frac{3x^{-1}y^1}{4} = \boxed{\frac{3y^1}{4x}}$

2. $\frac{2ab^3}{-2a^3b^2}$

$= \frac{1a^{1-3}b^{3-2}}{-1}$

$= \frac{1a^{-2}b^1}{-1} = \boxed{\frac{1b^1}{-1a^2}}$

3. $\frac{-2x^2y^{-5}}{-3x^{-4}y^3}$

$= \frac{+2x^2x^4}{+3y^5y^3}$

$= \frac{2x^{2+4}}{3y^{5+3}} = \boxed{\frac{2x^6}{3y^8}}$

4. $\frac{(n^2)^4(-n^0)^3}{-n^2}$

$= \frac{n^8(-1)^3}{-n^2}$

$= \frac{n^8(-1)}{-n^2}$

$= \frac{+n^8}{+n^2}$

$= n^{8-2}$

$= n^6$

5. $\frac{2a^2bc^{-4}}{5^{-1}a^{-3}b^3c^2}$

$= \frac{2a^2b\cdot5^1a^3}{b^3c^2c^4}$

$= \frac{10a^5b}{b^3c^6}$

$= \frac{10a^5b^{1-3}}{c^6}$

$= \boxed{\frac{10a^5}{b^2c^6}}$

6. $\left(\frac{x^2y}{mp^8}\right)^5$

$= \frac{x^{2\cdot5}y^5}{m^5p^{8\cdot5}}$

$= \boxed{\frac{x^{10}y^5}{m^5p^{40}}}$

7. $\left(\frac{3c}{5d}\right)^{-2}$

$= \left(\frac{5d}{3c}\right)^2 = \frac{5^2d^2}{3^2c^2} = \boxed{\frac{25d^2}{9c^2}}$

8. $\left(\frac{15m^8y}{3my^{-5}}\right)^{-3}$

$= \left(\frac{3my^{-5}}{15m^8y}\right)^3$

$= \left(\frac{1m^{-7}y^{-6}}{5}\right)^3$

$= \left(\frac{1}{5m^7y^6}\right)^3$

$= \boxed{\frac{1}{125m^{21}y^{18}}}$

9. $\left(\frac{27m^2n}{9m}\right)^{-2}$

$= \left(\frac{\cancel{9}^{1}mn}{\cancel{27}^{3}m^2}\right)^2$

$= \left(\frac{1n}{3m}\right)^2 = \frac{1n^2}{9m^2}$

10. $\left(\frac{2a^2b^{-2}}{8a^{-3}b}\right)^{-4}$

$= \left(\frac{\cancel{8}^{4}a^{-3}b}{\cancel{2}^{1}a^2b^{-2}}\right)^4$

$= \left(\frac{4a^{-3-2}b^{1-(-2)}}{1}\right)^4$

$= \left(4a^{-5}b^1\right)^4$

$= 4^4a^{-20}b^4$ $\boxed{\frac{256b^4}{a^{20}}}$

Warm-Up #2: Use your calculator to complete the following tables:

1.

① ② ③ $\boxed{4}\ \boxed{\wedge}\ \boxed{(}\ \boxed{1}\ \boxed{\div}\ \boxed{2}\ \boxed{)}$ $\boxed{y^x}$ $\boxed{x^y}$

x	$x^{\frac{1}{2}}$	
1	1	$\leftarrow 1^{1/2}$
4	2	$\leftarrow 4^{1/2}$
9	3	$\leftarrow 9^{1/2}$
16	4	$\leftarrow 16^{1/2}$
25	5	
36	6	

Explain the effect the exponent $\frac{1}{2}$ has on the value of x.

$\sqrt{x}$

Write a rule to describe this relationship:

$$x^{\frac{1}{2}} = \sqrt{x}$$

2.

y	$y^{\frac{1}{3}}$
1	1
8	2
27	3
64	4
125	5
216	6

Explain what effect the exponent $\frac{1}{3}$ has on the value of y.

$\sqrt[3]{y}$

Write a rule to describe this relationship:

$$y^{\frac{1}{3}} = \sqrt[3]{y}$$

3. What do you think $x^{\frac{1}{4}}$ means? Test your prediction on your calculator, letting $x = 16$.

$\sqrt[4]{x}$ $\qquad 16^{1/4} = \sqrt[4]{16} = 2$ ✓

4. What would $x^{\frac{1}{n}}$ mean (as a radical)?

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

Exponent Law:

Exponent Laws	**Example #1 (simplify & evaluate where possible)**
Rational Exponents ***(with numerator = 1)*** $x^{\frac{1}{n}} = \sqrt[n]{x}$	a) $100^{\frac{1}{2}} = \sqrt[2]{100} = 10$ b) $(-8)^{\frac{1}{3}} = \sqrt[3]{-8} = -2$ c) $1024^{\frac{1}{5}} = \sqrt[5]{1024} = 4$ d) $(625m^4)^{\frac{1}{4}} = \sqrt[4]{625m^4} = \sqrt[4]{625} \times \sqrt[4]{m^4} = 5m$ e) $(81m)^{\frac{1}{4}} = \sqrt[4]{81m} = \sqrt[4]{81} \times \sqrt[4]{m} = 3\sqrt[4]{m}$ f) $-343^{\frac{1}{3}} = -\sqrt[3]{343} = -7$ g) $(-49)^{\frac{1}{2}} = \sqrt[2]{-49}$ = NO SOLUTION h) $16^{-\frac{1}{4}} = \frac{1}{16^{1/4}} = \frac{1}{\sqrt[4]{16}} = \frac{1}{2}$ i) $1000^{-\frac{1}{3}} = \frac{1}{1000^{1/3}} = \frac{1}{\sqrt[3]{1000}} = \frac{1}{10}$

Example #2: Simplify the following in ~~radical~~ exponent form.

1. $\sqrt[2]{121} = 121^{1/2}$
2. $\sqrt[5]{-32} = (-32)^{1/5}$

   ← brackets!
3. $\frac{1}{\sqrt[3]{125}} = (\sqrt[3]{125})^{-1} = 125^{-1/3}$
4. $10\sqrt{3xy}$

   $= 10 \times (3xy)^{1/2}$

PW: pg. 17-19 (including 19)

3

QUIZ THURS pgs. 1-16 (not today)

97. ## Challenge #15

If $\sqrt{9} \times \sqrt{9} = 9$,

and $9^a \times 9^a = 9$

Then what is the value of 'a'?

$\frac{1}{2}$

Explain:

$9^{\frac{1}{2}} = \sqrt[2]{9^1} = \sqrt[2]{9} \times \sqrt[2]{9} = 3 \times 3 = 9$

98. ## Challenge #16

If $\sqrt[3]{2} \times \sqrt[3]{2} \times \sqrt[3]{2} = 2$,

and $2^a \times 2^a \times 2^a = 2$

Then what is the value of 'a'?

$\frac{1}{3}$

Explain:

$2^{\frac{1}{3}} = \sqrt[3]{2^1} = \sqrt[3]{2} \times \sqrt[3]{2} \times \sqrt[3]{2} = 2$

99. Write a "rule" that relates a **rational** (fraction) **exponent** to an equivalent **radical expression.**

A rational (fraction) exponent can be written as an equivalent radical expression by making the denominator the index and the numerator the exponent of the radicand.

Ex. $x^{\frac{1}{2}} = \sqrt[2]{x^1} \rightarrow \sqrt[2]{x}$

$\sqrt{\ }$ square root "2" is implied

index $\sqrt[3]{\ }$ The denominator in a rational exponent is the index.

☆ DENOMINATOR IS THE INDEX! ☆

Rational Exponents in the form: $x^{\frac{1}{n}}$

radical form

$\sqrt[5]{x} = x^{\frac{1}{5}} \rightarrow$ exponential form.

Remember, *rational* often refers to fractions.

What does a rational exponent mean?

Recall: $\sqrt{9} \times \sqrt{9} = 9$.

But $9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 9$

And $3 \times 3 = 9$

So, $\sqrt{9} = 9^{\frac{1}{2}} = 3$

If $\sqrt[3]{2} \times \sqrt[3]{2} \times \sqrt[3]{2} = 2$

But $2^{\frac{1}{3}} \times 2^{\frac{1}{3}} \times 2^{\frac{1}{3}} = 2$

So, $\sqrt[3]{2} = 2^{\frac{1}{3}}$

100. Write another statement like the one to the left.

Recall: $\sqrt{16} \times \sqrt{16} = 16$

But: $16^{\frac{1}{2}} \times 16^{\frac{1}{2}} = 16$

And: $4 \times 4 = 16$

So: $\sqrt{16} = 16^{\frac{1}{2}} = 4$

If: $\sqrt[3]{8} \times \sqrt[3]{8} \times \sqrt[3]{8} = 8$

But: $8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}} = 8$

SO: $\sqrt[3]{8} = 8^{\frac{1}{3}}$

The Rule...

$$a^{\frac{1}{n}} = \sqrt[n]{a} \qquad and \qquad a^{-\frac{1}{n}} = \frac{1}{\sqrt[n]{a}}$$

Evaluate or simplify the following.

101. $49^{\frac{1}{2}}$

$\sqrt[2]{49} = \boxed{7}$

102. $-(16^{\frac{1}{2}})$

$-1 \times \sqrt[2]{16} = \boxed{-4}$

103. $(-16)^{\frac{1}{2}}$

$\sqrt[2]{-16} = \boxed{4i}$

(no real solution)

104. $64^{\frac{1}{3}}$

$\sqrt[3]{64} = \boxed{4}$

105. $27^{-\frac{1}{3}}$

$\frac{1}{\sqrt[3]{27}} = \boxed{\frac{1}{3}}$

106. $32^{-\frac{1}{5}}$

$\frac{1}{\sqrt[5]{32}} = \boxed{\frac{1}{2}}$

107. $10000^{\frac{1}{4}}$

$\sqrt[4]{10000} = \boxed{10}$

108. $(4x^2)^{\frac{1}{2}}$

$\sqrt[2]{4x^2} = \boxed{2x}$

109. $(27x^6)^{-\frac{1}{3}}$

$\frac{1}{\sqrt[3]{27x^6}} = \boxed{\frac{1}{3x^2}}$

✓

Write in radical form.

110. $7^{\frac{1}{2}}$ $\boxed{\sqrt[2]{7}}$ ☆ $\sqrt{7}$ ☆	111. $(3x)^{\frac{1}{3}}$ $\boxed{\sqrt[3]{(3x)}}$ ☆ $\sqrt[3]{3x}$ ☆	112. $4^{\frac{1}{5}}$ $\boxed{\sqrt[5]{4}}$
113. $4^{-\frac{1}{5}}$ $\boxed{\frac{1}{\sqrt[5]{4}}}$	114. $-64^{\frac{1}{3}}$ $-1 \times \sqrt[3]{64}$ $= \boxed{-\sqrt[3]{64}}$	115. $64^{-\frac{1}{3}}$ $\boxed{\frac{1}{\sqrt[3]{64}}}$

Write in exponential form.

116. $\sqrt[2]{13}$ $\boxed{13^{\frac{1}{2}}}$	117. $-3\sqrt{x}$ $\boxed{-3x^{\frac{1}{2}}}$	118. $\sqrt[2]{2y}$ $\boxed{(2y)^{\frac{1}{2}}}$
119. $\sqrt[4]{4}$ $\boxed{4^{\frac{1}{4}}}$	120. $\sqrt[7]{4}$ $\boxed{4^{\frac{1}{7}}}$	121. $\frac{1}{\sqrt[5]{3x}}$ $\boxed{(3x)^{-\frac{1}{5}}}$

Consider the following...

Step 1: $32^{\frac{3}{5}} = \left(32^{\frac{1}{5}}\right)^3$

Step 2: $32^{\frac{3}{5}} = (\sqrt[5]{32})^3$

Step 3: $32^{\frac{3}{5}} = (2)^3$

Step 4: $32^{\frac{3}{5}} = 8$

122. **Challenge #17. Complete the following as shown above.**

Step 1: $27^{\frac{2}{3}} = \left(27^{\frac{1}{3}}\right)^2$

Step 2: $27^{\frac{2}{3}} = (\sqrt[3]{27})^2$

Step 3: $27^{\frac{2}{3}} = (3)^2$

Step 4: $27^{\frac{2}{3}} = 9$

Explain: - Make $\frac{2}{3} \to \left(\frac{1}{3}\right)^2 = \frac{1}{3} \times \frac{2}{1} = \frac{2}{3}$
- Turn into equivalent radical expression
- solve radical
- solve exponent.

✓

Math 10

Unit 2: Exponents

Lesson 5: pages 20-24

Warm-Up #1: Simplify or evaluate as far as possible (#1-6), or re-write radicals as exponents (#7-10). Express answers with positive exponents.

1. $16^{\frac{1}{4}} = \sqrt[4]{16}$
$= 2$

2. $27^{-\frac{1}{3}} = \frac{1}{27^{1/3}}$
$= \frac{1}{\sqrt[3]{27}} = \frac{1}{3}$

3. $-25^{\frac{1}{2}} = -\sqrt[2]{25}$
$= -5$

4. $(-25)^{\frac{1}{2}} = \sqrt[2]{-25}$
NO SOLUTION

5. $1024^{0.5} = 1024^{1/2}$
$= \sqrt[2]{1024}$
$= 32$

6. $((-2)^{-2})^{\frac{1}{2}} = (-2)^{-1}$
$= \frac{1}{(-2)^1} = \boxed{\frac{1}{-2}}$

7. $8\sqrt[3]{a} = \boxed{8 \times a^{1/3}}$

8. $\sqrt[2]{(16y^8)} = (16y^8)^{1/2}$
$= 16^{1/2}\, y^4$ ← $\frac{8}{1} \times \frac{1}{2}$
$= \boxed{4y^4}$ $= \frac{8}{2}$

9. $\frac{50}{\sqrt[3]{xy}}$
$= 50\,(xy)^{-1/3}$ ← $\sqrt[3]{\ }$
on bottom

10. $(\sqrt[3]{\sqrt[7]{z}})^6$
$= (\sqrt[3]{z^{1/7}})^6$
$= (z^{\frac{1}{7}\cdot\frac{1}{3}})^6$ $\frac{1}{7} \times \frac{1}{3} \times \frac{6}{1}$
$= \boxed{z^{6/21}}$ $= \frac{6}{21}$

1

Warm-Up #2:

1. Re-write the exponents below as a product of two fractions, remembering that $\frac{a}{b} = \frac{a}{1} \times \frac{1}{b}$. Then, evaluate. The first one is done as an example ☺

   a. $9^{\frac{3}{2}} = \left(9^{\frac{3}{1}}\right)^{\frac{1}{2}} = (729)^{\frac{1}{2}} = \sqrt{729} = 27$

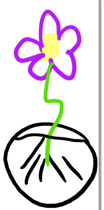
   b. $100^{\frac{5}{2}} = \left(100^{\frac{5}{1}}\right)^{\frac{1}{2}} = (10000000000)^{1/2} = 100000$

   c. $216^{\frac{2}{3}} = \left(216^{\frac{2}{1}}\right)^{\frac{1}{3}} = (46656)^{1/3} = 36$

This works, but there's an easier way!

Exponent Law:

Exponent Laws	Example #1 (simplify & evaluate where possible)
$m \swarrow$ power $n \leftarrow$ root **Rational Exponents** ***(with numerator ≠ 1)*** $x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m$ OR $\sqrt[n]{x^m}$ $1\frac{1}{2} = \frac{3}{2}$	a) $32^{\frac{3}{5}} = \left(\sqrt[5]{32}\right)^3 = (2)^3 = \boxed{8}$ b) $(-32)^{\frac{3}{5}} = \left(\sqrt[5]{-32}\right)^3 = (-2)^3 = \boxed{-8}$ c) $16^{\frac{7}{4}} = \left(\sqrt[4]{16}\right)^7 = (2)^7 = \boxed{128}$ d) $(-27)^{\frac{2}{3}} = \left(\sqrt[3]{-27}\right)^2 = (-3)^2 = \boxed{9}$ e) $(-25)^{\frac{5}{2}} = \left(\sqrt[2]{-25}\right)^5 =$ NO SOLUTION f) $-25^{\frac{5}{2}} = -\left(\sqrt[2]{25}\right)^5 = -(5)^5 = \boxed{-3125}$ g) $-25^{-\frac{5}{2}} = -\left(\sqrt[2]{25}\right)^{-5} = -(5)^{-5} = -\frac{1}{5^5} = \boxed{-\frac{1}{3125}}$ h) $16^{1.5} = 16^{3/2} = \left(\sqrt[2]{16}\right)^3 = (4)^3 = \boxed{64}$ i) $1000^{-\frac{2}{3}} = \frac{1}{1000^{2/3}} = \frac{1}{\left(\sqrt[3]{1000}\right)^2} = \frac{1}{10^2} = \boxed{\frac{1}{100}}$



2

Exam

ple #2: Write the following with exponents. Then use exponent laws and evaluate.

1. $\sqrt[2]{8}\times\sqrt{8^3}$ $= 8^{1/2} \times 8^{3/2} = 8^{\frac{1}{2}+\frac{3}{2}} = 8^{\frac{4}{2}} = 8^2 = \boxed{64}$

2. $\sqrt{g^5}\times\sqrt{g^7}$ $= g^{5/2} \times g^{7/2} = g^{12/2} = \boxed{g^6}$

3. $\sqrt{\sqrt{16^3}}$ $= \left(16^{\frac{3}{2}}\right)^{\frac{1}{2}} = 16^{\frac{3}{2}\times\frac{1}{2}} = \boxed{16^{3/4}}$ $\quad (a^m)^n = a^{m\times n}$

4. $\sqrt[3]{x^2}\cdot(\sqrt[4]{x})^1$ $= x^{\frac{2}{3}} \cdot x^{\frac{1}{4}} = x^{\frac{2\cdot 4}{3\cdot 4}+\frac{1\cdot 3}{4\cdot 3}} = x^{\frac{8}{12}+\frac{3}{12}} = \boxed{x^{11/12}}$

5. $(\sqrt[5]{18})^2\cdot\sqrt[5]{18^3}$ $= 18^{2/5}\cdot 18^{3/5} = 18^{\frac{2}{5}+\frac{3}{5}} = 18^{\frac{5}{5}} = 18^1 = \boxed{18}$

6. $\sqrt[3]{64}\cdot\sqrt[4]{16^3}$ $= 64^{1/3}\cdot 16^{3/4} = 64^{\frac{1\cdot 4}{3\cdot 4}+\frac{3\cdot 3}{4\cdot 3}} = 64^{\frac{4}{12}+\frac{9}{12}} = \boxed{64^{\frac{13}{12}}}$

Quiz Mon?

PW: pgs. 20-23

Ch. 2 TEST THURS 26th

Example #3: Find the area of a triangle that has a base of $82^{\frac{4}{5}}cm$ and a height of $82^{\frac{11}{5}}cm$. (Hint: $A = \frac{b\times h}{2}$)

$A = \frac{b\cdot h}{2}$

$= \frac{82^{4/5}\cdot 82^{11/5}}{2}$

$= \frac{82^{\frac{4}{5}+\frac{11}{5}}}{2}$

$= \frac{82^{15/5}}{2}$

$= \frac{82^3}{2}$

$= 275684\,cm^2$

Rational Exponents in the form: $x^{\frac{m}{n}}$ where m is not 1.

Consider the power $27^{\frac{2}{3}}$. To understand the meaning of the rational exponent we can use the exponent law:
$(a^m)^n = a^{m\times n}$.

If we take $27^{\frac{2}{3}}$ and split the exponent into two parts we get the following...

$$27^{\frac{2}{3}} = \left(27^{\frac{1}{3}}\right)^2$$

This can then be written as...

$$\left(\sqrt[3]{27}\right)^2$$

The power can be evaluated from this point...

$$\left(\sqrt[3]{27}\right)^2 = (3)^2 = 9$$

The Rule...

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m \qquad and \qquad a^{-\frac{m}{n}} = \frac{1}{\sqrt[n]{a^m}} = \frac{1}{\left(\sqrt[n]{a}\right)^m}$$

Two more examples:

Eg.1 Evaluate $8^{\frac{2}{3}}$ without using a calculator.

$$8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$$

Means square of the cube root of 8.

Eg.2 Evaluate $9^{-\frac{3}{2}}$ without using a calculator.

$$9^{-\frac{3}{2}} = \left(\frac{1}{9}\right)^{\frac{3}{2}} = \frac{\left(1^{\frac{1}{2}}\right)^3}{\left(9^{\frac{1}{2}}\right)^3} = \frac{1}{(\sqrt{9})^3} = \frac{1}{(3)^3} = \frac{1}{27}$$

Means "the reciprocal" of the cube of the square root of 9.

Write each of the following using radicals. (Do not evaluate)

123. $4^{\frac{2}{5}}$

$\boxed{\sqrt[5]{4^2}}$

124. $4^{\frac{3}{5}}$

$\boxed{\sqrt[5]{4^3}}$

125. $4^{\frac{4}{5}}$

$\boxed{\sqrt[5]{4^4}}$

126. $4^{-\frac{2}{5}}$

$\boxed{\frac{1}{\sqrt[5]{4^2}}}$

127. $4^{-\frac{3}{5}}$

$\boxed{\frac{1}{\sqrt[5]{4^3}}}$

128. $4^{-\frac{4}{5}}$

$\boxed{\frac{1}{\sqrt[5]{4^4}}}$

Evaluate each of the following.

129. $4^{\frac{1}{2}}$

$4^{\frac{1}{2}} = \sqrt{4} = \boxed{2}$

130. $125^{\frac{1}{3}}$

$125^{\frac{1}{3}} = \sqrt[3]{125} = \boxed{5}$

131. $8^{\frac{2}{3}}$

$(8^{\frac{1}{3}})^2$

$(\sqrt[3]{8})^2 = (2)^2 = \boxed{4}$

132. $81^{\frac{3}{4}}$

$(81^{\frac{1}{4}})^3$

$(\sqrt[4]{81})^3$

$(3)^3 = \boxed{27}$

133. $4^{\frac{3}{2}}$

$(4^{\frac{1}{2}})^3$

$(\sqrt[2]{4})^3 = (2)^3 = \boxed{8}$

134. $16^{-\frac{3}{4}}$

$\frac{1}{(16^{\frac{1}{4}})^3} = \frac{1}{(\sqrt[4]{16})^3} = \frac{1}{(2)^3}$

$= \boxed{\frac{1}{8}}$

135. $(-27)^{-\frac{2}{3}}$

$\frac{1}{((-27)^{\frac{1}{3}})^2}$

$\frac{1}{(\sqrt[3]{-27})^2} = \frac{1}{(-3)^2} = \boxed{\frac{1}{9}}$

136. $(-8)^{-\frac{5}{3}}$

$\frac{1}{((-8)^{\frac{1}{3}})^5} = \frac{1}{(\sqrt[3]{-8})^5} = \frac{1}{(-2)^5}$

$= \boxed{\frac{1}{-32}}$

137. $9^{2.5}$

$9^{\frac{5}{2}} = (9^{\frac{1}{2}})^5 = (\sqrt[2]{9})^5$

$= (3)^5 = \boxed{243}$

138. $(-1)^{-\frac{8}{5}}$

$= \frac{1}{((-1)^{\frac{1}{5}})^8}$

$= \frac{1}{(\sqrt[5]{-1})^8} = \frac{1}{(-1)^8}$

$= \frac{1}{1} = \boxed{1}$

139. $\left(\frac{100}{9}\right)^{\frac{3}{2}}$

$(100^{\frac{1}{2}})^3$ $\sqrt[2]{100^3}$

$(\sqrt[2]{100})^3$

$(10)^3 = 1000$

$(9^{\frac{1}{2}})^3$

$(\sqrt[2]{9})^3$

$(3)^3$

$= 27 \rightarrow \boxed{\frac{1000}{27}}$

140. $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$ → JUST FLIP ☆

$\cancel{\frac{1}{((\frac{27}{8})^{\frac{1}{3}})^2} = \frac{1}{(\sqrt[3]{\frac{27}{8}})^2}}$

$\cancel{\frac{1}{((\frac{3}{2}))^2} = \frac{1}{(\frac{9}{4})} = \boxed{\frac{1}{\frac{9}{4}}}}$

$(\sqrt[3]{\frac{8}{27}})^2 = (\frac{2}{3})^2$

$= \boxed{\frac{4}{9}}$

$\frac{1}{(\sqrt[3]{27})^2} = \frac{1}{3^2} = \frac{1}{9}$

$\frac{1}{(\sqrt[3]{8})^2} = \frac{1}{2^2} = \frac{1}{4}$

$= \frac{1}{9} \div \frac{1}{4} = \frac{1}{9} \times \frac{4}{1} = \boxed{\frac{4}{9}}$

✓

Write each of the following using exponents. (Do not evaluate)

Eg. $\sqrt{12} = 12^{\frac{1}{2}}$ Eg. $(\sqrt[3]{7})^4 = 7^{\frac{4}{3}}$ Eg. $\frac{1}{(\sqrt[3]{7})^2} = 7^{-\frac{2}{3}}$

141. $\sqrt{7}$

$\boxed{7^{\frac{1}{2}}}$

142. $\sqrt[3]{34}$

$\boxed{34^{\frac{1}{3}}}$

143. $\sqrt[3]{-11}$

$\boxed{(-11)^{\frac{1}{3}}}$

144. $\sqrt[5]{a^2}$

$\cancel{(a^2)}^{\frac{1}{5}} = \boxed{a^{\frac{2}{5}}}$

145. $\sqrt[3]{6^4}$

$\boxed{6^{\frac{4}{3}}}$

146. $(\sqrt[3]{x})^2$

$(x^{\frac{1}{3}})^2 = \boxed{x^{\frac{2}{3}}}$

147. $(\sqrt[5]{6})^3$

$(6^{\frac{1}{5}})^3 = \boxed{6^{\frac{3}{5}}}$

148. $(\sqrt[4]{2x})^5$

$((2x)^{\frac{1}{4}})^5$

$= \boxed{(2x)^{\frac{5}{4}}}$

149. $\frac{1}{\sqrt[3]{a}}$

$\boxed{a^{-\frac{1}{3}}}$

$(2b^3)^{\frac{1}{3}} = 2^{\frac{1}{3}} \cdot b$

$2^{\frac{1}{3}} \times b^{\frac{3}{1} \times \frac{1}{3}} = 2^{\frac{1}{3}}\, b^{\frac{3}{3}}$

$= 2^{\frac{1}{3}}\, b^{\frac{1}{1}}$

150. $\frac{1}{(\sqrt[5]{x})^4}$

$(x^{-\frac{1}{5}})^4$

$= \boxed{x^{-\frac{4}{5}}}$

151. $\frac{1}{\sqrt[4]{x^3}}$

$\boxed{x^{-\frac{3}{4}}}$

152. $\sqrt[3]{2b^3}$

$2b^{\frac{3}{3}}$

$\boxed{2b^{\frac{1}{1}}}$? $\boxed{2^{\frac{1}{3}} b}$

Evaluate if possible.

153. $(-9)^{\frac{1}{2}}$

$\sqrt[2]{-9} = 3i$

$\boxed{\text{no solution}}$

154. $100000^{\frac{3}{5}}$

$(\sqrt[5]{100000})^3$

$= (10)^3 = \boxed{1000}$

155. $\left(\frac{27}{8}\right)^{\frac{2}{3}}$

$27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = (3)^2 = 9$

$8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = (2)^2 = 4$

$= \boxed{\frac{9}{4}}$

156. $3^{\frac{1}{2}} \times 3^{\frac{1}{2}}$

$\sqrt[2]{3} \times \sqrt[2]{3} = \sqrt{3} \times \sqrt{3}$

$= \sqrt{9} = \boxed{3}$

157. $-9^{\frac{1}{2}}$

$-1 \times \sqrt[2]{9} = -1 \times 3$

$= \boxed{-3}$

158. $(2^5)^{0.4}$

$(2^5)^{\frac{4}{10}} = (2^5)^{\frac{2}{5}}$

$(\sqrt[5]{32})^2 = (2)^2 = \boxed{4}$

$\sqrt[2]{7} \times \sqrt[2]{7}$

Evaluate if possible.

159.

a. $-8^{\frac{4}{3}}$

$-1 \times (\sqrt[3]{8})^4$

$= -1 \times (2)^4 = -1 \times 16$

$= \boxed{-16}$

b. $(-8)^{\frac{4}{3}}$

$(\sqrt[3]{(-8)})^4 = (-2)^4 = \boxed{16}$

What important rule is explored above? The exponent only effects the thing closest to it.

160. $4^{\frac{3}{2}} \div 16^{\frac{1}{4}}$

$(\sqrt[2]{4})^3 \div \sqrt[4]{16}$

$(2)^3 \div 2$

$8 \div 2 = \boxed{4}$

161. $(-1)^{-\frac{3}{2}}$

$\frac{1}{(\sqrt[3]{-1})^3} = \frac{1}{i^3}$

no real solution

162. $(\sqrt[3]{5^2})(\sqrt[3]{5})$

$\cancel{\sqrt[3]{5^2}}$? $\sqrt{5}$?

$(\sqrt[3]{5})^2(\sqrt[3]{5})$?

$= \sqrt[3]{5^2} = \sqrt[3]{25} \times \sqrt[3]{5} = \sqrt[3]{5} \times \sqrt[3]{5} \times \sqrt[3]{5} = \boxed{5}$ ✓

163. $(\sqrt[4]{16})(\sqrt[5]{32})$

$(2)(2) = \boxed{4}$

164. $\sqrt{\sqrt[3]{729}}$

$\sqrt{9} = \boxed{3}$

165. Evaluate to two decimal places using a calculator

$\frac{1}{\sqrt[5]{300}}$

$\boxed{\frac{1}{3.13}} = \boxed{0.32}$

166. Evaluate to two decimal places using a calculator

$\frac{5}{\sqrt[6]{256}}$

$\boxed{1.98}$

167. Evaluate to two decimal places using a calculator

$\frac{1}{\sqrt[13]{2500}}$

$\boxed{0.55}$

168. Challenge

Write the following radicals as a single power.

$(\sqrt[2]{x^3})(\sqrt[3]{x})$

$(x^{\frac{3}{2}})(x^{\frac{1}{3}})$

$(x^{\frac{9}{6}})(x^{\frac{2}{6}})$

$\boxed{x^{\frac{11}{6}}}$

$\cancel{(x^{\frac{3}{2}})(x^{\frac{1}{3}})}$

$\cancel{x^{\frac{3}{3}} = \boxed{x^1}}$

$\cancel{(x^{\frac{3}{2}})(x^{\frac{1}{3}})}$

$\cancel{x^{\frac{3}{6}} = x^{\frac{1}{2}}}$

✓

Write each of the following radicals as a single power.

169. $(\sqrt{x^3})(\sqrt[3]{x})$

$\left(x^{\frac{3}{2}}\right)\left(x^{\frac{1}{3}}\right)$ Write as powers (both base-x).

$\left(x^{\frac{9}{6}}\right)\left(x^{\frac{2}{6}}\right)$ Create common denominators.

$\left(x^{\frac{9+2}{6}}\right)$ Add numerators.

$\left(x^{\frac{11}{6}}\right)$

170. $(\sqrt[3]{x^2})(\sqrt[4]{x^3})$

$x^{\frac{2}{3}} \times x^{\frac{3}{4}}$

$= x^{\frac{8}{12}} \times x^{\frac{9}{12}}$

$= \boxed{x^{\frac{17}{12}}}$

171. $(\sqrt[5]{x^3})(\sqrt[3]{x^2})$

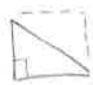
$= x^{\frac{3}{5}} \times x^{\frac{2}{3}}$

$= x^{\frac{9}{15}} \times x^{\frac{10}{15}}$

$= \boxed{x^{\frac{29}{15}}}$

More rational exponents...

172. The height and the base of a triangle each measure $2^{\frac{3}{2}}$ cm. Without using a calculator, what is the area of the triangle?

$2^{\frac{3}{2}}$  $2^{\frac{3}{2}}$

$\frac{2^{\frac{3}{2}} \times 2^{\frac{3}{2}}}{2} = \frac{2^{\frac{6}{2}}}{2} = \frac{2^3}{2}$

$= \frac{8}{2} = \boxed{4cm^2}$

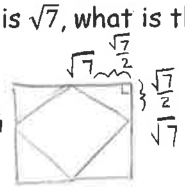
173. Find the area of a rectangle if the length is $5^{\frac{2}{3}}$ and the width is $5^{\frac{2}{5}}$. Write your answer in exponential form, then approximate to two decimal places.

$5^{\frac{2}{3}}$  $5^{\frac{2}{3}}$

$= 5^{\frac{2}{5}} \times 5^{\frac{2}{3}} = 5^{\frac{6}{15}} \times 5^{\frac{10}{15}}$

$= \boxed{5^{\frac{16}{15}} cm^2}$ $= \boxed{5.57cm^2}$

174. Inscribe a square inside another square such that the corners of the internal square contact the midpoint of sides of the larger square. If the side length of the larger square is $\sqrt{7}$, what is the area of the inscribed square? Answer in exact form.



$= \left(\frac{\sqrt{7}}{2}\right)^2 + \left(\frac{\sqrt{7}}{2}\right)^2 = 1.75 + 1.75 = \sqrt{3.5} = 1.870828693^2 = \boxed{3.5 \text{ units}^2}$

175. Simplify(write as a single power.)

$\left[(\sqrt[3]{x^4})(\sqrt[5]{x})\right]^{-2}$

$\frac{1}{\left[(\sqrt[3]{x^4})(\sqrt[5]{x})\right]^2}$

$\frac{1}{\left[(x^{\frac{4}{3}})(x^{\frac{1}{5}})\right]^2} = \frac{1}{\left[(x^{\frac{20}{15}})(x^{\frac{3}{15}})\right]^2}$

$= \frac{1}{(x^{\frac{23}{15}})^2} = \boxed{\frac{1}{x^{\frac{46}{15}}}}$

$\frac{23}{15}$ 2

176. Simplify (write as a single power.)

$\left[(\sqrt[4]{x^9})(\sqrt[3]{x^6})\right]^{\frac{2}{3}}$

$\left[(x^{\frac{9}{4}})(x^{\frac{6}{3}})\right]^{\frac{2}{3}}$

$\left[(x^{\frac{9}{4}})(x^{\frac{2}{1}})\right]^{\frac{2}{3}} = \left[(x^{\frac{9}{4}})(x^{\frac{8}{4}})\right]^{\frac{2}{3}}$

$= \left[(x^{\frac{17}{4}})\right]^{\frac{2}{3}} = x^{\frac{34}{12}} = \boxed{x^{\frac{17}{6}}}$

177. Ei-Q evaluated $64^{\frac{3}{2}}$ using the following steps. In which step did she make her first error?

Step 1: $64^{\frac{3}{2}} = (\sqrt{64})^3$

Step 2: $64^{\frac{3}{2}} = (8)^3$

Step 3: $64^{\frac{3}{2}} = 24$ $= \boxed{512}$

a) In step 1.
b) In step 2.
c) In step 3.
d) She made no error.

178. Flinflan started to evaluate $81^{-\frac{3}{4}}$ in two different ways shown below. Which of the following statements is correct?

Method 1: $81^{-\frac{3}{4}} = (\sqrt[4]{81})^{-3}$

Method 2: $81^{-\frac{3}{4}} = \frac{1}{\sqrt[4]{81^3}}$

a) Method 1 will produce the correct answer but method 2 will not.
b) Method 2 will produce the correct answer but method 1 will not.
c) Both methods will produce the correct answer.
d) Neither method will produce the correct answer.

179. Simplify: $[(\sqrt[3]{x^4})(\sqrt[5]{x^2})]^{-1}$

$\frac{1}{(\sqrt[3]{x^4})(\sqrt[5]{x^2})} = \frac{1}{(x^{\frac{4}{3}})(x^{\frac{2}{5}})} = \frac{1}{(x^{\frac{20}{15}})(x^{\frac{6}{15}})}$

$\boxed{\frac{1}{x^{\frac{26}{15}}}}$

180. Simplify: $[(\sqrt[3]{a^5})(\sqrt[4]{a^3})]^{-2}$

$\frac{1}{[(a^{\frac{5}{3}})(a^{\frac{3}{4}})]^2} = \frac{1}{[(a^{\frac{20}{12}})(a^{\frac{9}{12}})]^2} = \frac{1}{(a^{\frac{29}{12}})^2} = \frac{1}{a^{\frac{58}{12}}} = \boxed{\frac{1}{a^{\frac{29}{6}}}}$

181. Simplify:

$$\sqrt[3]{\left(a^{\frac{2}{3}}\right)^{\frac{1}{4}}}$$

$\sqrt[3]{a^{\frac{2}{12}}} = \sqrt[3]{a^{\frac{1}{6}}}$ $6\times3=18$

$\boxed{a^{\frac{1}{18}}}$?

182. Simplify:

$$\sqrt[4]{\left(x^{\frac{1}{3}}\right)^{\frac{1}{5}}}$$

$\sqrt[4]{x^{\frac{1}{15}}}$ $4\times15=60$

$\boxed{x^{\frac{1}{60}}}$?

Match each item in column 1 with an equivalent item in column 2

Column 1	Column 2
183. $\left(\frac{t}{j}\right)^{\frac{2}{3}}$ = F	A. $\sqrt[3]{\frac{j^2}{t^2}}$
184. $\left(\frac{j}{t}\right)^{\frac{3}{2}}$ = C	B. $-\left(\frac{j}{t}\right)^{\frac{3}{2}}$
185. $\left(\frac{t}{j}\right)^{-\frac{2}{3}}$ $\sqrt[3]{\frac{j^2}{t^2}}$ = A	C. $\sqrt{\frac{j^3}{t^3}}$
186. $\left(\frac{j}{t}\right)^{-\frac{3}{2}}$ = $\sqrt{\frac{t^3}{j^3}}$ = E	D. $-\left(\frac{t}{j}\right)^{\frac{2}{3}}$
187. $\left(\frac{t}{j}\right)^{-\frac{3}{2}}$ = $\sqrt{\frac{j^3}{t^3}}$ = C	E. $\sqrt{\frac{t^3}{j^3}}$
	F. $\sqrt[3]{\frac{t^2}{j^2}}$
	G. $-\left(\frac{t}{j}\right)^{\frac{3}{2}}$

188. Which of the following is equivalent to $3a^{\frac{1}{2}} \times (5a)^{\frac{1}{2}}$

$15a^{\frac{2}{2}} = 15a$

~~a. $15a$~~
b. $a\sqrt{15}$
c. $3\sqrt{5a}$
d. $3a\sqrt{5}$

$3\sqrt{a} \times 1\sqrt{5a}$

$= 3$

$3 \times a^{\frac{1}{2}} \times 5^{\frac{1}{2}} \times a^{\frac{1}{2}}$

$= 3 \times a^{\frac{1}{2}} \times a^{\frac{1}{2}} \times 5^{\frac{1}{2}}$

$= 3 \times a^{\frac{2}{2}} \times 5^{\frac{1}{2}}$

$= 3 \times a \times 5^{\frac{1}{2}}$

$= 3a\sqrt{5}$

189. Which of the following is equivalent to $2x^{\frac{1}{2}} \times (3x)^{\frac{1}{2}}$

a. $6x$
b. $x\sqrt{6}$
c. $2\sqrt{3x}$
d. $2x\sqrt{3}$

$2 \times x^{\frac{1}{2}} \times 3^{\frac{1}{2}} \times x^{\frac{1}{2}}$

$= 2 \times x^{\frac{1}{2}} \times x^{\frac{1}{2}} \times 3^{\frac{1}{2}}$

$= 2 \times x^{\frac{2}{2}} \times 3^{\frac{1}{2}}$

$= 2 \times x \times 3^{\frac{1}{2}}$

$= 2x\sqrt{3}$

190. Which of the following is not equivalent to $x^{\frac{2}{3}}$?

a. $\sqrt[3]{x^2}$ ✓ $= \boxed{x^{\frac{2}{3}}}$ ✓

b. $(\sqrt[6]{x})^4$ ✓ $= x^{\frac{4}{6}} = \boxed{x^{\frac{2}{3}}}$ ✓

c. $(x^2)(\sqrt[3]{x})$ ✓ why? $= x^{\frac{2}{1}} \times x^{\frac{1}{3}} = x^{\frac{6}{3}} \times x^{\frac{1}{3}} = \boxed{x^{\frac{7}{3}}}$ ×

d. $\sqrt{x^3}$ $= \boxed{x^{\frac{3}{2}}}$ ×

191. Which of the following is not equivalent to $a^{\frac{3}{2}}$?

a. $\sqrt[4]{a^6}$ ✓ $= a^{\frac{6}{4}} = \boxed{a^{\frac{3}{2}}}$ ✓

b. $\sqrt[3]{a^2}$ × $= a^{\frac{2}{3}}$ ×

c. $a\sqrt{a}$ ✓ $= a^{\frac{2}{2}} \times a^{\frac{1}{2}} = a^{\frac{2}{2}} \times a^{\frac{1}{2}} = \boxed{a^{\frac{3}{2}}}$ ✓

d. $\sqrt{\sqrt{a^6}}$ ✓ $= \sqrt[2]{a^{\frac{6}{2}}} = \sqrt[2]{a^3} = \boxed{a^{\frac{3}{2}}}$ ✓

192. Evaluate. Answer in simplest fraction form.

$$\frac{3^0 + 2^{-1}}{3^2 + 2^{-2}}$$

$$\cancel{\frac{2}{3^2 + 2^3} = \frac{2}{9+8} = \frac{2}{17}}$$

$$\frac{3^0 + 2^{-1}}{3^2 + 2^2} = \frac{1}{3^2 + 2^2 \times 2^1} = \frac{1}{9 + 4 \times 2} = \frac{1}{13 \times 2} = \boxed{\frac{1}{26}} \text{ ?}$$

$$\frac{3^0 + 2^{-1}}{3^2 + 2^2} = \frac{1 + \frac{1}{2}}{9+4} = \frac{1^{\frac{1}{2}}}{13} = \frac{\sqrt[2]{1}}{13}$$

$$\frac{3^0 + 2^{-1}}{3^2 + 2^2} = \frac{1 + \frac{1}{2}}{9+4} = \frac{1\frac{1}{2}}{13} = \frac{\frac{3}{2}}{13} = \frac{3}{2} \div \frac{13}{1} = \frac{3}{2} \times \frac{1}{13} = \boxed{\frac{3}{26}}$$ ✓

193. Evaluate. Answer in simplest fraction form.

$$\frac{3^{-2} + 3^2}{3^{-2} + 2^0}$$

$$\frac{3^2 \times 3^2}{2^0 \times 3^2} = \frac{3^4}{1 \times 9} = \frac{81}{9} = \frac{9}{1}$$

$$\frac{3^{-2} + 3^2}{3^{-2} + 2^0} = \frac{\frac{1}{3^2} + 9}{\frac{1}{3^2} + 1} = \frac{\frac{1}{9} + 9}{\frac{1}{9} + 1} = \frac{9\frac{1}{9}}{1\frac{1}{9}} = \frac{\frac{82}{9}}{\frac{10}{9}} = \frac{82}{9} \div \frac{10}{9} = \frac{82}{\cancel{9}} \times \frac{\cancel{9}}{10} = \frac{82}{10} = \boxed{\frac{41}{5}}$$

Answers:

1. 81
2. 2
3. x^8
4. $2x$
5. $9 \times 9 = 81$ or
   $3 \times 3 \times 3 \times 3 = 81$ or $3^4 = 81$
6. Answers vary. Similar to above.
7. $16,8,4,2,1,\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{16}$
8. Divide by 2 as you go down the list
9. Fits the pattern above.
10. Yes follows the division pattern.
11. Decreasing exponent value is like dividing by two in this case.
12. 4
13. 2^5
14. 2
15. -4^2
16. -9^2
17. $\frac{2x^3}{2x^3}, (5x)^0$
18. $(-3)^2$
19. -64
20. -27
21. -16
22. $\frac{1}{16}$
23. $-\frac{1}{16}$
24. $\frac{1}{81}$
25. $\frac{1}{81}$
26. $-\frac{1}{81}$
27. 16
28. 16
29. -16
30. 1
31. -1
32. 1
33. a^9
34. g^4
35. $15m^6$
36. a^9
37. a^{-2}
38. f^{2+x}
39. x^1
40. 2^{-2}
41. g^4
42. m^4
43. t^5
44. x^{10}
45. $15m^6$
46. $5x^6$
47. $-\frac{1}{2}a^2 = -\frac{a^2}{2}$
48. $\frac{4x^7}{5}$
49. $\frac{a^3}{3}$
50. $\frac{2}{3}$
51. 15625
52. m^6
53. $8m^{12}$
54. m^6
55. 1
56. $x^{-6}y^{-9} = \frac{1}{x^6y^9}$
57. $8m^{12}$
58. $2^{-3}c^{-12}d^{-9} = \frac{1}{8c^{12}d^9}$
59. $(-3)^{-4}x^8y^{-12} = \frac{x^8}{81y^{12}}$
60. $3^{-3}x^6y^9 = \frac{1}{27}x^6y^9$ or $\frac{x^6y^9}{27}$
61. $-18x^5y^9$
62. $128a^{12}b^2$
63. $\frac{8}{125}$
64. $\frac{125}{8}$
65. $\frac{x^3}{8}$
66. $\frac{16y^2}{9x^{10}}$
67. $\frac{x^3}{8}$
68. $\frac{a^4}{b^4}$
69. $\frac{x^{10}}{y^{15}}$
70. $\frac{-8a^6}{27y^9}$
71. $\frac{a^6}{b^4}$
72. $\frac{16x^2}{9y^2}$
73. $\frac{16y^2}{9x^{10}}$
74. $\frac{25a^6b^4c^{12}}{4}$
75. $\frac{n^3}{8m^3}$
76. $27b^6$
77. $\frac{4x^{10}}{y^{12}}$
78. $\frac{2a^2}{b^3} = \frac{2a^2}{1} \times \frac{1}{b^3}$ and $\frac{1}{b^3} = b^{-3}$
79. $\frac{12x^3}{y} = \frac{12x^3}{1} \times \frac{1}{y}$ and $\frac{1}{y} = y^{-1}$
80. $\frac{3a^2}{b^5}$
81. $\frac{3a^2}{b^5}$
82. $\frac{1}{8x^3y^3}$
83. $\frac{3a^2}{b^5}$
84. $\frac{a^2}{b^3}$
85. $2x^5y^5$
86. $\frac{3a^2}{b^3c^5}$
87. $\frac{y^6z^2}{x^8}$
88. $\frac{9}{2x^7y^{11}}$
89. $\frac{1}{8x^3y^3}$
90. $\frac{4}{a^{15}b^9}$
91. $\frac{2}{m^2n}$
92. Remember that a negative exponent can be evaluated by reciprocating the base, therefore expressions like a^{-3} become $\frac{1}{a^3}$. Notice the exponent became positive.
93. $\frac{4y^{12}}{9x^8}$
94. $\frac{27b^3}{8a^3}$
95. $\frac{1}{8x^9y^6}$
96. $\frac{4x^6}{3y^7}$
97. $\frac{1}{2}$
98. $\frac{1}{3}$
99. $x^{\frac{1}{n}} = \sqrt[n]{x}$
100. Possible answer:
     $\sqrt[4]{3} \times \sqrt[4]{3} \times \sqrt[4]{3} \times \sqrt[4]{3} = 3$
     $3^{\frac{1}{4}} \times 3^{\frac{1}{4}} \times 3^{\frac{1}{4}} \times 3^{\frac{1}{4}} = 3$
     $\therefore \sqrt[4]{3} = 3^{\frac{1}{4}}$
101. 7
102. -4
103. *no real number*
104. 4
105. $\frac{1}{3}$
106. $\frac{1}{2}$
107. 10
108. $2x$
109. $\frac{1}{3x^2}$
110. $\sqrt{7}$
111. $\sqrt[3]{3x}$
112. $\sqrt[5]{4}$
113. $\frac{1}{\sqrt[5]{4}}$
114. $-\sqrt[3]{64}$
115. $\frac{1}{\sqrt[3]{64}}$
116. $13^{\frac{1}{2}}$
117. $-3x^{\frac{1}{2}}$
118. $(2y)^{\frac{1}{2}}$
119. $4^{\frac{1}{4}}$
120. $4^{\frac{1}{7}}$
121. $(3x)^{-\frac{1}{5}}$
122. $27^{\frac{2}{3}} = \left(27^{\frac{1}{3}}\right)^2$
     $27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$
     $27^{\frac{2}{3}} = (3)^2$
     $27^{\frac{2}{3}} = 9$
123. $\sqrt[5]{4^2}$ or $\left(\sqrt[5]{4}\right)^2$
124. $\sqrt[5]{4^3}$ or $\left(\sqrt[5]{4}\right)^3$
125. $\sqrt[5]{4^4}$ or $\left(\sqrt[5]{4}\right)^4$
126. $\frac{1}{\sqrt[5]{4^2}}$ or $\frac{1}{\left(\sqrt[5]{4}\right)^2}$
127. $\frac{1}{\sqrt[5]{4^3}}$ or $\frac{1}{\left(\sqrt[5]{4}\right)^3}$
128. $\frac{1}{\sqrt[5]{4^4}}$ or $\frac{1}{\left(\sqrt[5]{4}\right)^4}$
129. $\sqrt{4} = 2$
130. $\sqrt[3]{125} = 5$
131. $\left(\sqrt[3]{8}\right)^2 = 4$
132. $\left(\sqrt[4]{81}\right)^3 = 27$
133. $\left(\sqrt{4}\right)^3 = 8$
134. $\frac{1}{\left(\sqrt[4]{16}\right)^3} = \frac{1}{8}$
135. $\frac{1}{\left(\sqrt[3]{-27}\right)^2} = \frac{1}{9}$
136. $\frac{1}{\left(\sqrt[3]{-8}\right)^5} = -\frac{1}{32}$
137. $9^{\frac{5}{2}} = \left(\sqrt{9}\right)^5 = 243$

138. 1
139. $\frac{1000}{27}$
140. $\frac{4}{9}$
141. $7^{\frac{1}{2}}$
142. $34^{\frac{1}{3}}$
143. $(-11)^{\frac{1}{3}}$
144. $a^{\frac{2}{5}}$
145. $6^{\frac{4}{3}}$
146. $x^{\frac{2}{3}}$
147. $6^{\frac{3}{5}}$
148. $(2x)^{\frac{5}{4}}$
149. $a^{-\frac{1}{9}}$
150. $x^{-\frac{4}{5}}$
151. $x^{-\frac{3}{4}}$
152. $2^{\frac{1}{3}}b$
153. *no real solution*
154. 1000
155. $\frac{9}{4}$
156. 3
157. -3
158. 4
159. a)-16 b) 16
160. 4
161. *no real solution*
162. 5
163. 4
164. 3
165. 0.32
166. 1.98
167. 0.55
168. $x^{\frac{11}{6}}$
169. *Answered on page.*
170. $x^{\frac{17}{12}}$
171. $x^{\frac{19}{15}}$
172. $4\ cm^2$
173. $5^{\frac{16}{15}}\ \text{cm}^2 \cong 5.57\ \text{cm}^2$
174. $\frac{7}{2}$ *or* $3.5\ cm^2$
175. $x^{-\frac{46}{15}}$ or $\frac{1}{x^{\frac{46}{15}}}$
176. $x^{\frac{17}{6}}$
177. c
178. c
179. $x^{-\frac{26}{15}} = \frac{1}{x^{\frac{26}{15}}}$
180. $a^{-\frac{29}{6}} = \frac{1}{a^{\frac{29}{6}}}$
181. $a^{\frac{1}{18}}$
182. $x^{\frac{1}{60}}$
183. F
184. C
185. A
186. E
187. C
188. D
189. D
190. C,D
191. B
192. $\frac{3}{26}$
193. $\frac{41}{5}$