Foundations \& Pre-Calculus 10 Homework \& Notebook

Name: $\qquad$ Teacher: Miss Zukowski

## Unit 5: Relations \& Functions

Submission Checklist: (make sure you have included all components for full marks)
$\square$ Cover page \& Assignment Log
$\square$ Class Notes

- Homework (attached any extra pages to back)
$\square$ Quizzes (attached original quiz + corrections made on separate page)
- Practice Test/ Review Assignment

| Assignment Rubric: Marking Criteria |  |  |  |
| :---: | :---: | :---: | :---: |
| Excellent (5) - Good (4) - Satisfactory (3) - Needs Improvement (2) - Incomplete (1) - NHI (0) |  | Self <br> Assessment | Teacher Assessment |
| Notebook | - All teacher notes complete <br> - Daily homework assignments have been recorded \& completed (front page) <br> - Booklet is neat, organized \& well presented (ie: name on, no rips/stains, all pages, no scribbles/doodles, etc) | /5 | /5 |
| Homework | - All questions attempted/completed <br> - All questions marked (use answer key, correct if needed) | /5 | /5 |
| Quiz <br> (1mark/dot point) | - Corrections have been made accurately <br> - Corrections made in a different colour pen/pencil ( $+1 / 2$ mark for each correction on the quiz) | /2 | /2 |
| Practice <br> Test <br> (1mark/dot <br> point) | - Student has completed all questions <br> - Mathematical working out leading to an answer is shown <br> - Questions are marked (answer key online) | /3 | /3 |
| Punctuality | - All checklist items were submitted, and completed on the day of the unit test. (-1 each day late) | /5 | /5 |
| Comments: |  | /20 | /20 |



## Homework Assignment Log

\& Textbook Pages $\qquad$

| Date | Assignment/Worksheet | Due Date | Completed? |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Quizzes \& Tests:

| What? | When? | Completed? |
| :--- | :--- | :--- |
| Quiz 1 |  |  |
| Quiz 2 |  |  |
| Unit/ Chapter test |  |  |

## 1) RELATIONS E FUNCTIONS: INTRODUCTION

1. Using the following graph, answer the questions below. The graph shows the distance a rock climber is from the base of the cliff as time passes.
a) Place each line segment in the appropriate section of the table. OA, AB, BC, CD, DE, EF, FG.

| Climbing | Resting | Descending |
| :---: | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |


b) Describe one property a line segment has if the climber is climbing.
c) Describe one property a line segment has if the climber is resting.
d) Describe one property a line segment has if the climber is descending.
e) How would the graph of the line segment be different if he increased his speed for the first time he climbed?
f) What would you add to the graph to show the climbers return to the bottom of the cliff?
2. Match each graph below with a situation from the list given. Then, draw each graph carefully labeling each axis to show the quantities being compared.

a) the temperature of a cup of hot chocolate over time
b) a car accelerating to a constant speed
c) the distance a person walks during a hike
d) the height of a soccer ball kicked across a field

| 而 |  |
| :--- | :--- | :--- |

3. Create a speed-time graph for the following scenario. Label each section of your graph with capital letters, and write a description of what is happening at each line segment.

> Connor is riding his skateboard along a path. Almost immediately after leaving home, Connor travels down a short steep hill. At the bottom, the path makes a turn. The remainder of the trip is on relatively flat land. Connor kicks to keep moving. He then stops before a railway crossing. He also practises a few tricks along the way. He completes a basic "ollie" and performs a second ollie over a speed bump. Finally, after travelling at a constant rate for the last part of the trip, Connor arrives at his destination.

## Graph:

> Hint: make sure to graph the independent variable on the horizontal axis, and the dependent variable on the vertical axis.

## Explanation:

## Summary Ideas:

$>$ A graph represents the relationship between two quantities.
$>$ Straight lines are used to indicate a constant rate of change.
$>$ Horizontal lines are used if one quantity is NOT changing relative to the change in the other quantity.

$>$ A steeper line indicates a $\qquad$ rate of change. This line could represent either an increase or a decrease.
$>$ A curve shows that the rate of change is $\qquad$ .

Key Terms

| Term | Definition | Example |
| :--- | :--- | :--- |
| Relation |  |  |
| Function |  |  |
| Ordered pair |  |  |
| Coordinate Plane |  |  |
| x-axis |  |  |
| y-axis |  |  |
| Domain |  |  |
| Range |  |  |
| Element |  |  |
| Permissible values |  |  |
| Dependent |  |  |
| Variable |  |  |
| Independent <br> Variable |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

## Introduction to Relations

Relationships exist everywhere we look...

- There is a relationship between the lengths of lineups at the fair and how exciting the rides are.
- There is a relationship between the height of a ball and how long ago it was kicked.
- There is a relationship between traffic and the time of day.
- There is a relationship between distance travelled and the speed of the car.

Some relationships don't even seem to have a mathematical relationship but are connected in some other way.
For example: The students in your class all have a birth month and height. We could write a list matching each student's birth month and height.

As ordered pairs... $(3,155),(5,138),(11,162),(12,135),(7,142), \ldots$
(March, 155 cm tall)

Some notes here...
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Challenge Question:

1. Give examples of three other relationships you see on an everyday basis:
2. Write a set of 3 ordered pairs for one of your relationships above. Explain what the ordered pair means.

Use the following information to answer questions below.
Consider the data given in following table:

| Student | Height (cm) | Arm Span (cm) | Written as ordered pairs |
| :---: | :---: | :---: | :---: |
| Lulu | 135 | 137 |  |
| Bones | 144 | 151 | $(135,137)$ |
| Phat Charlie | 150 | 148 | $(144,151)$ |
| Lucky | 150 | 156 | $(150,148)$ |
| Dizzy Dee | 165 | 165 | $(150,156)$ |
| Crash | 155 | 152 | $(165,165)$ |
| Anjohkinu | 160 | 164 | $(155,152)$ |
| Sam | 200 | 210 | $(160,164)$ |
| Talloola | 125 | 127 | $\begin{aligned} & (200,210) \\ & (125,127) \end{aligned}$ |

3. Why do you think the numbers in brackets are called "ordered pairs"?
4. The data above represents a relation between what two quantities?
5. Graph the data in the table above (Trouble graphing? See next page).

6. Describe the relationship you see on the graph above.
(What does it look like? What shape is it?)

## 2) RELATIONS छ FUNCTIONS: IRAPHING RELATIONS

Warm-Up \#1: You are walking to school at a rate of $100 \mathrm{ft} / \mathrm{min}$.
a) Copy and complete the table of values for this scenario.
b) Graph your data on the grid below.

| Time, $\boldsymbol{t}(\mathbf{s})$ | Distance Walked, <br> $\boldsymbol{d}(\mathrm{ft})$ |
| :---: | :---: |
| 30 |  |
| 60 |  |
| 90 |  |
| 120 |  |



Warm-Up \#2: You are walking to school at a rate of $150 \mathrm{ft} / \mathrm{min}$.
a) Copy and complete the table of values for this scenario.
b) On the same grid as warm-up \#1, graph the new walking data in a different colour.

| Time, $\boldsymbol{t}(\mathrm{s})$ | Distance Walked, <br> $\boldsymbol{d}(\mathrm{ft})$ |
| :---: | :---: |
| 30 |  |
| 60 |  |
| 90 |  |
| 120 |  |

Warm-Up \#3: You live 5 km from school. You sleep in for the first 10 min of class.
a) Complete the table of values for this scenario.
b) Graph your data on the grid below.

| Time (min), $\boldsymbol{t}$ | Distance From <br> School, $\boldsymbol{d}(\mathrm{km})$ |
| :---: | :---: |
| 0 |  |
| 2 |  |
| 4 |  |
| 6 |  |



## What is a Relation??

| Relation |  |
| :--- | :--- |

## Relations can be Represented in Many Ways:

1. In Words: $\qquad$
2. A Table of Values:
3. A Set of Ordered Pairs: $\qquad$
4. An Equation: $\qquad$
5. A Graph:

Example 1: The value of a car depreciates with each year. Using the information from the table of values, present the relation in each of the other ways.

| Time (years) | Value (thousands <br> of dollars) |
| :---: | :---: |
| 0 | 15 |
| 2 | 13 |
| 4 | 11 |
| 6 | 9 |
| 8 | 7 |

Ordered Pairs:


Equation: $\qquad$

Words: $\qquad$

Example 2: Using a table of values, plot the relation described by $y=x$.

| $x$ | $y$ |
| :---: | :---: |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |



Example 3: Using a table of values, plot the relation described by $y=3 x-4$.

| $x$ | $y$ |
| :---: | :---: |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |



Example 4: Using a table of values, plot the relation described by $y=-\frac{3}{2} x$.

| $x$ | $y$ |
| :---: | :---: |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |



## What is a relation?

## Definition 1:

If two groups of items are related, the set of all possible pairings is called a relation.
For example:
A person's height and their arm span.
Distance travelled and driving time.
Exam score and study time.

## Definition 2:

A relation is the set of ordered pairs that connects two sets.

## Definition 3:

7. Write your own...

| Domain: |  |
| :--- | :--- | :--- |
| The set of first items in a relation. | Range: |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

Some notes here (possibly)...
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Graphing Relations on a Coordinate Plane

Below are two examples of the Coordinate Plane
8. The vertical line with numbers on it is called the $\qquad$
9. The horizontal line with numbers on it is called the $\qquad$


10. What is the difference between each of the graphs shown above?
11. Describe a scenario where it is more appropriate to use the graph on the right.
12. Describe a scenario where it is more appropriate to use the graph on the left.
$\qquad$
$\qquad$
13. How could you describe to another student where to plot a point on the plane? For example: $(2,5)$
$\qquad$
14. Plot and label the following ordered pairs on each of the grids above (whenever possible): $A(1,2), B(-3,5), C(10,4), D(-3,-7), E(8,-2)$

Coordinate Geometry (Cartesian Coordinate Geometry)
Based on the coordinate plane.


The coordinate plane has two axes.
The vertical $\boldsymbol{y}$-axis.
The horizontal $\mathbf{x}$-axis.
The point where the axes meet is called the origin.

Every point on the plane can be located using two numbers called coordinates or an ordered pair.

The ordered pair is always given as $(x, y)$. The coordinates of the origin are $(0,0)$.
Example: Point A has coordinates (4,2).
4 is an element of the domain. 2 is an element of the range.
15. Challenge Question:

Using the graph below, plot the relation described by the equation $y=2 x$.


Graphing relations using a Table of Values.

Using the graph below, plot the relation described by the equation $y=2 x$.

| $y=2 x$ |  |
| :---: | :---: |
| x | y |
| -2 | -4 |
| -1 | -2 |
| 0 | 0 |
| 1 | -2 |
| 2 | -4 |

A Table of Values:
Choose a few reasonable input values ( x ), then calculate output values (y). This produces some ordered pairs to plot our relation.

17. Are there any input values that would not make sense? Are there any that are "not permitted"?

Using the graph below, plot the relation described by the equation $y=\sqrt{x+2}$.

| $y=\sqrt{x+2}$ |  |
| :---: | :---: |
| x | y |
| -2 | 0 |
| -1 | 1 |
| 2 | 2 |
| 7 | 3 |$\quad$| Can you see why I |
| :--- |
| chose these |
| particular input ( x ) |
| values? |


18. Are there any input values that would not make sense? Are there any that are "not permitted"?
19. Consider your answers to the previous two questions. What effect do "not permitted" input values have on the graph of the relation?

- Positive Inputs (domain)

When creating a table of values, you should consider:

- Negative Inputs (domain)
- Zeros
- Non-permitted input values

Graphing Relations continued...


Graphing Relations continued...
26. Using the table and graph below, plot the relation described by the equation $y=x^{2}$.


| $y=x^{2}$ |  |
| :---: | :---: |
| x | y |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

30. Using the table and graph below, plot the relation described by the equation $y=$ $-2 x-1$.

31. Using the table and graph below, plot the relation described by the equation $y=$ $\frac{3}{2} x$.


| $y=\frac{3}{2} x$ |  |
| :---: | :---: |
| x | y |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

Graphing Relations continued...
32. Using the table and graph below, plot the relation described by the equation $y=$ $1-2 x$.


| $y=1-2 x$ |  |
| :---: | :---: |
| x | y |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

34. Using the table and graph below, plot the relation described by the equation $x=$ $2 y$.


| $y=x$ |  |
| :---: | :---: |
| x | y |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

33. Using the table and graph below, plot the relation described by the equation $y=$ $x$.

34. ${ }^{* *}$ CHALLENGE**Using the table and graph below, plot the relation described by the equation $y=|x|$
(where $x$ is an absolute value, meaning the magnitude of a real number without regard to its sign)


| $y=\|x\|$ |  |
| :---: | :---: |
| x | y |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

## 3) RELATIONS ६ FUNCTIONS: DOMA IN AND RANGE

Warm-Up \#1: List all the values of $x$ in each relation.
a) $(1,3),(2,5),(9,4)$
b)

| $x$ | $y$ |
| ---: | ---: |
| -3 | 4 |
| -1 | 7 |
| 0 | 1 |

c) $y=2 x-3$

Warm- Up \#2: List all the values of $y$ in each relation.
a) $(1,3),(2,5),(9,4)$
b)

| $x$ | $y$ |
| :---: | :---: |
| -3 | 4 |
| -1 | 7 |
| 0 | 1 |

c) $y=2 x-3$

Warm-Up \#3: For each relation, write an inequality statement.
a)

b)

c)

d)


What is different about part d in the warm up?

Warm-Up \#4: Fill in the table below, with the correct symbol for each number set. Then, identify the number set represented in the examples from warm-up \#1.

| Number Set | Symbol |
| :---: | :---: |
| Real |  |
| Rational |  |
| Irrational |  |
| Integer |  |
| Whole |  |
| Natural |  |

Review your answer in the warm up. Identify the appropriate number set for each inequality.
a)
b)
c)
d) $\qquad$

When interpreting information to solve a problem, it is important to make sense of the possible values of each quantity being compared.

Example \#1: Determine the possible values for each quantity in the given relation.


When comparing two quantities, the words DOMAIN and RANGE are used to describe the values that are appropriate.

The $\qquad$ is the set of all possible values for the independent variable in a relation.

The $\qquad$ is the set of all possible values for the dependent variable in a relation.

There are a variety of ways to express the domain and range of a relation.

1. Words - A description of the value that are allowed.

Example: the range is the set of all whole numbers less than twenty
2. Number Line - A picture of the values that are allowed.

Example:
3. $\mathbf{A}$ List - used for discrete data

Example: For the relation $\{(3,1),(2,-3),(7,0.4)\}$
the domain is $\qquad$
the range is $\qquad$
4. Set Notation - a formal way to give the values of the domain and range.

Example: $\{x \mid x \geq-1, x \in Z\}$

```
What does it all mean???
{ } = type of brackets used for a set
    means "such that"
E means "is an element of"
    (or "belongs to")
This statement is read as:
```

5. Interval Notation - uses different brackets to indicate an interval

Example: $[0,10]$ means all numbers between zero to ten inclusive.
Example: $(0,10)$ means all numbers between zero and ten (not including 0 or 10)
Example: $(10, \infty)$ means all numbers greater than 10

Example \#2: Consider the Relation: all real numbers between -5 and 2 including -5 but not including 2.

| Number Line: |
| :--- | :--- |
| Set Notation: |
| Interval Notation: |
| Could I use a list? Explain. |
|  |

Example \#3: Complete the table.

| Words: your age from grade 1 until now |
| :--- | :--- |
| Number Line: |
| List: |
| Set Notation: |
| Interval Notation: |
|  |

Let's see how this applies to graphs!

Example \#4: Write the domain and range for each relation, as specified.
a)


Domain as a list:

Range in set notation:
c)


Domain in set notation:

Range in set notation:
e) $\{(4,-1),(-1,4),(-1,3),(4,-3),(-3,0)\}$

Domain in set notation:

Range in set notation:
b)


Domain in words:

Range in interval notation:
d)


Domain in interval notation:

Range in interval notation:

## Domain \& Range (continued)

Recall, $(2,5)$ and $(-3,7)$ are called ordered pairs because the order of the two elements is important.

- The first set of elements in the ordered pair is called the domain of the relation.
- The second set of elements in the ordered pair is called the range of the relation.

36. Challenge Question:

List the domain and range for the relation $(1,1),(2,4),(3,9),(4,16)$
Answer:
Domain: $\{1,2,3,4\}$
37. Which of the following is/are true?
a. The domain is the set of permissible values of $x$.
b. The domain is the set of permissible values of $y$.
c. The range is the set of permissible values of $x$.
d. The range is the set of permissible values of $y$.

Your notes here...

Remember, domain is all "first elements" and range is all "second elements".

| Since we are often <br> working with graphs <br> that have an x-axis <br> and a y-axis. | Domain is often described as all permissible values of $x$ |
| :--- | ---: | :--- |
|  | Range is often described as all permissible values of $y$. |

Find the domain and range:

## Example:

Find the domain and range of the relation:
$(2,3),(3,4),(4,5),(5,6)$

## Solution:

Simply list the first elements, then second:
domain: $\{2,3,4,5\} \quad$ range: $\{3,4,5,6\}$

## Example:

Find the domain and range of the following relation.


## Solution:

First, find the coordinates of the BIG points:
$(2,4),(7,9)$, $(10,-3)$

Domain: $\{2,7,10\}$
Range: $\{4,9,-3\}$
**It's OK that there is no apparent pattern...this is still a relation.

Find each of the following.

41. Challenge Question: Find the domain of the following graph.

42. How many items are there in the domain of the relation above?
43. What is the smallest item in the domain?
44. What is the biggest value in the domain?
45. How many items are there in the range?
46. What is the smallest item in the range?
47. What is the biggest item in the range?

When the graph of a relation is a line or curve, the domain and range cannot be expressed as a list of numbers as in the earlier questions. Why is this so?

Consider Example A and B.

Example A


Example B


| Use Inequalities | Use Interval Notation | Use a number line |
| :---: | :---: | :---: |
| Example A | Example A | Example A |
| Domain: $-4 \leq x<3$ | Domain:[-4,3) | Domain: $\longleftarrow$ |
| Range: $2<y \leq 4$ | Range: ( 2,4$]$ | $\text { Range: } \longleftrightarrow \underset{0}{\mathrm{O}_{2}^{1010}}$ |
| Example B | Example B | Example B |
| Domain: $x \geq-4$ | Domain: $[-4, \infty)$ | Domain: |
| Range: $y \leq 3$ | Range: $(\infty, 3]$ | Range: |
| The inequality symbols: $<,>, \leq, \geq, \neq$ | Brackets are used to show the interval. |  |
| Set Notation: <br> $\boldsymbol{x} \boldsymbol{\epsilon} \boldsymbol{R}$ : The domain is the set of real numbers. | [ if the number is included ( if the number is not included $\infty$ is used if the set does not end. | included. <br> Hollow circles indicate the number is not included. |
| $\{y \mid y \leq 0, y \epsilon R\}$ : The range is the set of real numbers less than or | $(-\infty, \infty)$ : No upper or lower limit, or, "all real numbers". |  |
|  | $(3, \infty)$ : All real numbers greater than 3. |  |


57. Try to match each of the following graphs with domain and range below. (There are three on each graph)


A. $x \in R, \quad y \in R$
B. $[1,9]$ and $[-7,1]$
C. $\{x \mid x \in R\},\{y \mid y \leq 0, y \in R\}$
D. domain $[4,9]$ range $[4,7]$
E. $\{x \mid x \geq-7, x \in R\}, \quad\{y \mid y \geq 1, y \in R\}$
F. Domain is all real numbers from -5 to 8 . Range is all real numbers from 3 to 8.

Find the domain and range for each of the following graphs.


Find the domain and range for each of the following graphs.

64. Use a list (discrete):

domain $\qquad$
range $\qquad$
65. Use a list (discrete):

domain $\qquad$
range
66. Use interval notation:

domain $\qquad$
range

## 4) RELATIONS \& FUNCTIONS: IRAPHING IN A DOMAIN

In Lesson 2, when we graphed functions, we graphed in the domain of all real numbers.
Now, when you graph a relation, the domain may be:

1. Given as $\mathbf{R}$ (all real numbers)
2. Given as a list, for example: $\{-2,-1,0,1,2\}$
3. Given as an inequality, for example: $\{x \geq 0\}$

The domain that you are given just tells you what values of $x$ you are allowed to plug into your table of values

Example \#1: Graph the relation $y=2 x+1$ in the following domains.
a) $\{x \mid x \in R\}$

| $x$ | $y$ |
| :--- | :--- |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |


b) $\{x \mid x=-4,-3,0,4\}$

| $x$ | $y$ |
| :--- | :--- |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |


c) $\{x \mid x<-1, x \in R\}$

| $x$ | $y$ |
| :---: | :---: |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |



Example \#2: Find the domain and range of the following relations.
a) $y=2 x-8$

Domain $=$

Range $=$
b) $y=\sqrt{x+3}$

Domain $=$

Range $=$
c) $y=x^{2}-1$

## Domain $=$

Range $=$

Hint: Think about this in one of two ways:

1. Visualize the graph
2. Consider if there are any $x$-values that you can't plug into the equation (ones that will give you an ERROR in your calculator)


## ASSIGNMENT \# 4

67. Draw a graph using the following domain and range.

$\{x \mid x \geq 4, x \in R\}$,
$\{y \mid y \leq-3, y \in R\}$
68. Draw a graph using the following domain and range.

$\{x \mid-3 \leq x \leq 2, x \in R\}$
$\{y \mid 5 \leq y \leq 8, y \in R\}$
69. Draw a graph using the following domain and range.

$\{x \mid x=5\}$,
$\{y \mid y \geq-4, y \in R\}$
70. What is the domain of $y=3 x$ ?

71. Challenge Question:

Graph the line represented by the equation $y=3 x$ if the domain is $x \geq-2$.


## Graphing Relations and Domain:

When graphing a relation, the domain may be:

- Given as $\boldsymbol{R}$ (all real numbers)
- Given as a list such as $\{-2,-1,0,1,2\}$
- Given as an inequality such as $x \geq 0$.


We will consider the impact each of these have when graphing the relation $y=2 x$

$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Graph the relation $y=2 x$. The domain is $\{x \mid x \in R\}$.


Any values of $x$ would be permissible. This results in a continuous line in both directions.

Two arrow heads!

Graph the relation $y=2 x$. The domain is $\{-\mathbf{2},-\mathbf{1}, \mathbf{0}, \mathbf{1}, \mathbf{2}\}$


Only the specified values of $x$ would be permissible $\{-2,-1,0,1,2\}$. This results in five discrete points on the graph.

Find the $y$ values that go with these five $x$ values.

Graph the relation $y=2 x$.
Domain: $(0, \infty)$.


Any values of $x$ greater than 0 would be permissible. This results in a continuous line starting at $x=0$ and moving in the positive direction.

One arrow head!

Graph each of the following for the given domain.

82. In your own words, describe the different ways a relation may look due to restrictions on the domain.

## Finding the domain and range of an equation.

Becoming more familiar with the equation of particular relations (assuming there is one) allows you to quickly determine the domain or range.

Possible Strategies:

- Visualize the graph from memory (or actually plot it).
- Consider possible restrictions based on the equation.

For example, $y=\sqrt{x}$ has a domain $x \geq 0$ because all negative values of $x$ produce a "not real" output.

| 83. Find the domain of the relation: $y=3 x$ | 84. Find the domain of the relation: $y=\sqrt{x-2}$ | 85. Find the domain of the relation: $y=x^{2}$ |
| :---: | :---: | :---: |
| 86. Find the range of the relation: $y=3 x$ | 87. Find the range of the relation: $y=\sqrt{x-2}$ | 88. Find the range of the relation: $y=x^{2}$ |

## 89. Challenge Question:

Consider the various ways graphs look because of the restrictions on their domain before you answer the following question.

Use the equation $C=10 n$ to graph the cost, C , of a family with ' n ' people to go to the movies.

90. Challenge Question:

Find a reasonable domain for the function above.

Find a reasonable range for the function above.

## Some notes here...

$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

# 5) RELATIONS छ FUNCTIONS: CONTINUOUS/DISCRETE \& VERTICAL LINE TEST 

Warm-Up: Students at Reynolds are selling t-shirts during lunch for $\$ 10$ each.
a) Complete the following table of values:

| Number of t- <br> shirts sold | Total amount of <br> money made, in <br> dollars |
| :---: | :---: |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |

b) Graph the relation:
c) Can the dots be connected? Explain.

d) Fill in the table below, and add in your own example in.

| Type of Data | Continuous | Discrete |
| :---: | :---: | :---: |
| Characteristics | - Graph will appear as $\qquad$ <br> - Occurs when quantities don't $\qquad$ values. <br> - Occurs when having $\qquad$ of quantities makes sense. | - Graph will appear as $\qquad$ <br> - Occurs when quantities can only be $\qquad$ <br> - Occurs when part numbers $\qquad$ |
| Example |  |  |

Introduction to Functions

## A function is

When a relation is presented as a graph, a quick method to determine whether or not it is a function is known as the VERTICAL LINE TEST.


If a vertical line intersects the graph at more than one point, the relation IS NOT a function.

Example \#1: Do these graphs represent functions?
a)

b)

c)


Example \#2: Do these relations represent functions? Justify your choice.
a) $\{(1,3),(2,4),(3,5),(4,3),(2,1)\}$
b)

| Name | Shoe Size |
| :--- | :---: |
| Andrew | 10 |
| Nathan | 11 |
| Joel | 12 |
| Aaron | 13 |
| Simon | 12 |

d) $y=3 x+5$
c)

| Name | Sibling |
| :--- | :---: |
| Anika | Jared |
| Anika | Joel |
| Anika | Nathan |
| Caroline | Aaron |
| Caroline | Simon |

e) $y^{2}=x$

## Discrete and Continuous Data

From previous page...
Use the equation $C=10 n$ to graph the cost, C , of a family with ' n ' people to go to the movies.

91. Why is the graph above a series of dots, not a continuous graph?

## Continuous Data:

- Graph will appear as a line.
- Occurs when quantities don't "skip" values...continuous things like time and temperature.


## Discrete Data:

- Graphs will appear as a series of dots.
- Quantities such as whole items (people, cars, hamburgers, etc.)
- When part numbers don't "make sense."

Answer the following questions.
92. A cup of coffee sits on the counter for several hours. Describe what would happen to the temperature of the coffee in the cup.
94. A high school student is surveying the volume of traffic in the school parking lot. The number of cars in the parking lot is a function of the time of day.
a) What would be a reasonable range for this function?
b) Discrete or Continuous (Circle one)
96. The Cost of Energy Bars:

Cost is a function of $\qquad$ .

Discrete or Continuous (Circle one)
93. A cup of coffee sits on the counter for several hours. Temperature is a function of the time the coffee is on the counter.
a) What would be a reasonable domain for this function?
b) Discrete or Continuous (Circle one)
95. Dalleep is plotting his height as a function of time.
a) What quantity would represent the domain? Height or Time (Circle one)
b) Height:

Discrete or Continuous (Circle one)
c) Time:

Discrete or Continuous (Circle one)
d) What would be a reasonable range?
97. Total Earnings ( with hourly wage):

Earnings are a function of $\qquad$ .

Discrete or Continuous (Circle one)

## Functions:

A special class of relation in which there is only one output $(y)$ for every valid input ( $x$ ).

Consider the following tables:


| Table 1 |  |
| :---: | :---: |
| $x$ | $y$ |
| -1 | 3 |
| 0 | 0 |
| 2 | -6 |
| 4 | -12 |

Domain ( $x$ ): $\{-1,0,2,4\}$
Range (y): $\{3,0,-6,-12\}$

| Table 2 |  |
| :---: | :---: |
| $x$ | $y$ |
| -2 | 4 |
| -1 | 1 |
| 0 | 0 |
| 1 | 1 |

Domain $(x):\{-2,-1,0,1\}$
Range (y): $\{4,1,0,1\}$

| Table 3 |  |
| :---: | :---: |
| $x$ | $y$ |
| 4 | 2 |
| 3 | 7 |
| 2 | 12 |
| 4 | 17 |

Domain ( $x$ ): $\{4,3,2,4\}$
Range(y): $\{2,7,12,17\}$

Notice in Table 3, when the input $(x)$ is 4, there are two possible outputs, 2 or 17 . This is NOT a function.
Tables 1 and 2 are both functions. Each element in the domain produces only one element in the range.

Some notes here possibly...

Which of the following relations are functions? Indicate why or why not.

| 98. $(2,1),(3,5),(7,8),(9,12)$ | 99. $(1,3),(5,3),(7,3),(12,3)$ |  | 100. $(3,1),(3,5),(2,6),(1,4)$ |  |
| :---: | :---: | :---: | :---: | :---: |
| YES or NO | YES or NOWhy? |  | YES or NO |  |
| Why? |  |  | Why? |  |
| 101. $y= \pm \sqrt{x}$ | 102. |  | 103. |  |
|  | - x | 7 y | x | y |
|  | -3 | 7 | -3 | 11 |
|  | 2 | 3 | 2 | 5 |
|  | 5 | 5 | -3 | -6 |
|  | 1 |  | 8 | 7 |
| YES or NO | YES or NO |  | or NO |  |
| Why? | Why? |  | ? |  |

## The Vertical Line Test:

You can test whether a relation is a function by using the vertical line test.
If you move a vertical line through the relation from left to right, the vertical line will only ever contact a function once. If the vertical line contacts the graph more than once at a given time, it is not a function.

Eg.1.


Yes, it is a function.

A vertical line will contact the function only once.

Eg. 2.


No, not a function.
A vertical line may contact the function twice.

Determine if each of the following relations is a function or not.


## Some notes here possibly...

$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## 6) RELATIONS छ FUNCTIONS: FUNCTION NOTATION

Warm-Up \#1: Complete the table of values for the equation $y=3 x+2$

Does this equation represent a function? $\qquad$

| $x$ | $y$ |
| :---: | :---: |
| -3 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 3 |  |

Warm-Up \#2: Evaluate $y=2 x^{2}-3 x+5$ for each of the given values.
a) $x=-3$
b) $x=3$

Warm- Up \#3: Evaluate $y=3 x-5$ for each of the given values.
a) $y=10$
b) $y=-26$

Investigation: If $f(x)=3 x+2$ determine the following.
a) $f(-3)$
b) $f(-1)$
c) Predict the value of $f(0), f(3)$.

Example \#1: If $p(a)=-2 a+5$, determine the following:
a) $p(-2)$
b) $p(1)$
c) $a$, if $p(a)=1$
d) $a$, if $p(a)=-5$
e) Use your results to create a table of values.

f) Graph the function.

g) Is this discrete or continuous data? Explain.

Example \#2: If $g(x)=5 x-1$, determine a simplified expression for each of the following:
a) $g(2 x)$
b) $g(x-5)$

Example \#3: Use the graph below to determine the values.
a) $f(-1)$
b) $f(0)$

d) $x$ when $f(x)=4$

## Function Notation:

There is a special way to write functions. This is called function notation.
Consider the following comparisons:

## Equation Function Notation

$\begin{array}{ll}y=x+2 \longrightarrow & \\ y=3 x-5 \longrightarrow & f(x)=x+2 \\ C=20 t+1200 \longrightarrow & \\ C=3 x-5 \\ G=3 h^{2}-2 \longrightarrow & \longrightarrow \\ & \\ & \end{array}$
Notice the $f(x)$ part simply replaces the $y$. There is
no new operation; it is only a new way of writing the
equation. It immediately tells you "this is a
function."
Notice the letter in brackets is always the same as the
one on the right.

- Function notation allows us to use letters appropriate to our function and differentiate between several functions (give them unique names).
- Also the notation tells us which variable is dependent on the other.

Eg. $g(h)=3 h^{2}-2 \quad$ tells us that function $g$ is written in terms of $h$. That is, $g$ depends on $h$.

Function notation can also be used to tell us to perform an operation.
Evaluate $f(2), f(-3), f(x+2)$ for the function $f(x)=3 x+7$
$f(2)=3(2)+7$
$f(2)=13$

$$
\begin{aligned}
& f(-3)=3(-3)+7 \\
& f(-3)=-2
\end{aligned}
$$

$$
\begin{gathered}
f(x+2)=3(x+2)+7 \\
f(x+2)=3 x+6+7 \\
f(x+2)=3 x+13
\end{gathered}
$$

If $f(x)=5 x-6$, find
110. $f(4)$
111. $f(-1)$
112. $f(-3+x)$

If $g(x)=2 x-4$, find



Use the graph of $h(t)$ to answer the following questions.


132. $h(t)=4$

Find both values of $t$.

Use the graph of $d(x)$ to answer the following questions.



Use the graph of $f(x)$ to answer the following questions.

136. Find $f(-3)$.
137. Find $f(5)$.
138. Find $x$ if $f(x)=0$.

## Describing the same relation in various ways.

Below are three descriptions of the same relation.


Fill in remaining two cells below for the given relation.


The following two examples demonstrate a relationship between two quantities.

A computer service technician charges a fee of $\$ 120$ to assess a problem and a fee of $\$ 60$ per hour to fix the problem. If the high school network requires 12 hours of work, what will the total cost be?

Cost:
$=\$ 120+12(\$ 60)$
$=\$ 120+\$ 720$
$=\$ 840$

This is a relationship between time worked and cost.

We could show this as $(12,840)$.
146. Graph the data you created in the question above.


The height of a thrown object can be modeled as a function of time (since it was thrown) by the following equation.

$$
h(t)=-5 t^{2}+12.5 t+100
$$

Find the height of the object 2 seconds after it has been thrown.

Height is found by substituting 2 into the right side of the equation.
$h(2)=-5(2)^{2}+12.5(2)+100$
$h(2)=-20+25+100$
$h(2)=105 m$

This is a relationship between height and time.
We could show this as $(2,105)$.
147. Create a set of data for the relation above.
148. Graph the data you created in the question above.


Solve each problem using any strategy that works.

A computer service technician charges a fee of $\$ 120$ to assess a problem and a fee of $\$ 60$ per hour to fix the problem.
149. If the high school network requires 7 hours of work, what will the total cost be?
150. What are the two variables in this problem?
151. Which variable would be the "dependent variable?" (see pg.30)
152. Does the dependent variable correspond to the domain or range?
$\qquad$
153. Do you think this problem models discrete or continuous data? Explain.
154. What is significant about the point $(0,120)$ ?

The height of a thrown object can be modeled as a function of time since thrown by the following equation.

$$
h(t)=-5 t^{2}+12.5 t+100
$$

155. Find the height of the object 3 seconds after it has been thrown.
156. Can you think of any values for time $(t)$ that don't make sense?
157. What does time represent domain or range?
158. Can you think of any values for height ( $h$ ) that don't make sense?
159. Is height the dependent or independent variable?
160. Use the graph on the previous page to estimate the time it takes the object to reach maximum height.

BONUS: Can you calculate the time it takes the object to land?

Solve each problem using any strategy that works.
161. A bike technician charges $\$ 40$ for a basic tune-up and $\$ 20 / \mathrm{h}$ for any additional work.

Write an equation that relates cost (C) to time ( t ) for the scenario above.
162. Create a table for the scenario above.

163. Graph the relation above.

164. The population of a colony of bacteria grows through cell division. The doubling time for the population is 30 minutes. Complete the table below for the growth of bacteria starting with one bacterium.

165. Graph the relation above.

166. What numbers are acceptable values for the horizontal axis (domain) of the graph above? (Think about what numbers would not make sense.)
167. Going to the movies. The cost of going to the movies for a group of grade 10 students is represented by the equation $C=10.5 n$.
a) What is a reasonable range for this function?
b) What is the dependent variable?
c) Write the equation using function notation.
168. Driving Distance. JJ leaves Nanaimo driving north. At the time he left, he was 105 km from home. The following graph represents the relationship between distance from home and elapsed driving time.

The equation for the relation $d(t)=50 t+100$.
a) Explain why the function is called $d(t)$.
b) Suggest a reasonable domain for the function $d(t)$.

c) Find $d(3)$.
d) Why is the graph a line and not a series of dots?
169. Halloween dance. Student's Council plans on hiring DJ-Jae-Sun for this year's Halloween dance. Jae-Sun appreciates what he remembers of math functions and sends the council the following pricing information.

$$
C(n)=2000+17.50 n
$$

a) Explain what you think the equation above means.
c) What is a reasonable range for your school?
b) What would be a reasonable domain at your school?
d) What does the range represent?
e) Is this the dependent or independent variable?
170. Wedding banquet. Lin-Karen is planning her dream wedding. Catering costs are a function of the number of people that attend the wedding. A high end caterer quoted Lin-Karen a set-up cost of 1500 dollars plus 75 dollars per guest.
a) Write the cost as a function of the number of guests using function notation.

## b) Is this Discrete or Continuous data?

c) Graph the relation above using a reasonable domain. Use a ruler to mark your axes. Label your axes with "Number of Guests" on the horizontal axis.

## General relations

When considering some relationships, it is solely the pattern or trend that we are interested in.

Can you visualize a graph for the following relationships?

- The height above the ground of a passenger on a Ferris Wheel as a function of time.
- The number of cars in a parking lot as a function of the time of day.
- Temperature of a cup of coffee as a function of time since it was poured.
- The cost of mailing a package as a function of its mass.
- The height of a football as a function of time since it was kicked.

Match each of the following with an example from above. Then describe below why you made that choice.




Some notes here possibly...
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Answer the questions associated with each graph.

177. A hot cup of coffee was left on the table to cool. Graph the data below.

178. To hire a plumber to fix his drain, Mr. J had to pay an initial "call-out" fee of $\$ 60$ then he had to pay the plumber $\$ 45$ per hour. Graph the Cost as a function of Time in hours for this service.


## Answers:

1. Answers may vary.

Possible relations:
Age and height, cost and time, wage and hours,
2. (27 years, 180 cm ),
( 9 years, 110 cm )
( 0.6 years, 55 cm )
3. Their order is important.
4. Height and Arm Span.
5.

6. Points appear to resemble a line.
7. Check with a classmate or teacher.
8. $y$-axis
9. x -axis
10. Graph on left includes negative coordinates.
11. Graphs of data where negatives are not included. (eg. Distance vs. Time)
12. Graphs of data where negatives are appropriate. (eg. Altitudes, temperatures)
13. Two units right and five units up of the origin (middle).
14.


15.

16.

17. Same as Q15. All input values "make sense" mathematically.
18. Same as $Q 16$. Any value of ' $x$ ' less than -2 will make the equation (function) undefined over the set of real numbers.
19. The graph will have breaks or stopping points where values are not permitted.
20.

21.

22. $-2,5$. Any real value is a possible element of the domain.
23. $-7,1$. Any real value is a possible element of the range.
24.

25.

26.

27.

28. No negative values can be contained in the range.
29. No negative values can be contained in the range.
30.
31.

32.

33.

34.

35.

36. Domain: $\{1,2,3,4\}$, Range: $\{1,4,9,16\}$
or $\{x \mid 1,2,3,4\},\{y \mid 1,4,9,16\}$
37. a and d
38. $\{x \mid-2,3,5,8\}$
39. $\{y \mid 3\}$
40. $\{x \mid-1,0,1,2,3\}$
41. $\{x \geq 0, x \in \mathbb{R}\}$ The domain is the real numbers greater than or equal to zero.
42. Infinite
43. 0
44. There is not a largest value. Infinity.
45. Infinite
46. 2
47. There is not a largest value. Infinity.
48. $(-\infty, \infty),(-\infty, \infty)$
49. $\{x \mid x \geq-4, x \in R\},\{y \mid y \geq 1, y \in R\}$
50.
x:

$y$ :

51. The domain consists of all real numbers from 6 to 5 inclusive.
The range consists of all real numbers from -1 to 2 inclusive.
52. $\{x \mid-5<x<7, x \in R\},\{y \mid y=4\}$
53. $(-\infty, \infty),(-\infty, \infty)$
54. Find the upper and lower limits. Separate them with a comma. Use a square bracket if the limit is included, a curved bracket if it is not.
55. Plot the limit(s) on a number line with a solid dot if included or hollow dot if excluded. Draw an arrow/line in the appropriate direction (unless the data is only a point or points).
56. Find the limit(s). Choose the correct symbol, $<, \leq,>, \geq, \neq$. Fill out the inequality using one of the following as a guide:
$x \geq$
$\qquad$
$\qquad$
57. A. 4 B. 5 C. 3 D. 6 E. 1 F. 2
58. $\{x \in R\},\{y \in R\}$
59. $[-7,6],[-5,7]$
60. x :

61. $\{x \mid-5 \leq x \leq 8, x \in R\},\{y \mid-3 \leq y \leq 5, y \in R\}$
62. $(-6, \infty),(\infty, 3)$
63. x :

64. $\{x \mid 2\},\{y \mid-3,-1,1,3\}$
65. $\{x \mid-7,-5,-3,6\},\{y \mid-4,5,7\}$
66. $(-8,9),[-5,7)$
67.

68.

69.

70.

71. $\{x \mid x \in R\}$
72.

73.

74.

75.

76.

77.

78.

79.

80.

81.

82. Consider, points, lines, breaks, ...
83. $\{x \mid x \in R\}$ or $[-\infty, \infty]$
84. $x \geq 2$ or $[2, \infty)$ or All real numbers greater than or equal to 2 .
85. $\{x \mid x \in R\}$ or $(-\infty, \infty)$ or All real numbers.
86. $\{y \mid y \in R\}$ or $(-\infty, \infty)$ or All real numbers.
87. $y \geq 0$ or $[0, \infty)$ or All real numbers greater than or equal to 0 .
88. $\{y \mid y \geq 0\}$ or $([0, \infty)$ or Real numbers greater than zero.
89.

90. $\{x \mid 2,3,4,5,6,7,8,9\},\{y \mid 20,30,40,50,60,70,80,90\}$
91. The space between points represents "fractions of people" and the corresponding cost. The domain is limited to whole numbers in this case.
92. The temp. of the coffee will cool until it reaches room temperature.
93. a) Several (3 or 4) hours.
b) Continuous
94. a) $\{n \mid 0 \leq n \leq 75, n \in W\}$ b) discrete
95. a) Time
b) Continuous
c) Continuous
d) An human's height would range from about 45 cm to about 200 cm . There are exceptions of course!
96. Cost is a function of number of bars. Discrete data.
97. Earnings are a function of hours worked. Discrete data.
98. Yes. Each x-value has only one corresponding $y$-value.
99. Yes. Each x -value has only one corresponding $y$-value.
100. No. There are two possible outputs when $x$ is 3.
101. No. The $\pm$ indicates that each input, except zero, will have two outputs.
102. Yes. Each x-value has only one corresponding $y$-value.
103. No. There are two possible outputs when $x$ is -3 .
104. No.
105. No.
106. Yes.
107. Yes. Each input value ( $x$ ) will produce only one output value ( $y$ ).
108. Yes. Each input value ( $x$ ) will produce only one output value $(y)$.
109. No. There will be two outputs (one positive, one negative) for each input. Except when the input is zero.
110. 14
111. -11
112. $5 x-21$
113. 4
114. -6
115. $2 x-6$
116. 74
117. -1
118. $5 x^{2}+10 x-1$
119. 27
120. $x^{2}+4 x+6$
121. $4 x^{2}-8 x+6$
122. -7
123. $x$ is independent.
124. 3
125.

126. 32
127. $f(x)$ which has replaced $y$.
128. 2
129.

130. 4
131. 3
132. $2,-8$
133. 6
134. 10
135. $x=-1,1$
136. 9
137. Not possible. 5 is not an element of the domain in this function.
138. $x=7$
139.

140. Each element in the range is one less than triple an element in the domain.
141. $f(x)=2 x$
142. Each element in the range is double an element in the domain.
143. $f(x)=x^{2}$
144.

145. $(1,180),(2,240),(3,300),(4,360),(5,420), \ldots$
146.

147. $(1,107.5),(2,105),(3,92.5), \ldots$
148.

149. $\$ 540$
150. $C$ : total cost $h$ : hours worked
151. Cost (C)
152. Range
153. Explain your answer.
154. The technician has not performed any hours of repair work but there is still a cost of $\$ 120$.
155. 92.5 m
156. Negative values.
157. Domain
158. Negative values (unless the object thrown can fall below the height that we have called height zero. For example, it may land in a hole or crevace.
159. Dependant
160. Approximately 1.5 seconds.
161. $C=40+20 h$ or

$$
C(h)=20 h+40 .
$$

162. 

| Time (hours) | Cost (\$) |
| :---: | :---: |
| 0 | 40 |
| 1 | 60 |
| 2 | 80 |
| 3 | 100 |
| 4 | 120 |
| 5 | 140 |

163. 


164.

165

166. Real numbers greater than or equal to zero. 167. a)
$\{C \mid 10.5,21,31.5,42,52.5,63,73.5,84,94.5,105\}$
b) $C$, cost.
c) $C(n)=10.5 n$
168. a) The function is called $d(t)$ beacure "distance" is a function of "time".
b) Real numbers between 0 and 5 inclusive.
c) 250
d) This data is a line because both distance and time are continuous (no gaps).
169. a) $C(n)=2000+17.50 n$ The DJ charges is a $\$ 2000$ fixed cost in addition to $\$ 17.50$ per hour or possibly per student. It is not clear what $n$ represents. I will work assuming $n$ represents hours.
b) eg. $\{n \mid 0 \leq n \leq 4, n \in W\}$
c) eg. $\{C \mid 2000,2017.50,3035,2052.50,2070\}$
d) Cost of the DJ.
e) Dependent
170. a) $C(n)=1500+75 n$
b) Discrete
c)

171. Many answers. Eg. Wage (\$/hr)
172. Many answers. Eg. Cost to hire a taxi.
173. The heating element must turn on and off to maintain an approximately constant temperature.
174. The ball is kicked from a height above zero (ground).
175. The rise and fall of a stock's price over time.
176. Each line on the graph represents a range of masses. There is not a different price for every possible mass. For example, it currently costs $\$ 0.64$ to send any letter under 30 g from one part of Victoria, B.C. to another.
177.

178.


