# Unit #6 Part I: Slope, Intercepts & Linear Relations

**Submission Checklist:** (make sure you have included all components for full marks)

- Cover page & Assignment Log
- Class Notes
- Homework (attached any extra pages to back)
- Quizzes (attached original quiz + corrections made on separate page)
- Practice Test/ Review Assignment

<table>
<thead>
<tr>
<th><strong>Assignment Rubric: Marking Criteria</strong></th>
<th>Self Assessment</th>
<th>Teacher Assessment</th>
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<tbody>
<tr>
<td><strong>Notebook</strong></td>
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<tr>
<td>- All teacher notes complete</td>
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<tr>
<td>- Daily homework assignments have been recorded &amp; completed (front page)</td>
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<td>- Booklet is neat, organized &amp; well presented (ie: name on, no rips/stains, all pages, no scribbles/doodles, etc)</td>
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<tr>
<td><strong>Homework</strong></td>
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<td>- All questions attempted/completed</td>
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<tr>
<td>- All questions <strong>marked</strong> (use answer key, correct if needed)</td>
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<tr>
<td><strong>Quiz</strong> (1mark/dot point)</td>
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<td>- Corrections have been made <strong>accurately</strong></td>
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<td>- Corrections made in a different colour pen/pencil (+½ mark for each correction on the quiz)</td>
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<td><strong>Practice Test</strong> (1mark/dot point)</td>
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<td>- Student has completed all questions</td>
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<td>- Mathematical working out leading to an answer is shown</td>
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<td>- Questions are <strong>marked</strong> (answer key online)</td>
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<td><strong>Punctuality</strong></td>
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<td>- All checklist items were submitted, and completed on the day of the unit test. (-1 each day late)</td>
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<td><strong>Comments:</strong></td>
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</table>
# Homework Assignment Log

& Textbook Pages: ___________________________

<table>
<thead>
<tr>
<th>Date</th>
<th>Assignment/Worksheet</th>
<th>Due Date</th>
<th>Completed?</th>
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## Quizzes & Tests:

<table>
<thead>
<tr>
<th>What?</th>
<th>When?</th>
<th>Completed?</th>
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<tbody>
<tr>
<td>Quiz 1</td>
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<td>Quiz 2</td>
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<td>Unit/Chapter test</td>
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</tbody>
</table>
One of the most important properties of straight lines is their angle from the horizontal (ie. steepness). The mathematical term for steepness is “slope”.

**Investigation #1:**
1) On the coordinate grid below, graph the points $A(-3, -5)$ and $B(5, 7)$ and join them to form a line segment.

2) Create a right angle triangle where AB is the hypotenuse and C is the third vertex.

3) Count the squares from A to C and B to C and record your numbers.

   Vertical change (sometimes called rise):
   
   Horizontal change (sometimes called run):

A vertical change represents the change in y-values of your coordinate points. This is represented by $\Delta y$ and is calculated by subtracting the two y-values of the coordinates, $y_2 - y_1$.

A horizontal change represents the change in x-values of your coordinate points. This is represented by $\Delta x$ and is calculated by subtracting the two x-values of the coordinates, $x_2 - x_1$.

4) Calculate $\Delta y$ and $\Delta x$ for the line segment AB.
**Investigation #2:**

1) On the coordinate grid below, graph the points C(−4, 7) and D(3, −8) and join them to form a line segment.

2) Create a right angle triangle where CD is the hypotenuse and E is the third vertex.

3) Count the squares from C to E and D to E and record your numbers.

   Vertical change (sometimes called rise):

   Horizontal change (sometimes called run):

4) Calculate $\Delta y$ and $\Delta x$ for the line segment CD.

---

**The slope, $m$, of a line segment joining $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by**

$$m = \frac{\text{Rise}}{\text{Run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

When is it most appropriate to use $m = \frac{\text{Rise}}{\text{Run}}$ to calculate the slope of a line segment?

When is it most appropriate to use $m = \frac{y_2 - y_1}{x_2 - x_1}$ to calculate the slope of a line segment?
**Example #1:** Determine the slope of each line segment.

\[ m_{AB} = \]

\[ m_{CD} = \]

\[ m_{EF} = \]

Which line segments have a positive slope? __________

Which line segments have a negative slope? __________

**Example #2:** Determine the slope of each line segment.

a) \( G(0, -7) \) to \( H(4, 0) \)

b) \( M(5, -2) \) to \( N(-1, 4) \)

**SUMMARY:**

<table>
<thead>
<tr>
<th>Slope</th>
<th>Diagram</th>
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</table>
## Characteristics of Linear Relations

### Key Terms

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
<th>Example</th>
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<tr>
<td>Line</td>
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<td>Line segment</td>
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<tr>
<td>Linear relation</td>
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<tr>
<td>Slope</td>
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<td>Positive slope</td>
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<td>Negative slope</td>
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<tr>
<td>Zero slope</td>
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<tr>
<td>Undefined slope</td>
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<td>Intercepts</td>
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<td>Parallel lines</td>
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<td>Parallel slopes</td>
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<td>Perpendicular lines</td>
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<td>Perpendicular slopes</td>
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<td>Midpoint formula</td>
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<tr>
<td>Distance formula</td>
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<tr>
<td>Parallelogram</td>
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</tbody>
</table>
Linear Relations:

- A relationship between two quantities that when graphed will produce a **straight line**.
- One quantity **increases or decreases at a constant rate** with respect to another.

\[ y = -3x \quad C = 50n + 1000 \quad p = 40q \]

**LINE SEGMENT**: A part of a line that has two endpoints and includes all the points between the endpoints.

1. Using a dashed or coloured line, graph the relation represented by the equation \( y = -3x \).
2. Using a solid or different coloured line graph the same relation if the domain is \( 0 \leq x \leq 2 \).

The solid section you just plotted is a line segment, a section of the dashed line.

3. What are the endpoints of the line segment? ______________
4. What are the endpoints of the dashed line? ____________

5. What are 5 properties you could use to describe the line segment above?

6. Which of these properties are also true for the dashed line above?
Slope of a Line (or Line Segment): (Rate of Change)

Consider the line segment below.

7. What is the vertical change (rise) between the endpoints?

8. What is the horizontal change between the two endpoints?

9. What is the ratio of rise to run as a fraction?

10. How fast does the relationship change in the vertical direction when compared to the horizontal direction?

Your notes here...
11. Challenge Question:
   Find the slope (rate of change) of the line below.

12. Challenge Question:
   Find the slope (rate of change) of the line segment with end points at A(-4,0) and B(0,3).
Find the slope (rate of change) of the line below.

Recall:
Slope is the ratio of $\frac{\text{Rise}}{\text{Run}}$. We can count gridlines from point-to-point to get $\frac{\text{rise}}{\text{run}} = \frac{3}{4}$.

NOTE:
If you started at the right point...
$\frac{\text{rise}}{\text{run}} = \frac{-3}{-4} = \frac{3}{4}$, we would be moving in the “negative” direction but the slope calculated would be the same.

Find the slope (rate of change) of the line segment with end points at A(-4,0) and B(0,3).

Strategy 1: Plot the points on a grid and follow the same solution strategy to the left.

Strategy 2:
We can see the rise is actually a change in the y-direction...a difference in the y-values.
For the points: A(-4,0) and B(0,3)

\[
\text{rise}: y - y = 3 - 0 = 3 \\
\text{run}: x - x = 0 - (-4) = 4
\]

Therefore slope $= \frac{\text{Rise}}{\text{Run}} = \frac{3}{4}$.

**IMPORTANT**
TO USE THIS STRATEGY...you must be consistent with your “starting” x and y values in calculating rise and run.

Note the formula on the next page to help you do this.
Slope of a Line (or Line Segment)

Slope is the measure of the “steepness” of a line. It is represented with the symbol \( m \).
Slope also describes the direction of the line.

The slope is found by dividing the vertical change (the rise or fall) by the horizontal change (the run).

\[
m = \frac{\text{rise}}{\text{run}} \quad \text{or} \quad m = \frac{y_2 - y_1}{x_2 - x_1}
\]

Positive Slope

Eg. \( m = 2 \)

Rises from left to right.

Negative Slope

Eg. \( m = -3 \)

Falls from left to right.

Zero Slope

Eg. \( m = 0 \)

Rise is 0. 0 divided by any “run” will still = 0.

Undefined Slope

Eg. \( m = \infty \)

Think... the run is 0. Division by 0 is undefined.

13. Describe, in your own words, how you find the slope of a line segment.

14. How does a line segment differ from a line?
Find the rise, the run and the slope for the following lines by counting units.
In most cases, you will need to pick two points on the line to use.

<p>| | | |</p>
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>15.</td>
<td>rise=</td>
<td>run=</td>
</tr>
<tr>
<td>16.</td>
<td>rise=</td>
<td>run=</td>
</tr>
<tr>
<td>17.</td>
<td>rise=</td>
<td>run=</td>
</tr>
<tr>
<td>18.</td>
<td>rise=</td>
<td>run=</td>
</tr>
</tbody>
</table>

Use the formula \( m = \frac{y_2 - y_1}{x_2 - x_1} \) to find the slopes of line segments with the following endpoints.

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<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>23. ((0,0)) and ((2,3))</td>
<td>24. ((1,3)) and ((2,7))</td>
<td>25. ((-5,7)) and ((-4, -2))</td>
</tr>
</tbody>
</table>
Use the formula \( m = \frac{y_2 - y_1}{x_2 - x_1} \) to find the slopes of line segments with the following endpoints.

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<table>
<thead>
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<tbody>
<tr>
<td>26. ((5,7)) and ((5,3))</td>
<td>28. ((-4,5)) and ((6,5))</td>
<td>30. ((\frac{1}{2},4)) and ((2,-6))</td>
</tr>
</tbody>
</table>

27. Find the coordinates of any another point on this line.  
29. Find the coordinates of any another point on this line.  
31. Find the coordinates of any another point on this line.

32. The slope of a line is -2. If the line passes through \((t, -1)\) and \((-4,9)\), find the value of \(t\).  
33. The slope of a line is \(-\frac{3}{2}\). If the line passes through \((5,2)\) and \((b,-4)\), find the value of \(b\).

34. Challenge  
Given a point on the line and the slope, sketch the graph of the line.  
\((2,3), m = -2\)
2) Applications of Slope

If you know one point on the line, you can use the slope to find any other point on the same line.

Steps:

Example #1:
   a) Draw a line segment passing through \(A(0, 0)\) with a slope of \(\frac{2}{3}\).

   b) Write the coordinates of 4 other points on the line.

Example #2:
   a) Draw a line segment passing through \(B(1, 2)\) with a slope of \(-\frac{3}{4}\).

   b) Write the coordinates of 4 other points on the line.
**Example #3:**

a) Draw a line segment passing through $C(-4,-2)$ with a slope $0$.

b) Write the coordinates of 4 other points on the line.

**Example #4:**

a) Draw a line segment passing through $D(-5,3)$ with a slope that is undefined.

b) Write the coordinates of 4 other points on the line.
Example #5: Calculate the slope of the following diagrams.

a) Stairs

b) Symmetrical Roof

Example #6: Determine if the following represents a positive, negative, or zero rate of change. What are the units of the slope?

a) A baby’s height over time.

Units?

b) The number of fans seated when the hockey game ends.

Units?

c) Driving at a steady speed of 100 km/h.

Units?

d) The population of Europe during the Black Plague.

Units?
Given a point on the line and the slope, sketch the graph of the line.

35. \((2,3), m = -2\)

1. Plot the point: (2,3)
2. Use \(\frac{\text{rise}}{\text{run}} = \frac{-2}{1}\) to get a second point...and a third.
3. Connect with a line.

36. \((-3,-2), m = \frac{2}{3}\)

37. \((-4,5), m = 0\)

38. \((-3,4), m = -\frac{4}{3}\)

39. \((-1,1), m = 3\frac{1}{2}\)

40. \((-5,7), m \text{ is undefined}\)
Slope is a measure of **Rate of Change** for a relation. That is, how fast one quantity increases or decreases in respect to another.

Answer the following questions regarding slope and rate of change.

<table>
<thead>
<tr>
<th>Question</th>
<th>Details</th>
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<tbody>
<tr>
<td>41. A fallen tree leans against a vertical cliff. The tree was 15 m from the cliff and now rests against the cliff 25 m from the ground.</td>
<td>Find the positive slope of the fallen tree.</td>
</tr>
<tr>
<td>42. A section of roller coaster falls 52 m in a horizontal distance of 4 m.</td>
<td>Find the slope of this section of track?</td>
</tr>
<tr>
<td>43. The cost for 8 students to go to the movies is $80.</td>
<td>What is the cost per student, or rate?</td>
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<tr>
<td>44. Write two ordered pairs for this relation.</td>
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<tr>
<td>45. To fill my gas tank that holds 70 litres, I paid $68.53.</td>
<td>What is the rate for gasoline per litre (in cents to the nearest tenth)?</td>
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<tr>
<td>46. TSpray drove 735 kilometres in 7 hours.</td>
<td>Find his rate of travel per hour.</td>
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<tr>
<td>47. What name is given to this quantity?</td>
<td></td>
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</tbody>
</table>
Answer the following questions regarding slope and rate of change.

48. A round of golf for a group of hackers consists of the “green fee” and the club rental fee. Clubs are rented on a fee per club basis. Jack pays $72.25 for his green fee and 3 clubs, and Jill pays $95 for her green fee and 10 clubs. What is the rate to rent one club?

49. Pro-lectic charges their customers a fixed cost plus an hourly rate. To work in my basement, they charged me $210 for 5 hours work. To complete my upstairs renovations they charged me $720 for 22 hours work. What is the hourly rate?

50. Plot the relation above.

51. Plot the relation above.
52. Below is a scale drawing of a bridge support. Perform the necessary measurements to determine the slope of the indicated beam.

53. Below is a scale diagram of a section of road between Sidney and Victoria. Measure and calculate the slope of the road.

54. A fishing boat moving at 12 knots is shown below. Calculate the slope of the line in the water behind the boat.

55. Terraced landscapes are used by farmers to create usable space from seemingly unusable geography. Calculate the slope of the hill that has been terraced to support crops.

56. The pitch of a roof is a measure of its "steepness". Calculate the height of the roof truss below if its total span is 20 feet and the pitch (slope) is 6/12.

57. Mr. J is building a hide-away cabin with a roof that has a pitch of 9/12. T-spray is also building a hut but his roof is one-third as steep. If both roofs have the same total height, how many times wider is T-spray's roof?
Since slope compares two quantities, it is a *measure of rate of change*.

For each of the following scenarios, what rate does the slope represent?

58. Rate:______________________
What are the units of the slope?___________

59. Rate:______________________
What are the units of the slope?___________

60. Rate:______________________
What are the units of the slope?___________
Warm-Up:

1. Find the slope of the following lines using the graphs below:
   
   a)  
   \[ \text{rise: } \quad \text{run: } \quad \text{m = } \quad \]
   
   b)  
   \[ \text{rise: } \quad \text{run: } \quad \text{m = } \quad \]

2. a) Calculate the slope of the line that passes through A (2, 6) and B (8, 15). Give your answer in lowest terms.

   b) Find the co-ordinates of any other point on this line.

3. Graph the line that passes through the point F (-5, 4) and has a slope of \( m = -3 \). (Plot at least 4 points)
Part 1: Parallel Lines

Parallel lines NEVER ______________________________________

Parallel lines have EQUAL ________________________________

Example #1: State the slope that is parallel:

a) \( m = \frac{-3}{8} \)   

b) \( m = 4 \)

Example #2: Determine if AB is parallel to CD.

<table>
<thead>
<tr>
<th>A (17, 82)</th>
<th>B (21, 92)</th>
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<tbody>
<tr>
<td>C (6, 20)</td>
<td>D (10, 30)</td>
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</tbody>
</table>

Example #3: Find “k” if the following slopes are parallel.

a) \( m_1 = \frac{4}{3} \) and \( m_2 = \frac{k}{2} \)   

b) \( m_1 = -\frac{4}{5} \) and \( m_2 = \frac{10}{k} \)

Example #4: Determine the co-ordinates of Point D, on the y-axis, so that MN is parallel to CD.

<table>
<thead>
<tr>
<th>M (-3, 3)</th>
<th>N (1, 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C (4, -3)</td>
<td>D (     ,     )</td>
</tr>
</tbody>
</table>
Part 2: Perpendicular Lines

Perpendicular lines intersect at ____________________________
Perpendicular lines have ____________________________ slopes
Perpendicular slopes will always multiply to ____________

Example #1: State the slope that is perpendicular to the following:

a) \( m = \frac{-12}{5} \)  
b) \( m = 20 \)

Example #2: Determine if AC is perpendicular to BD.

<table>
<thead>
<tr>
<th>A (1,10)</th>
<th>B (-3,7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C (-2,-1)</td>
<td>D (8,10)</td>
</tr>
</tbody>
</table>

Example #3: Find “k” if the following slopes are perpendicular.

a) \( m_1 = \frac{12}{5} \) and \( m_2 = \frac{k}{2} \)  
b) \( m_1 = \frac{-4}{5} \) and \( m_2 = \frac{10}{k} \)

Example #4: Is the triangle with vertices \( A (-2, -3), B (2, 1) \) and \( C (-4, -2) \) a right triangle?
61. **Challenge # 5**
Determine if AB is parallel to CD given the following points: A(1,2), B(5,4), C(0,-2), D(6,1).

62. What can you say about the slopes of parallel line segments?
Slopes of Parallel Lines (or segments)

Recall two lines are parallel if they do not ever intersect.

Parallel lines have *equal slopes*.

Any two horizontal lines are parallel.
Any two vertical lines are parallel.

To determine if line segments are parallel, calculate their slopes.

Eg.1. Determine if $AB$ is parallel to $CD$.  $A(1,2)$, $B(5,4)$, $C(0,-2)$, $D(6,1)$.

Slope of $AB$: $m_{AB} = \frac{4-2}{5-1} = \frac{2}{4} = \frac{1}{2}$
Slope of $CD$: $m_{CD} = \frac{1-(-2)}{6-0} = \frac{3}{6} = \frac{1}{2}$  \(\text{SAME SLOPES} \Rightarrow \text{PARALLEL}\)

Eg.2. The following are slopes of two lines. Find the value of $k$ so that the two lines are parallel.

$m_1 = 2$ and $m_2 = -\frac{6}{k}$

Since the lines are parallel, slopes must be equal.  $2 = -\frac{6}{k}$

Cross Multiply: $\frac{2}{1} = -\frac{6}{k} \quad 2k = -6 \quad k = -3$
Determine if the following pairs of line segments are parallel.

63. A(-2,-1), B(1,5) and C(2, -1), D(4,3)

64. E(-3, 0), F(1, 5) and G(0, -6), H(2, -1)

65. I(-4,0), J(8,2) and K(2, 8), L(-2, 4)

The following are slopes of two lines. Find the value of \( k \) so that the two lines are parallel.

66. \( m_1 = \frac{-2}{3} \) and \( m_2 = \frac{k}{9} \)

67. \( m_1 = -3 \) and \( m_2 = \frac{k}{4} \)

68. \( m_1 = \frac{k}{3} \) and \( m_2 = \frac{1}{2} \)

70. The points A(6,3), B(2,9), and C(2,3) are given. Determine the coordinates of point D so that CD is parallel to AB and D is on the y-axis.
Slopes of Perpendicular Line Segments.

- The slopes of perpendicular lines are negative reciprocals.
- The product of perpendicular slopes is -1.

71. Plot the right triangle with vertices:
    A(2,2), B(5,7), and C(10,4).

72. Find the slope of AB. \( m = \)

73. Find the slope of BC. \( m = \)

These segments form the right angle in the triangle.

74. What do you notice about the slopes of the two segments.

75. Multiply the two slopes. What is the result?

76. Is the triangle with vertices X(-9,-1), Y(-7,7), Z(3,-4) a right triangle?
Perpendicular Lines will have slopes that are NEGATIVE RECIPROCALs.

Examples of perpendicular slopes are: \( m_1 = 5, \ m_2 = -\frac{1}{5} \).

Examples of perpendicular slopes are: \( m_1 = -\frac{5}{3}, \ m_2 = \frac{3}{5} \).

Perpendicular slopes will have a product of \(-1\).

Look at the example above... \( -\frac{5}{3} \times \frac{3}{5} = -\frac{15}{15} = -1 \)

Determine the slope of a line segment perpendicular to a segment with each given slope.

| 77. \( m = -3 \) | 78. \( m = -\frac{2}{3} \) | 79. \( m = \frac{4}{5} \) |

The following are slopes of two lines. Find the value of \( k \) so that the two lines are perpendicular.

| 80. \( m_1 = -\frac{2}{3} \) and \( m_2 = -\frac{k}{9} \) | 81. \( m_1 = -3 \) and \( m_2 = \frac{k}{4} \) | 82. \( m_1 = \frac{k}{3} \) and \( m_2 = \frac{1}{2} \) |

Graph each pair of line segments. Determine if they are perpendicular or not.

| 83. A(0,0), B(6,4) and C(7,3), D(-11,1) | 84. G(2,10), H(-7,-2) and J(7,0), K(-5,9) |
**Warm-Up:** Use the table of values method to graph the line \( y = -3x + 6 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( a) \) What is the \( x \)-intercept?

\( b) \) What is the \( y \)-intercept?

\[ \text{The } x \text{-intercept is the point where graph crosses the } x\text{-axis.} \]

\[ \text{The } y \text{-intercept is the point where the graph crosses the } y\text{-axis.} \]

**Notes:**

- Non vertical/horizontal lines are called ______________________ lines
- These lines cross both the ______________________ and the ______________________
- The point where the line crosses the _____ axis is called the ______________________
  - Coordinates: _______________
- The point where the line crosses the _____ axis is called the ______________________
  - Coordinates: _______________

Every \( y \)-intercept has an \( x \)-coordinate of _______.

Every \( x \)-intercept has a \( y \)-coordinate of _______.
Example #1: Find the x and y intercepts from the following graphs:

a) [Graph]

x-intercept = ______ y-intercept = ______

b) [Graph]

x-intercept = ______ y-intercept = ______

Example #2: Consider the line defined by $2x - 3y = 6$?

a) Determine the x-intercept and write the coordinates of this point.

b) Determine the y-intercept and write the coordinates of this point.

c) Graph the function using the intercepts

➢ One way of graphing linear relations without using a table of values is finding the x-intercept and y-intercept and connecting the two points.
Example #3: Graph the line $2x + y = 8$

$x$-intercept:

$y$-intercept:

Example #4: Graph the line $2x + 6y = 18$

$x$-intercept:

$y$-intercept:
Example #5: Graph the line $x = -4$

$x$-intercept:

$y$-intercept:

Example #6: Graph the line $y - 5 = 0$

$x$-intercept:

$y$-intercept:
Intercepts

Non-vertical and non-horizontal lines are called **oblique** lines.

Oblique lines will cross both the x-axis and the y–axis.

These points are called the x-intercept and the y-intercept.

85. **Challenge Question:**
Find the intercepts for the line $y = 2x + 4$.

86. **Challenge Question:**
Find the intercepts for the line $3x + 4y - 12 = 0$. 
Finding the Intercepts from a graph.

The location where a line passes through the x-axis is called the \textbf{x-intercept}. This point will have the coordinates \((x, 0)\).

The location where a line passes through the y-axis is called the \textbf{y-intercept}. This point will have the coordinates \((0, y)\).

Consider: \(2x + 4y = 16\)

This line has an x-intercept at \((8, 0)\). And a y-intercept at \((0, 4)\).

You may see this written as:

\begin{align*}
\text{x-intercept is } & 8. \\
\text{y-intercept is } & 4.
\end{align*}

Find the x- and y-intercepts from the graph below.

87. x-intercept:_____ y-intercept:_____ 
88. x-intercept:_____ y-intercept:_____
Finding the Intercepts from an equation.

The x-intercept will have coordinates $(x, 0)$. This means we can substitute 0 in for $y$ and solve to find the x-intercept. The y-intercept will have coordinates $(0, y)$.

<table>
<thead>
<tr>
<th>Eg. Find the x-intercept for</th>
<th>Find the y-intercept:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x + 4y = 16$</td>
<td>$2x + 4y = 16$</td>
</tr>
<tr>
<td>$2x + 4(0) = 16$</td>
<td>$2(0) + 4y = 16$</td>
</tr>
<tr>
<td>$2x = 16$</td>
<td>$4y = 16$</td>
</tr>
<tr>
<td>$x = 8$</td>
<td>$y = 4$</td>
</tr>
</tbody>
</table>

Calculate the x- and y-intercepts.

<table>
<thead>
<tr>
<th>89.  $2x + 3y = 12$</th>
<th>90.  $3x + 5y = 30$</th>
</tr>
</thead>
<tbody>
<tr>
<td>91.  $3x - 4y + 24 = 0$</td>
<td>92.  $4x + 5y = 10$</td>
</tr>
<tr>
<td>93.  $5y = 10x$</td>
<td>94.  $0.04x + 0.02y = 1400$</td>
</tr>
</tbody>
</table>
Using and Interpreting Intercepts

95. Find the intercepts and graph the line $2x + 6y = -18$.

96. Find the intercepts and graph the line $10x - 8y = -80$.

97. Based on the equation for the linear relation, when do you think it is most appropriate to graph the relation using intercepts?

98. The cost of a new pair of shoes at Shoelnc is reduced at a constant rate. The graph below shows the profit Shoelnc makes on each sale. In what month does Shoelnc “break even” on these shoes?

99. Use the graph below to plot the fuel consumed on Sandy’s last road trip. She started out with 72 litres of fuel and drove for 2 hours. At that point she had 54 litres left. After driving another 1.5 hours she had 40.5 litres remaining.

At this rate, when will she run out of fuel?
Mixed Practice:

100. A triangle has vertices $A(-2,3)$, $B(8,-2)$, and $C(4,6)$. Determine whether it is a right triangle.

101. $P(5,4)$ and $Q(1,-2)$ are points on a line. Find the coordinates of a point, $R$, so that $PR$ is perpendicular to $PQ$.

102. Find the value of $k$ so that the two slopes are perpendicular.

\[ m_1 = \frac{k}{2} \text{ and } m_2 = \frac{1}{4} \]

103. Two vertices of an isosceles triangle are $A(-5,4)$ and $B(3,8)$. The third vertex is on the $x$-axis. What are the possible coordinates of the third vertex, $C$?
Part I Answers

1. 

2. On graph above.
3. $(0,0)$ and $(2, -6)$
4. There are none.
5. Many answers. Eg. Definite endpoints, straight line, no gaps, decline to the right, etc.
6. Eg. Same "direction", that is decline to the right.
7. $\text{rise} = 4$
8. $\text{run} = 2$
9. $\frac{4}{2} = \frac{2}{1} = 2$
10. Twice as fast.
11. $m = \frac{2}{4}$
12. $m = \frac{3}{4}$
13. Choose any two points on the line and calculate or count rise/run.
14. A line segment has definite endpoints.
15. Rise: 3
   Run: 4
   Slope: $\frac{3}{4}$
16. Rise: 7
   Run: 0
   Slope: undefined
17. Rise: 0
   Run: 8
   Slope: $\text{undefined}$
18. Rise: $-1$
   Run: 3
   Slope: $-\frac{1}{3}$
19. Rise: 2
   Run: 2
   Slope: $\frac{2}{2} = 1$
20. Rise: $-4$
   Run: 4
   Slope: $-\frac{4}{4} = -1$
21. Rise: 4
   Run: 7
   Slope: $\frac{4}{7}$
22. Rise: $-6$
   Run: 3
   Slope: $-\frac{6}{3} = -2$
23. $\frac{3}{2}$
24. $4$
25. $-9$
26. Undefined
27. $(5,6)$, many other answers.
28. 0
29. $(3,5)$, many other answers.
30. $-\frac{29}{3}$
31. $(5, -26)$, many other answers.
32. $t = 1$
33. $b = 9$
34. Answered on next page in booklet.
35. 
36. 
37. 
38.
39. Use the positive slope in these applications.
40. 13 Use the positive slope in these applications.
41. $10
42. (1, 10), (2, 20), ...
43. $3.25 per club and $62.50 for green fees
44. $30/hour,
45. Fixed cost: $60
46. $3 cm = 3/4 or 0.75
47. Speed
48. 105 km/hr
49. Speed, km/h, or m/s, or mph
50. Speed, km/h, m/s, or mph
51. Salary, $/hour
52. $30/hour.
53. Density, g/ml
54. 4 feet
55. 1/2 or 0.5
56. 5 feet
57. Three times as wide.
58. Speed, km/h, m/s, or mph
59. Salary, $/hour
60. Density, g/ml
61. Yes. Lines through those points will never intersect.
62. Slopes are equal. Both segments have a slope of 1/2.
63. Yes. Slope: 2
64. No. Slope $\frac{1}{2}$ and slope $\frac{3}{2}$.
65. No. Slope $\frac{1}{2}$ and slope 1.
66. $k = 6$
67. $k = -12$
68. $k = \frac{3}{2}$
69. $(7, -7)$, $(-1, 5)$, and $(-3, -3)$ also produce a parallelogram but the naming would then be out of order.
70. (0, 6)
71. On graph
72. $\frac{5}{3}$
73. $\frac{-3}{5}$
74. Opposite signs, reciprocated.
75. $\frac{5}{3} \times -\frac{3}{2} = -\frac{15}{6} = -1$
76. Yes. Two of the side lengths have slopes that are negative reciprocals.
77. $\frac{1}{3}$
78. $\frac{2}{3}$
79. $\frac{-5}{4}$
80. $k = -\frac{27}{2}$
81. $k = \frac{4}{3}$
82. $k = -6$
83. No. $\frac{2}{3}$ and $\frac{1}{9}$
84. Perpendicular. $\frac{4}{3}$ and $-\frac{3}{4}$
85. y-intercept: (0.4)
86. x-intercept: (−2.0)
87. y-intercept: (0.3)
88. x-intercept: (4.0)
89. y-intercept: (0.4)
90. x-intercept: (0.0)
91. y-intercept: (0.0)
92. x-intercept: (0.0)
93. We can choose to state the intercepts without coordinates.
94. y-intercept: 4
95. x-intercept: 6
96. y-intercept: 6
97. x-intercept: 10
98. y-intercept: 6
99. x-intercept: −8
92. y-intercept: 2  
x-intercept: \(\frac{5}{2}\)

93. y-intercept: 0  
x-intercept: 0

94. y-intercept: 70,000  
x-intercept: 35,000

95. y-intercept: -3  
x-intercept: -9

96. y-intercept: 10  
x-intercept: -8

97. When the intercepts are integers (which results if the constant is divisible by the coefficients of the x and y terms).

98. September

99. 4.5 more hours.

100. Yes. AC is perpendicular to BC.

101. R could be an infinite number of points. Eg. (-1,8) or (2,6) or (8,2)...

102. \(k = -8\)

103. (7,0), (-1,0), (3,0), (2,0)

**Additional Material**

104. Create a right triangle using the points and intersecting grid lines then use Pythagoras Theorem to calculate a distance of 5 units.

105. Create a right triangle using the points and intersecting grid lines then use Pythagoras Theorem to calculate a distance of \(\sqrt{85}\) units.

106. 

107. See above.

108. \(a^2 + b^2 = c^2\)

\[c = \sqrt{a^2 + b^2}\]

This is the distance formula where “a” represents the horizontal difference and “b” the vertical difference.

109. \(2\sqrt{10}\)

110. 3

111. \(\sqrt{29}\)

112. \(\sqrt{137}\)

113. \(4\sqrt{5}\)

114. \(\sqrt{29}\)

115. \(\sqrt{101}\)

116. \(\sqrt{65}\)

117. \(\sqrt{181}\)

118. Length: \(3\sqrt{10}\)  
Width: \(\sqrt{10}\)  
Perimeter: \(8\sqrt{10}\)

119. 14.6 km

120. (3,2)

121. (2,1)

122. (4,8)

123. (5,5)

124. (0, -3)

125. (1,9,0,9)

126. \(\left(-1, \frac{1}{2}\right)\)

127. (301,5,149.5)

128. (-4,3)

129. (-6, -10)

130. (10,4)

131. (1, -6)

132. \(\left(\frac{3}{2}, 2\right)\) or \((1,5,2)\)

133. 4.92

134. \(m = 4\)

135. \(3\sqrt{17}\)

136. \(\left(1, \frac{1}{2}\right)\) or \((1,3,5)\)

137. Infinite possibilities. The x-coordinate must be -5. Eg. (-5, 0), (-5,1),...