Foundations & Pre-Calculus 10
Homework & Notebook

Name:________________________________ **Block:**__________
Teacher: Miss Zukowski **Date Submitted:** ____/______/ 2019

Unit #7 Systems of Linear Equations + Linear Sequences

Submission Checklist: *(make sure you have included all components for full marks)*

- ❏ Cover page & Assignment Log
- ❏ Class Notes
- ❏ Homework *(attached any extra pages to back)*
- ❏ Quizzes *(attached original quiz + corrections made on separate page)*
- ❏ Practice Test/ Review Assignment

Assignment Rubric: Marking Criteria			
Excellent (5) - Good (4) - Satisfactory (3) - Needs Improvement (2) - Incomplete (1) - NHI (0)		**Self Assessment**	**Teacher Assessment**
Notebook	• *All* teacher notes complete • Daily homework assignments have been recorded & completed *(front page)* • Booklet is neat, organized & well presented *(ie: name on, no rips/stains, all pages, no scribbles/doodles, etc)*	/5	/5
Homework	• *All* questions attempted/completed • *All* questions **marked** *(use answer key, correct if needed)*	/5	/5
Quiz *(1mark/dot point)*	• Corrections have been made **accurately** • Corrections made in a *different colour pen/pencil* *(+½ mark for each correction on the quiz)*	/2	/2
Practice Test *(1mark/dot point)*	• Student has completed all questions • Mathematical working out leading to an answer is shown • Questions are **marked** *(answer key online)*	/3	/3
Punctuality	• *All checklist items were submitted, and completed on the day of the unit test. (-1 each day late)*	/5	/5
Comments:		/20	/20

Homework Assignment Log

& Textbook Pages:____________________________________

Date	Assignment/Worksheet	Due Date	Completed?

Quizzes & Tests:

What?	When?	Completed?
Quiz 1		
Quiz 2		
Unit/ Chapter test		

1. Introduction to systems of equations

A **system of linear equations** is ____

____

The **solution** to a system of linear equations can be represented three ways:

1. ______________________________
2. ______________________________
3. ______________________________

Example #1: Is the point (4, -1) a solution to the system of equations? Justify your answer.

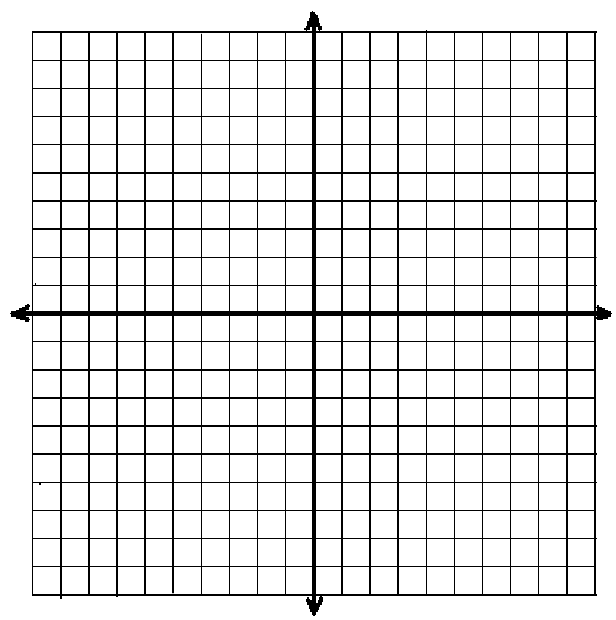
$$3x + y = 11$$
$$x - 2y = 6$$

Example #2:

a) Graph the following system of linear equations.

$3x + 2y = -12$

$-2x + y = 1$



b) From your graph, identify the point of intersection – this is the solution to the system of equations.

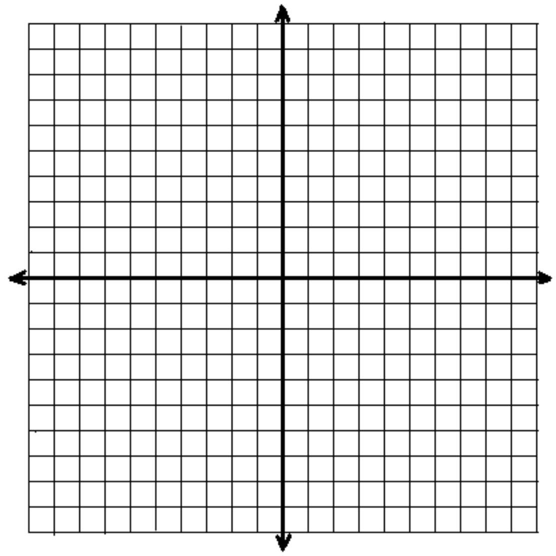
c) Verify your solution algebraically.

The point that satisfies all of the equations in a system of equations is said to be the solution to the system.

Example #3:
Solve the system of equation **and** verify your solution.

$x + y = 8$

$3x - 2y = 14$



ASSIGNMENT # 1
pagPages 4-8 questions #1-25

Introduction: Systems of Linear Equations

Challenge

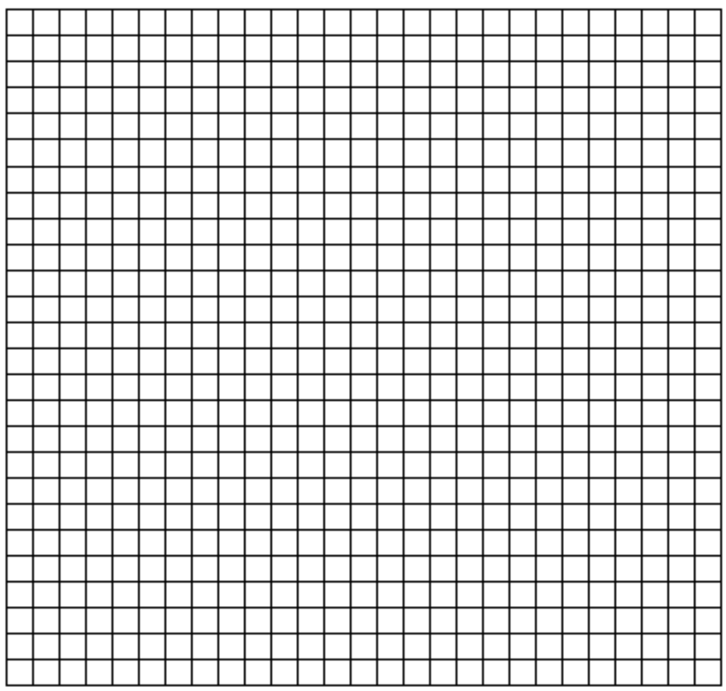
Jazhon is considering two job offers. Concrete Emporium will pay Jazhon a base monthly salary of $500 plus a commission rate of 5% on all sales each month. All Things Cement offers him a job that pays straight salary, $2500 per month.

Jazhon wants to consider the two jobs mathematically before he makes his decision. He writes the following equations to represent each job offer.

Concrete Emporium: $E = 0.05s + 500$
All Things Cement: $E = 2500$

1. What does Jazhon need to consider before he can make an educated decision?

2. Graph the two equations on the grid below.

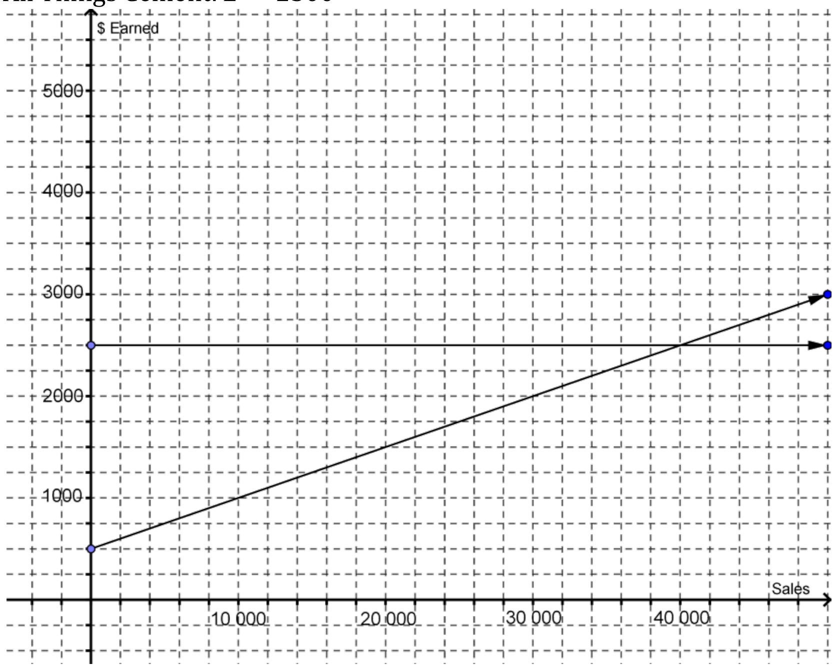


3. What is the significance of the point where the two lines cross?

4. When does the job offered by Concrete Emporium pay more?

Challenge

Concrete Emporium: $E = 0.05s + 500$
All Things Cement: $E = 2500$



Where the lines cross→ earnings are equal.

Concrete Emporium will pay more if Jazhon sells more than $40 000 worth of concrete.

We call the scenario to the left a **System of Linear Equations**.

The point (40000, 2500) is on both lines.

We say (40000, 2500) is the **solution to the system.**

That is…it is the point that **satisfies both equations.**

5. ## Challenge

Is (1,3) a solution to the following system?

$$y = -2x + 5$$
$$y = x + 2$$

Explain your reasoning.

Determine if the given point is a solution to the system of equations. Show your work.

6. Is (1,3) a solution to the following system?

① $y = -2x + 5$
② $y = x + 2$

Substitue $x = 1$ and $y = 3$ into both equations.

Equation ①	equation ②
$y = -2x + 5$	$y = x + 2$
$3 = -2(1) + 5$	$3 = 1 + 2$
$3 = -2 + 5$	$3 = 3$
$3 = 3$	

Since the point "satisfies" both equations…it IS the solution.
Answer: **YES**

7. Is (-1,1) a solution to the following system?

$5x + 6y = 1$
$6x + 2y = -3$

8. Is (2,1) a solution to the following system?

$x + 2y = 4$
$x - y = 1$

9. Is (3,3) a solution to the following system?

$3y = x + 6$
$3y = -4x + 21$

10. Is (1,2) a solution to the following system?

$2x + 2y = 6$
$y = 4x - 2$

11. Is (-1,1) a solution to the following system?

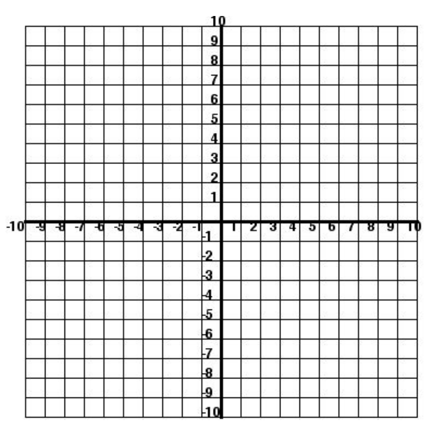
$7x = 3y + 10$
$6x + 5y = -1$

12. Explain how you can determine if a given point is the solution to a system of linear equations.

Challenge

13. Find the solution to the following system of equations.

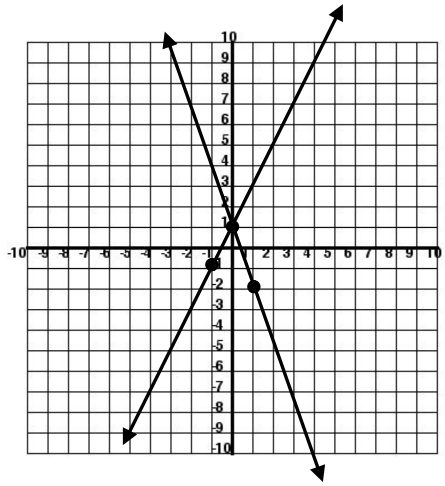
$$y = 2x + 1$$
$$y = -3x + 1$$



Explain your steps and/or thinking.

Find the solution to the following system of equations.

$$y = 2x + 1$$
$$y = -3x + 1$$



Explain your steps and/or thinking.

I graphed each of the lines.

I found the coordinates of the point that is on both lines

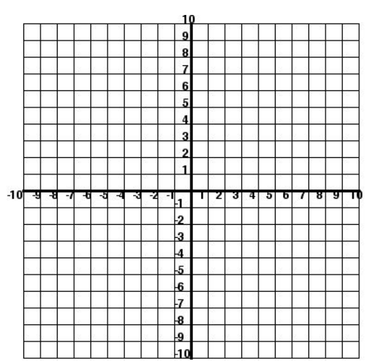
→ where the lines cross!

(0,1)

Solve the following systems by graphing:

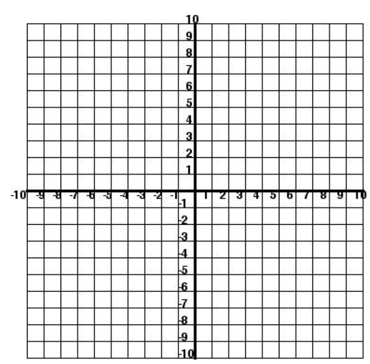
14. Solve:

$$y = 3x - 1$$
$$y = -2x + 4$$



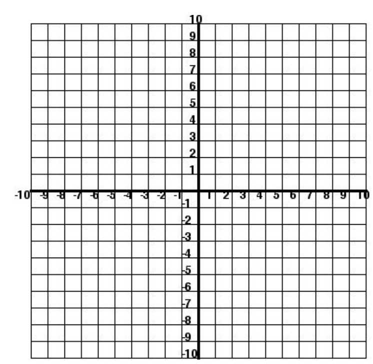
15. Solve:

$$x - y = -2$$
$$4x + 2y = 16$$



16. Solve:

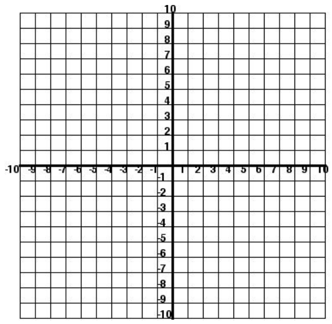
$$x + y = 5$$
$$3x - y = 3$$



Solve the following systems by graphing:

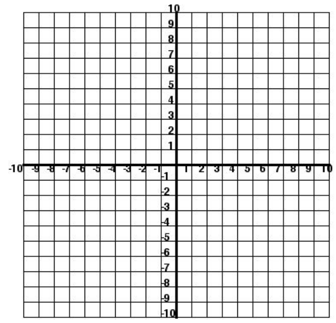
17. Solve:

$x + y = 4$ and $x - y = 2$



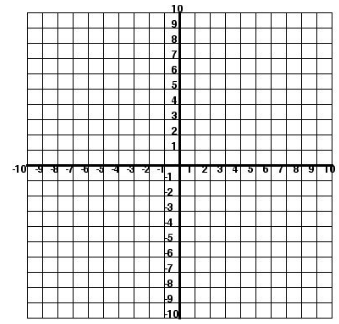
18. Solve:

$y = x - 2$ and $y = \frac{2}{5}x + 1$



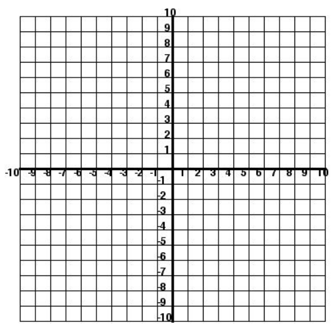
19. Solve:

$y = -3x + 5$ and $x - 2y = 4$



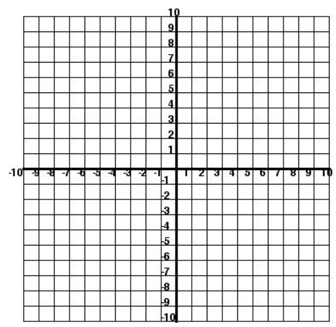
20. Solve:

$x + 2y = 8$ and $3x - y = 3$



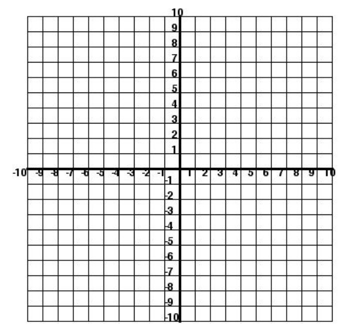
21. Solve:

$5x + 4y = 40$ and $5x + 6y = 60$



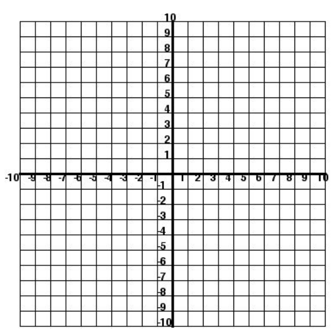
22. Solve:

$x = 5$ and $y + 4 = 10$



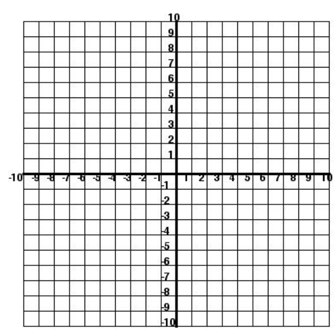
23. Solve:

$y = 2x - 3$ and $y = 2x + 3$



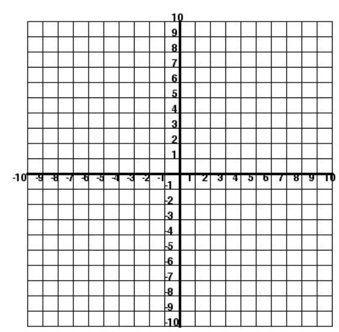
24. Solve:

$x - y = 1$ and $3y = 3x - 3$



25. Solve:

$2y = 3x - 2$ and $4y + 4 = 6x$



26. What do you notice about the equations above?

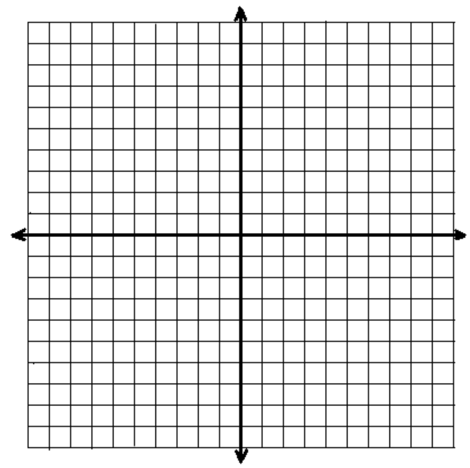
27. What do you notice about the equations above?

28. What do you notice about the equations above?

2) consistent & Inconsistent solutions

Warm-Up: Solve each system of equations graphically and verify algebraically..

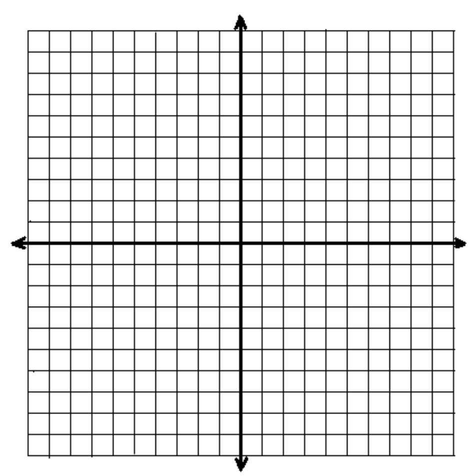
a) $\begin{Bmatrix} y = 3x + 2 \\ 2x - y = -4 \end{Bmatrix}$



Solution: _______

Verification:

b) $\begin{Bmatrix} 3x - y - 4 = 0 \\ 6x + 2y = -8 \end{Bmatrix}$

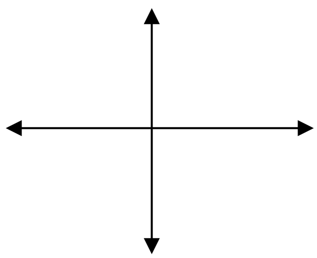
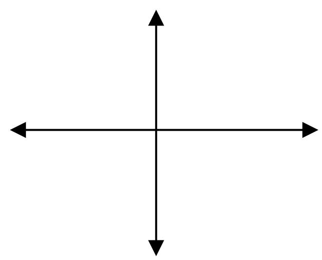
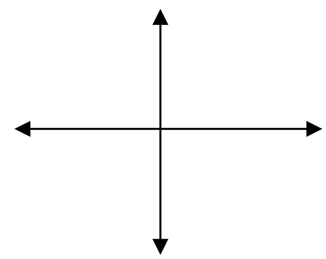


Solution: _______

Verification:

IMPORTANT IDEAS:

A system of linear equations can have _________ solution, _________ solution, or an _______________ number of solutions. Before solving, you can predict the number of solutions for a linear system by comparing the ____________________ and ____________________ of the equations.

Intersecting Lines	**Parallel Lines**	**Coincident Lines**
_____ solution(s)	_____ solution(s)	_____ solution(s)
		
_________ slopes	_________ slopes	_________ slopes
_________ y-intercepts	_________ y-intercepts	_________ y-intercepts

Example #1: Predict the number of solutions for each linear system. Justify your answer.

a) $\begin{Bmatrix} x + y = 3 \\ -2x - y + 2 = 0 \end{Bmatrix}$

b) $\begin{Bmatrix} 4x + 6y + 10 = 0 \\ -2x - 3y = 5 \end{Bmatrix}$

c) $\begin{Bmatrix} 2x - 4y + 1 = 0 \\ 3x - 6y - 2 = 0 \end{Bmatrix}$

Example #2: Given the equation $2x - y + 4 = 0$ write another linear equation that will form a linear system with the following number of solutions.

a) Exactly one solution

b) No solution

c) Infinite solutions.

Example #3: For the linear system $x - 2y + 4 = 0$ and $7x - 14y + C = 0$, what value(s) of C would give:

a) No solution

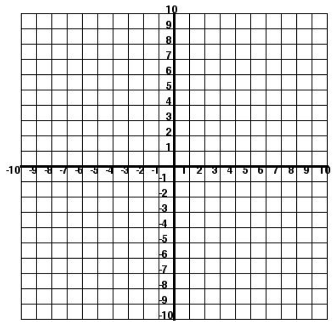
b) An infinite number of solutions

c) Exactly one solution

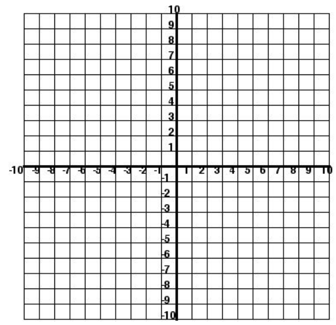
ASSIGNMENT # 2
pPages 9-11 Questions #25-50

29. ## Challenge

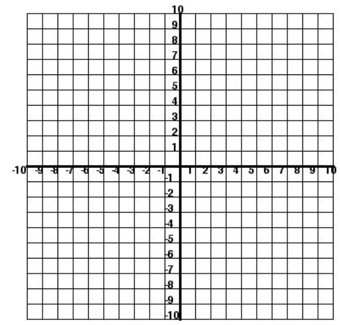
On the three graphs below, draw a system of linear equations with . . .



a) One solution



b) No solutions



c) Infinite Solutions

30. ## Challenge

How many solutions are there to the system

$y = 3x + 3$

$y = x + 1$

Explain your reasoning.

Types of Solution Sets:

One solution	No Solutions	Infinite Solutions
• Lines intersect once. • Different Slopes.	• Parallel Lines • Same Slopes • Different y-intercepts	• Same Lines • Same Slopes • Same y-intercepts
We say the system is **CONSISTENT**	We say the system is **INCONSISTENT** (no solution)	We say the system is **CONSISTENT**

Determine if the following systems have one solution, no solutions, or infinite solutions.

31. $y = 3x + 3$
$y = x + 1$

One solution because the slopes are different.

Lines will intersect once.

32. $y = 2x + 5$
$y = 3x - 5$

33. $3y = 9x + 12$
$3x - 9y = 12$

34. $6x + 4y = 1$
$3x - 2y = 4$

35. $2x + y = 5$
$y = -2x - 5$

36. $y = \frac{2}{3}x + 5$
$3y = 2x - 5$

Find the value of *k* that makes each system **inconsistent**.

37. $y = kx - 3$
$2y = 2x + 6$

38. $2y = kx + 1$
$2x - y = 7$

39. $4kx = y - 2$
$5x + 3y - 12 = 0$

Find the value of *b* that will produce a system with **infinite solutions**.

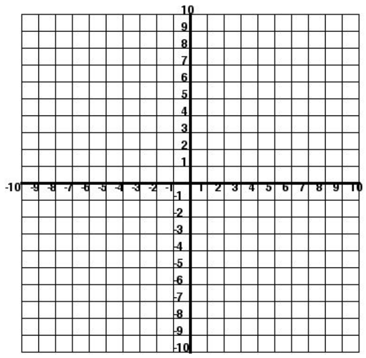
40. $y = x - b$
$2y = 2x - 4$

41. $3x - y = 7$
$4y = 12x + b$

42. $2x + 3y - 2b = 0$
$y = -\frac{2}{3}x + 1$

43. Solve:

$$2x + 3y - 6 = 0$$
$$3x - y + 2 = 0$$



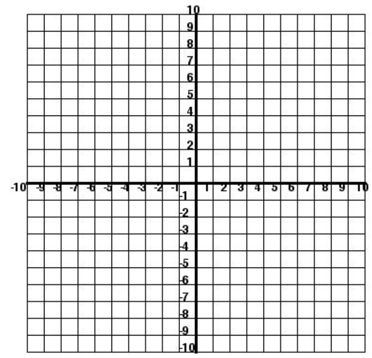
44. The system above is

a) Consistent

b) Inconsistent

45. Solve:

$$x - y = 1$$
$$5x + 2y = 5$$



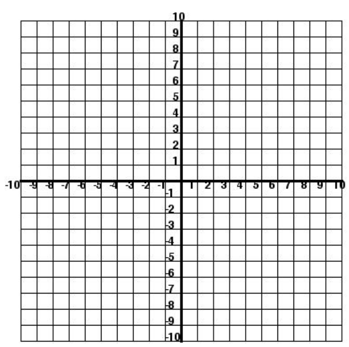
46. Add the two equations above and graph the new equation.

47. What do you notice?

48. Graph the system of equations:

$y = x + 2$

$3y = 2x - 5$



49. What is the problem when solving this system by graphing?

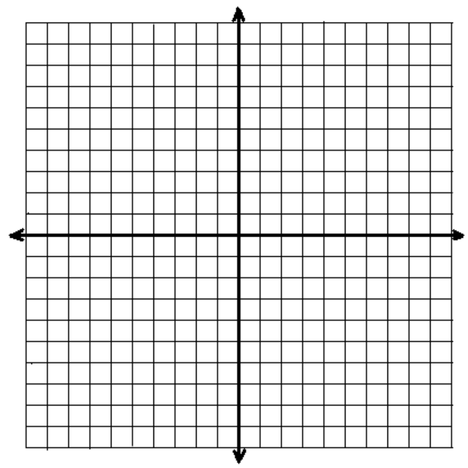
50. ## Challenge

Solve the system of linear equations: $y = x + 2$ and $3y = 2x - 5$.

3) Solving by Substitution

Warm-Up: Solve the system of equations graphically and verify algebraically..

a) $\begin{Bmatrix} 2x + y = 5 \\ x + y - 3 = 0 \end{Bmatrix}$



Solution: _______

Verification:

Solving by Substitution:

You already know how to verify your solution algebraically by substituting in the values for x and y. The process of graphing and verifying is often time consuming and an algebraic method could give the same results quicker and more accurately. This process is called **Solving by Substitution**

Process for Solving by Substitution

1. Choose one equation and solve for one variable (either x or y).

2. Substitute your equation from step 1 into the other equation. (You should have only 1 variable now!). Solve for your variable.

3. Substitute the value back into one of the original equations to solve for the second variable.

4. Identify your solution.

Example #1: Solve this system using substitution. $\begin{Bmatrix} 2x + y = 5 \\ x + y - 3 = 0 \end{Bmatrix}$

***Example #2*:** Solve this linear system using substitution. $\begin{Bmatrix} 2x - 4y = 7 \\ 4x + y - 5 = 0 \end{Bmatrix}$

a) How do you decide which variable to isolate? Explain.

b) Solve for that variable.

c) Substitute your solution into the unused equation. Then, solve.

d) You now have part of your solution, how do you get the other part? Explain.

e) Complete the solution.

f) Identify 2 different ways to verify your solution?

ASSIGNMENT # 3
pPages 12-15 Questions #50-67

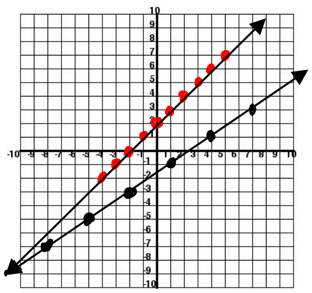
Solving Systems of Equations (without graphing)

Part 1: Solving By substitution.

Graph the system of equations:

$y = x + 2$

$3y = 2x - 5$



My thoughts…

If I graph each of these lines, I notice that they do not cross at a point that I can easily read on **this** graph.

Also, the second equation is not easily graphed.

I can use a different method.

Algebra! See My Solution Below.

51. What is the solution to a system of linear equations?

52. If a point is present on two lines, what values of that point are equal:
    a. x-values
    b. y-values
    c. both x- and y-values

Solve the system of equations:

"1" $y = x + 2$

"2" $3y = 2x - 5$

I will substitute (x+2) in to equation "2" for y.

3(x+2) = 2x – 5
3x+6 = 2x – 5
x = - 11

Then substitute x = - 11 into equation "1".

y = (-11) +2
y = - 9

Therefore the solution is (- 11, - 9)

53. Solve the following system of equation without graphing, consider the answers to the previous questions to guide you.

$$y = 2x - 1$$
$$y = -x + 1$$

54. Verify your solution above.

Solve the following systems of equations **by substitution**.

55. Solve.

$y = 2x - 1$

$y = -x + 1$

Since both $(2x - 1)$ and $(-x + 1)$ are equal to 'y', then they must be equal to each other.

$2x - 1 = -x + 1$

$3x = 2$

$x = \frac{2}{3}$

To find 'y', substitute your known 'x' into either equation.

$$y = -\left(\frac{2}{3}\right) + 1$$

$$y = \frac{1}{3}$$

Solution $\left(\frac{2}{3}, \frac{1}{3}\right)$

56. How can I check the solution to the left?

57. Check the solution to the left.

58. Solve.

$3x + y = 1$

$2x + 3y = 11$

59. Solve.

$a + c = 9$

$2a + c = 11$

60. Solve.

$3x - 4y = -15$

$5x + y = -2$

61. Solve.

$d + e = 1$

$3d - e = 11$

Solve the following systems of equations **by substitution**.

62. Solve.

$a + 6b = 9$

$3a - 2b = -23$

63. Solve.

$2t - w = 13$

$4t + 3w = 1$

64. Solve.

$3y = -6x + 15$

$5y = 5x + 10$

65. Solve.

$y = \frac{x}{3} + 2$

$3y + 4x = 21$

66. Solve.

$3x - 2y = 4$

$3x + 4y = 10$

67. Solve.

$\frac{1}{4}x + \frac{1}{2}y = 10$

$\frac{1}{4}x - \frac{1}{2}y = 0$

4) solving by elimination

Warm-Up #1: Identify the lowest common denominator for each pair of fractions.

a) $\frac{1}{3}+\frac{3}{4}$

b) $-\frac{2}{7}-\frac{5}{3}$

Warm-Up #2: Identify the lowest common multiple for each pair of numbers.

a) 5 and 15

b) 4 and 6

c) 12 and 5

Warm-Up#3: Simplify each expression without the use of a calculator.

a) $-3+(-5)=$

b) $-3+5=$

c) $-3+(+3)=$

d) $-2-(-4)=$

e) $-2-4=$

f) $-2-(+2)=$

If you **don't have a variable with a coefficient of 1** in a system of equations, substitution is difficult. ***There is another method you can use in these cases***.

You can solve a system of linear equations using the **ELIMINATION** method. ***To do this, a variable in both equations must have the same, or opposite, coefficients.*** It is often necessary to multiply one, or both, equations by a constant value to get the coefficients you need to eliminate.

Example #1: For each linear system, write an equivalent linear system where both equations have; (i) the opposite x-coefficients and (ii) the opposite y-coefficients.

a) $\begin{Bmatrix} x-2y=-6 \\ 3x+y=2 \end{Bmatrix}$

b) $\begin{Bmatrix} 14x+15y=16 \\ 21x+10y=-1 \end{Bmatrix}$

Example #2: Solve each system using the elimination method.

a) $\left\{\begin{array}{c} 3x - 5y + 9 = 0 \\ 4x + 5y - 23 = 0 \end{array}\right\}$

b) $\left\{\begin{array}{c} x - 2y = 7 \\ 3x + 4y - 1 = 0 \end{array}\right\}$

c) $\left\{\begin{array}{c} 3x + 4y = -5 \\ 2x + 8 = -5y \end{array}\right\}$

Example #3: Verify your solution for example #2b algebraically.

ASSIGNMENT # 4

p(skip pg 16) Pages 17-19 questions #74-97

68. Write a system of 2 linear equations for the following problem.

The sum of two numbers is 65. The first number is 17 greater than the second.

69. Find the numbers in the problem to the left.

70. Write a system of 2 linear equations for the following problem.

One number is 12 less than another number. Their sum is 102.

71. Find the numbers in the problem to the left.

72. Write a system of 2 linear equations for the following problem.

Mr. J bought a total of 12 pairs of socks. Athletic socks cost $5 per pair and dress socks cost $7 per pair. He spent $70 in total.

73. How many pairs of each type of socks did he buy?

Part 2: Solving By Elimination (Addition or Subtraction)

Challenge Questions

74. Is (3,1) a solution to the system $2x - y = 5$ and $2x - 4y = 2$?

75. Multiply each of the equations above by 2.
$2(2x - y = 5)\rightarrow$ $2(2x - 4y = 2)\rightarrow$

76. Is (3,1) still a solution to each of the equations above?

77. Add the two original equations together:
$2x - y = 5$
$\underline{2x - 4y = 2}$

78. Is (3,1) a solution to the new equation?

79. What conclusions can you draw about adding/subtracting equations together?

80. What conclusions can you draw about multiplying equations in a system by a constant?

81. Can you multiply the equations by different numbers without affecting the solution?

82. Graph equation ①:

① $2x + y = 8$

83. Graph equation ②:

② $y = 4x - 4$

84. Add equations① and ②.
Call this equation ③.

③ ________________

85. Graph equation ③.

86. Multiply ③ $\times$ 3 and call this equation ④.

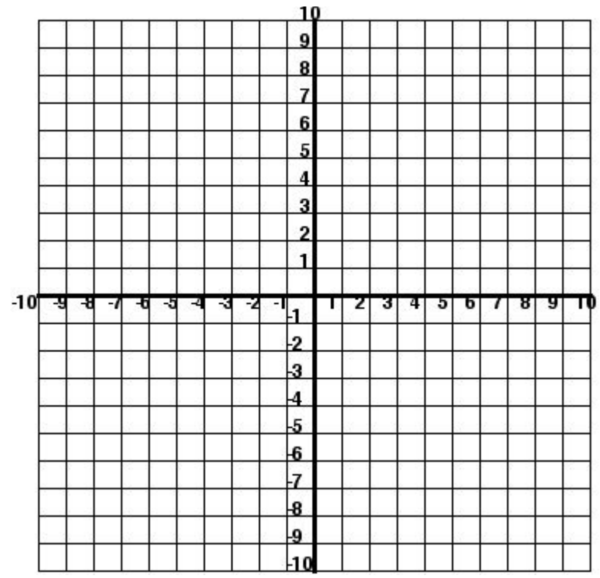
④ ________________

87. Graph equation ④.

88. Add ③and ④ , call this equation ⑤.

⑤ ________________

89. Graph equation ⑤.



90. Describe what you see happening above.

91. Write a set of rules describing what you may do to a system of equations in order to find the solution. That is, how can you manipulate the equations without affecting the solution?

92. Add the two equations together, then solve.

$3x - 6y = 21$

$\underline{-3x - 4y = -1}$

-10y = 20

y = - 2 → 3x – 6(- 2) = 21

3x + 12 = 21

3x = 9

x = 3

Solution: (3, - 2)

93. Solve.

$2x + 3y = 18$

$2x - 3y = -6$

94. Solve.

$8x + 2y = -20$

$2x - 2y = -30$

95. Solve.

$-4t + 3s = 2$

$8t - 6s = -4$

96. Solve.

$6x - 3y = 24$

$x + y = -2$

97. Solve.

$3b - a = 1$

$-12b + 4a = -4$

5a) word problems part I

Warm-Up #1: Solve this system of linear equations, using either substitution or elimination.

$$\begin{Bmatrix} 0.2x - 0.3y = 0.5 \\ 0.3x - 0.2y = 0.5 \end{Bmatrix}$$

Word Problems (Day 1):

1. The sum of two numbers is 752 and their difference is 174. Find the numbers.

2. The sum of five times one number plus three times a second number is eight. The sum of three times the first number plus five times the second number is 24.

To save time, let's just set up the following systems of equations. ***DO NOT SOLVE.***

3. Canada won 26 medals at the 2010 Winter Olympic Games, including 7 silver medals. The number of gold medals was 4 more than twice the number of bronze medals. How many gold and bronze medals did Canada win?

4. Each time Ms. A went to for lunch, she bought either a bowl of soup or a main course. During the school year, she spent $490 and bought 160 food items. How many times did she buy soup and a main course?

Soups............................*1.75*
Chicken
Green Pea
Main Course.................*4.75*
Spaghetti and Salad
Vegetarian Pizza

5. Nadine has a pink piggy bank of nickels and a blue piggy bank of dimes. The total number of coins is 300 and their value is $23.25. How many coins are in each piggy bank?

ASSIGNMENT # 5a
pPages 16 & 20

98. Solve.

$0.05x + 0.07y = 19$

$x + y = 300$

99. Solve.

$x + y = 1200$

$0.20x + 0.40y = 36$

100. Two numbers have a sum of 25 and a difference if 7. What are the two numbers?

101. Anya has a pocket full of loonies ($1 coins) and toonies ($2 coins). She has $41 in total. If she has 29 coins, how many of each does she have?

102. When three times one number is added to two times another number, the sum is 21. When 4 times the second number is subtracted from 10 times the first number, the difference is 38. What are the numbers?

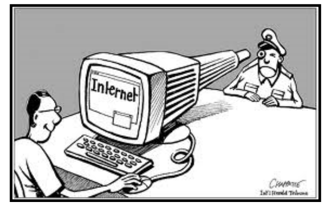
103. The total cost (before taxes) for three coffees and two cookies is $10.05. The cost for five coffees and three cookies is $16.10. Find the individual cost for each item.

5b) word problems part II

Word Problems (Day 2):

1. Ryan Kesler invested $2000, part of it at an annual interest rate of 8% and the rest at an annual interest rate of 10%. After one year, he earned $190 in interest. How much money did he have in each investment?

4. Forty-five high school students and adults were surveyed about how they use the internet. Thirty-one people reported using the internet heavily. This was 80% high school students and 60% of the adults. How many students were included in this survey?



5. A 50% acid solution is required in a chemistry lab. The instructor has a 20% stock solution and a 70% stock solution. He needs to make 20 litres of the 50% acid solution. How much of each stock solution should he use?

ASSIGNMENT # 5b
pPages 21-24 Questions #104- 120

Solving Problems with Systems of Equations. Use the method of your choice.

104. A job offered to Mr. Xu will pay straight commission at a rate of 6% on all sales. A second job offer will pay a monthly salary of \$400 and 2% commission. How much would Mr. Xu have to sell so that both jobs would pay him the same amount.

When would the job paying straight commission be a better choice?

105. In his 2004-05 season, Steve Nash scored 524 total baskets (not including free throws). He scored 336 more two point baskets than three point baskets. Write and solve a system of linear equations that represents this problem.

Interpret your solution:

106. Mr. J has a class with 30 students in it. 22 of those students own a cell phone. $\frac{4}{5}$ of the girls owned a cell phone and $\frac{3}{5}$ of the boys owned a cell phone. How many girls were in this class?

107. Daiki invested a total of \$12 000 in two stocks in 2009. One stock earned 4% interest and the other earned 7% interest. Daiki earned a total of \$615 in interest in 2009. How much did he invest in each stock?

For each of the following problems, write and solve a system of equations. Interpret solutions!

108. Breakers Volleyball sold 570 tickets to their home opener, some tickets cost \$2 and some cost \$5. The total revenue was \$1950. How many of each type of ticket were sold?

109. Mr. J is doing routine maintenance on his old farm truck. This month he spent \$26.50 on 6 litres of oil and 2 gaskets. Last month he spent \$25.00 on 4 litres of oil and 4 gaskets. Find the price of each gasket and one litre of oil.

110. Anya makes a trip to the local grocery store to buy some bulk candy. She chooses two of her favourite candies, gummy frogs and gummy penguins. Gummy frogs sell for \$1.10 per 100g and penguins sell for \$1.75 per 100g. Anya buys a total of 500g of candy for \$7.84 (no taxes). How much of each type did she buy?

111. For his Christmas party, Teems Prey is making a bowl of exotic punch for the kid's table. Imported lychee juice sells for \$12.50 per litre and guava nectar sells for \$18 per litre. He is making 8 litres and will need to pay \$126.40 for the perfect blend. How much of each type does he use?

③④

112. Jay Maholl swam 12 km downstream in Englishman River in two hours. The return trip upstream took 6 hours. Find the speed of the current in Englishman River.

113. (What assumption must you make?)

114. The Lucky-Lady dinghy travels 25 km upstream in five hours. The return trip takes only half an hour. Find the speed of the boat and the speed of the current.

115. A bumble bee travels 4.5 km into a headwind in 45 minutes. The return trip with the wind only takes 15 minutes. Assuming speeds are constant, find the speed of the bumble bee in still air.

116. A plane flew a distance of 650 km in 3.25 hours when travelling in a tailwind. The return trip took 6.5 hours against the same wind. Assume both speeds are constant. Find the speed of the plane and the wind speed.

117. A 50% acid solution is required for a chemistry lab. The instructor has a 20% stock solution and a 70% stock solution. She needs to make 20 litres of the 50% acid solution. How much of each stock solution should she use?

Let x = volume of 20% solution

Let y = volume of 70% solution.

$x + y = 20$

$0.2x + 0.7y = (0.5)(20)$

Solve the System:

118. A 65% acid solution is required for a chemistry lab. The instructor has a 20% stock solution and a 70% stock solution. She needs to make 20 litres of the 65% acid solution. How much of each stock solution should she use?

119. The karat (or carat) is a measure of the purity of gold in gold alloy. 18K gold is approximately 75% pure and 14K gold is approximately 58.5% pure. Using 18K and 14K stock, a goldsmith needs to produce 40g of gold alloy that is 70% pure. How much of each stock will he need to use? (round to nearest hundredth)

120. A goldsmith needs to make 50g of 14K gold (58.5%) from 18K (75%) and 10K (41.7%) stock alloys. How much of each does she need? (round to nearest hundredth)

6) Arithmetic Sequence

A ______________________________ is simply a list of numbers. In a sequence each number is called a ____________ of the sequence. There is a ***first*** term, ***second*** term, ***third t***erm, and so on. A sequence can be ____________ in which it is ***possible to count the number*** of terms, or ____________, in which the terms ***continue forever.***

For example: 1, 3, 6, 10 is a(n) ____________ sequence
1, 3, 6, 10, ... is a(n) ____________ sequence

A sequence is a function whose domain is a set of positive integers. However, a sequence is written using subscript notation rather than function notation.

For example: $a_1, a_2, a_3, \ldots, a_n$

The subscript identifies the term of the sequence. For instance a_3 is the third term, and a_n is the nth term of the sequence. The entire sequence is usually denoted by $\{a_n\}$.

Sequence

A **inite sequence** is ...

An **infnite sequence** is ...

Example 1 Write the first four terms of the sequence.

a) $a_n = \dfrac{n+1}{n}$

b) $b_n = 2n - 3$

c) $t_n = 2^n$

Another way of defining a sequence is to define the first term, or the first few terms, and specify the nth term by a formula involving the preceding term(s). Sequences defined in this manner are called ______________________

Example 2 Write the first four terms of the **recursive formula**: $a= 3$, $a_n = \dfrac{a_{n-1}}{n}$.

Sigma Notation

It is often important to find the **sum of a sequence**, $\{\_\_\_\_\} = a_1 + a_2 + a_3 + \cdots + a_n$.

The expanded notation $a_1 + a_2 + a_3 + \cdots + a_n$ can be written more compactly using ________________________.

The ***Greek letter Σ (sigma)*** is used as the summation symbol in sigma notation.

$$\underbrace{a_1 + a_2 + a_3 + \cdots + a_n}_{\textit{expanded notation}} =$$

The integer____is called the ***index of the sum***, which shows where the summation starts.

The integer ____ shows where the ***summation ends***.

The summation $\sum_{k=1}^{n} a_k$ has $n - k + 1$ terms.

Example 3 Find the sum of each sequence.

a) $\sum_{k=1}^{4}(2k+1)$

b) $\sum_{k=1}^{5}(k^2+1)$

c) $\sum_{k=1}^{3}(k^3-k)$

Example 4 Write the sum using sigma notation.

a) $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots + \frac{12}{12+1}$

b) $\frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \cdots + \left(\frac{2}{3}\right)^n$

Arithmetic Sequence

When the difference between successive terms of a sequence is ***always the same number***, the sequence is called ________________________________.

For example the sequence 3, 7, 11, 15, ... is arithmetic because adding 4 to any term produces the next term. The ***common difference****, d, of this sequence is 4.*

To develop a formula to find the general term of an arithmetic sequence, the first few terms need to be expanded.

1st term: $a_1 = a_1$

2nd term: $a_2 = a_1 + d$

3rd term: $a_3 =$

4th term: $a_4 =$

Notice that the coefficient of d is one less than the subscript of the term.

The *n*th Term of an Arithmetic Sequence

For an arithmetic sequence $\{t_n\}$ whose first term is a, with common difference d:

Example 5 For each arithmetic sequence, identify the common difference.

a) 3, 5, 7, 9, ...

b) 11, 8, 5, 2, ...

Example 6 Determine if the sequence $\{t_n\} = \{3 - 2n\}$ is arithmetic.

Example 7 Find the 12th term of the arithmetic sequence 2, 5, 8, ...

Example 8 Which term in the arithmetic sequence 4, 7, 10, ... has a value of 439?

Example 9 The 7th term of an arithmetic sequence is 78, and the 18th term is 45. Find the first term.

Example 10 Find x so that $3x + 2$, $2x - 3$, and $2 - 4x$ are consecutive terms of an arithmetic sequence.

ASSIGNMENT # 6
Questions #1, 2-11 (every 2nd Letter) + 12-19

Exercise Set

1. Fill in the blanks.

a) The domain of a sequence is the set of consecutive _____________ numbers.

b) A sequence with a last term is a(n) _____________ sequence.

c) A sequence with no last term is a(n) _____________ sequence.

d) The sequence $a_1 = 2,\ a_n = 2a_{n-1}$ is a _____________ sequence.

e) The formula for the nth term of an arithmetic sequence is $t_n =$ ________________ .

2. Write the first four terms of each sequence.

a) $\{n^2 - 2\}$

b) $\left\{\frac{n+2}{n+1}\right\}$

c) $\{(-1)^{n+1}n^2\}$

d) $\left\{\frac{3^n}{2^n + 1}\right\}$

e) $\left\{\frac{2^n}{n^2}\right\}$

f) $\left\{\left(\frac{2}{3}\right)^n\right\}$

3. Write the nth term of the suggested pattern.

a) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

b) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

c) $\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, \ldots$

d) $2, -4, 6, -8, \ldots$

4. Write the first four terms of the recursive sequence.

a) $a = 4,\ t_n = 2 + t_{n-1}$

b) $a = 3,\ t_n = n - t_{n-1}$

c) $a = 2,\ a_2 = 3,\ a_n = a_{n-1} + a_{n-2}$

d) $a_1 = -1,\ a_2 = 1,\ a_n = na_{n-1} + a_{n-2}$

5. Find the sum of each sequence.

a) $\sum_{k=1}^{5} 4$

b) $\sum_{k=1}^{4} (k^2 - 2)$

c) $\sum_{k=2}^{5} (k^2 - 1)$

d) $\sum_{k=0}^{3} (k^3 - 1)$

e) $\sum_{k=1}^{4} \frac{k^2}{2}$

f) $\sum_{k=6}^{8} (k + 1)^2$

6. Express each sum using summation notation with index $k = 1$.

a) $1 + 3 + 5 + 7$

b) $1^2 + 2^2 + 3^2 + 4^2 + 5^2$

c) $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots + \frac{n}{n+1}$

d) $5 + \frac{5^2}{2} + \frac{5^3}{3} + \cdots + \frac{5^n}{n}$

7. Write the first five terms of each arithmetic sequence.

a) $7, 11, 15,$ ____ , ____

b) $15, 12, 9,$ ____ , ____

c) $a = 4,\ d = 2$

d) $a = -1,\ d = -3$

e) $a = -5,\ d = -\frac{3}{4}$

f) $a = -\frac{2}{3},\ d = \frac{1}{5}$

8. Find the indicated arithmetic term.

a) $a = 5,\ d = 3;$ find t_{12}

b) $a = \frac{2}{3},\ d = -\frac{1}{4};$ find t_9

c) $a = -\frac{3}{4},\ d = \frac{1}{2};$ find t_{10}

d) $a = 2.5,\ d = -1.25;$ find t_{20}

e) $a = -0.75,\ d = 0.05;$ find t_{40}

f) $a = -1\frac{3}{4},\ d = -\frac{2}{3};$ find t_{37}

9. Find the number of terms in each arithmetic sequence.

a) $a = 6,\ t_n = -30,\ d = -3$

b) $a = -3,\ t_n = 82,\ d = 5$

c) $a = 0.6,\ t_n = 9.2,\ d = 0.2$

d) $a = -0.3,\ t_n = -39.4,\ d = -2.3$

e) $-1, 4, 9, \ldots, 159$

f) $23, 20, 17, \ldots, -100$

10. Find the first term in the arithmetic sequence.

a) 6th term is 10; 18th term is 46

b) 4th term is 2; 18th term is 30

c) 9th term is 23; 17th term is -1

d) 5th term is 3; 25th term is -57

e) 13th term is -3; 20th term is -17

f) 11th term is 37; 26th term is 32

11. Find x so that the values given are consecutive terms of an arithmetic sequence.

a) $x + 3,\ 2x + 1$, and $5x + 2$

b) $2x,\ 3x + 2$, and $5x + 3$

c) $x - 1,\ \frac{1}{2}x + 4$, and $1 - 2x$

d) $2x - 1,\ x + 1$, and $3x + 9$

e) $x + 4,\ x^2 + 5$, and $x + 30$

f) $8x + 7,\ 2x + 5$, and $2x^2 + x$

12. If t_n is a term of an arithmetic sequence, what is $t_n - t_{n-1}$ equal to?

13. List the first seven numbers of the Fibonacci sequence $a_1 = 1,\ a_2 = 1,\ a_n = a_{n-1} + a_{n-2},\ n > 2$.

14. The starting salary of an employee is \$23 750. If each year a \$1250 raise is given, in how many years will the employee's salary be \$50 000?

15. An auditorium has 8 seats in the first row. Each subsequent row has 4 more seats than the previous row. What row has 140 seats?

16. A well drilling company charges \$8.00 for the first meter, then \$8.75 for the second meter, and so on in an arithmetic sequence. At this rate, what would be the cost to drill the last meter of a well 120 meters deep?

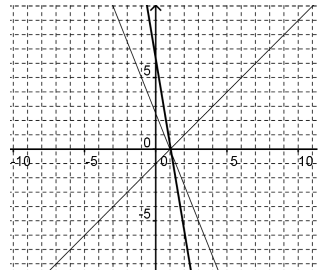
17. It is said that during the last weeks of his life Abraham deMoivre needed 15 minutes more sleep each night, and when he needed 24 hours sleep he would die. If he needed 8 hours sleep on September 1, what day did he die?

18. The first three terms of an arithmetic sequence are $x - 3$, $\frac{x^2}{25} + 9$, and $3x - 11$. Determine the fourth term.

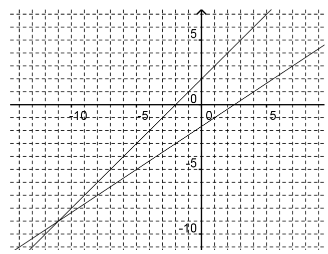
19. The first, third, and fifth terms of an arithmetic sequence are $2x - 1$, $x^2 - 3$, and $11 - x^2$ respectively. Determine the second term.

Answers:

1. He should estimate his earnings from sales.
2. On graph pg 5.
3. This is the ordered pair that represents the sales that would produce equivalent earnings.
4. When Jazhon sells more than \$40 000 of concrete.
5. (2,4) is a solution because it **satisfies** both equations in the **system**.
6. yes
7. no
8. yes
9. yes
10. yes
11. no
12. If the coordinates satisfy both (all) equations in the system. Also, the point will be on both lines when graphed.
13. (0,1). Plot each equation using slope and y-intercept. Find the coordinates of the point of intersection.
14. (1,2)
15. (2,4)
16. (2,3)
17. (3,1)
18. (5,3)
19. $(2,-1)$
20. (2,3)
21. (0,10)
22. (5,6)
23. No solution. Parallel lines never intersect.
24. Both lines share all points. We say there are infinite solutions.
25. Both lines share all points. We say there are infinite solutions.
26. Same slope, different y-intercept.
27. Same slope, same y-intercept. Same line.
28. Same slope, same y-intercept. Same line.
29. Answers will vary.
    One solution: lines will have diff. slopes.
    No solutions: Parallel lines.
    Infinite solutions: same lines.
30. One. These equations have different slopes.
31. One solution.
32. One solution.
33. One solution.
34. One solution.
35. No solutions.
36. No solutions.
37. $k = 1$
38. $k = 4$
39. $k = -\frac{5}{12}$
40. $b = 2$
41. $b = -28$
42. $b = \frac{3}{2}$
43. (0,2)
44. Consistent
45. (1,0)
46. $6x + y = 6 \rightarrow y = -6x + 6$



47. The new line passes through the solution to the original system.
48.

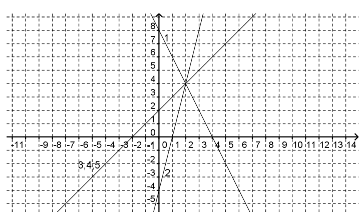


49. The intercept and intersection points are not integers therefore difficult to read on the graph. See page 12.
50. $(-11,-9)$
51. The point (or sometimes points) that satisfies all the equations.
52. C
53. $\left(\frac{2}{3},\frac{1}{3}\right)$
54. $\frac{1}{3} = 2\left(\frac{2}{3}\right) - 1 \rightarrow \frac{1}{3} = \frac{4}{3} - \frac{3}{3} \rightarrow \frac{1}{3} = \frac{1}{3}$

    $\frac{1}{3} = -\left(\frac{2}{3}\right) + 1 \rightarrow \frac{1}{3} = \frac{-2}{3} + \frac{3}{3} \rightarrow \frac{1}{3} = \frac{1}{3}$

    Both equations satisfied by the point $\left(\frac{2}{3},\frac{1}{3}\right)$.
55. Answered on page.
56. Substitute the point back into the original equations.
57. See #54 above.
58. $\left(-\frac{8}{7},\frac{31}{7}\right)$
59. (2,7)
60. $(-1,3)$
61. $(3,-2)$
62. $\left(-6,\frac{5}{2}\right)$
63. $(4,-5)$
64. (1,3)
65. (3,3)
66. (2,1)
67. (20,10)
68. $x + y = 65$
    $x = y + 17$
69. (41,24)
70. $x + y = 102$
    $x = y - 12$
71. (45,57)

72. $a + d = 12$
$5a + 7d = 70$
73. 7 athletic, 5 dress
74. Yes, it satisfies both equations.
75. $4x - 2y = 10$ and $4x - 8y = 4$
76. Yes
77. $4x - 5y = 7$
78. Yes
79. The solution to the original system will be a solution to the new equations too.
80. The solution to the original system will be a solution to the new equations too.
81. Yes. You must multiply each term in an equation by the same constant, but different equations can be multiplied by different constants without affecting the solution.
82. See graph below (with Q89).
83. See graph below.
84. ③ $2y = 2x + 4$
or $y = x + 2$
85. See graph below.
86. ④ $3y = 3x + 6$
or $y = x + 2$
87. See graph below.
88. ⑤ $5y = 5x + 10$
or $y = x + 2$
89. See graph below.



90. All 5 equations share a common point. Manipulating the equations did not change the fact that (2,4) was a solution.
91. You words here…
92. $(3, -2)$
93. $(3,4)$
94. $(-5,10)$
95. $*$ *infinite solutions*
96. $(2, -4)$
97. $*$ *infinite solutions*
98. $(100,200)$
99. $(2220, -1020)$
100. $(16,9)$
101. 17 loonies, 12 toonies
102. 5 is the first number, 3 is the second number.
103. Coffee: $2.05, Cookie: $1.95
104. If sales were $10 000 he would earn the same at both jobs. Straight commission would earn more money when he sold more than $10 000 in merchandise.
105. 430 two-point baskets, 94 three-point baskets
106. There are 20 girls.
107. $4500 at 7%
$7500 at 4%
108. 300 at $2, 270 at $5
109. Oil: $3.50, Gasket: $2.75
110. Frogs: 1.40 x 100grams = 140g
Penguins: 3.60 x 100 grams = 360g
111. Leechi: 3.2 l, Guava: 4.8 l
112. Current: 2 km/h
113. Speeds of swimmer and current are constant.
114. Boat: 27.5 km/h
Current: 22.5 km/h
115. Bumble Bee: 12 km/h
116. Plane: 150 km/h
Wind: 50 km/h
117. 20% stock: 8 l
70% stock: 12 l
118. 20% stock: 2 l
70% stock: 18 l
119. 14K: 12.12 g
18K: 27.88 g
120. 10K: 24.77 g
18K: 25.23 g

Assignment #6 KEY

1. **a)** natural **b)** finite **c)** infinite **d)** recursive **e)** $t_n = a + (n-1)d$
2. **a)** $-1, 2, 7, 14$ **b)** $\frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}$ **c)** $1, -4, 9, -16$ **d)** $1, \frac{9}{5}, 3, \frac{81}{17}$ **e)** $2, 1, \frac{8}{9}, 1$ **f)** $\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}$
3. **a)** $\frac{1}{n}$ **b)** $\frac{1}{2^{n-1}}$ **c)** $\left(\frac{2}{3}\right)^n$ **d)** $(-1)^{n+1} \cdot 2n$
4. **a)** $4, 6, 8, 10$ **b)** $3, -1, 4, 0$ **c)** $2, 3, 5, 8$ **d)** $-1, 1, 2, 9$
5. **a)** 20 **b)** 22 **c)** 50 **d)** 32 **e)** 15 **f)** 194
6. **a)** $\sum_{k=1}^{4}(2k-1)$ **b)** $\sum_{k=1}^{5} k^2$ **c)** $\sum_{k=1}^{n} \frac{k}{k+1}$ **d)** $\sum_{k=1}^{n} \frac{5^k}{k}$
7. **a)** $7, 11, 15, 19, 23$ **b)** $15, 12, 9, 6, 3$ **c)** $4, 6, 8, 10, 12$ **d)** $-1, -4, -7, -10, -13$
e) $-5, -\frac{23}{4}, -\frac{13}{2}, -\frac{29}{4}, -8$ **f)** $-\frac{2}{3}, -\frac{7}{15}, -\frac{4}{15}, -\frac{1}{15}, \frac{2}{15}$
8. **a)** 38 **b)** $-\frac{4}{3}$ **c)** $\frac{15}{4}$ **d)** -21.25 **e)** 1.2 **f)** -25.75
9. **a)** 13 **b)** 18 **c)** 44 **d)** 18 **e)** 33 **f)** 42
10. **a)** -5 **b)** -4 **c)** 47 **d)** 15 **e)** 21 **f)** $40\frac{1}{3}$
11. **a)** $-\frac{3}{2}$ **b)** 1 **c)** -4 **d)** -2 **e)** $-3, 4$ **f)** $-3, \frac{1}{2}$
12. $d =$ difference
13. 1, 1, 2, 3, 5, 8, 13
14. $t = a + (n-1)d \rightarrow 50\,000 = 23\,750 + (n-1)(1250) \rightarrow 21 = n - 1 \rightarrow n = 22$ years
15. $t = a + (n-1)d \rightarrow 140 = 8 + (n-1)(4) \rightarrow 33 = n - 1 \rightarrow n = 34$, row 34
16. $t = a + (n-1)d \rightarrow t = 8 + (120-1)(0.75) \rightarrow t = 97.25$; $97.25
17. $t = a + (n-1)d \rightarrow 24 = 8 + (n-1)\left(\frac{1}{4}\right) \rightarrow 64 = n - 1 \rightarrow n = 65$ days
September has 30 days, October has 31 days, together they have 61 days. Therefore he died November 4th.
18. $\frac{(x-3)+(3x-11)}{2} = \frac{x^2}{25} + 9 \rightarrow x^2 - 50x + 400 = 0 \rightarrow (x-40)(x-10) = 0 \rightarrow x = 10, 40$
$x = 10$: $7, 13, 19, 25$; $x = 40$: $37, 73, 109, 145$; The fourth term is 25 or 145.
19. $\frac{(2x-1)+(11-x^2)}{2} = x^2 - 3 \rightarrow 3x^2 - 2x - 16 = 0 \rightarrow (3x-8)(x+2) = 0 \rightarrow x = -2, \frac{8}{3}$
$x = -2$: $-5, \_\_\_, 1, \_\_\_, 7$; $x = \frac{8}{3}$: $\frac{39}{9}, \_\_\_, \frac{37}{9}, \_\_\_, \frac{35}{9}$; The second term is -2 or $\frac{38}{9}$.