10

HW Mark:	10	9	8	7	6	RE-Submit

Trigonometry

This booklet belongs to: Marissa Period 4

LESSON #	DATE	QUESTIONS FROM NOTES	Questions that I find difficult
1	May 6/15	Pg. 5-12	27, 28
2	May 11/15	Pg. 13-18	
3	May 13/15	Pg. 19-25	87*, 104*, 105, 109*
4	May 14/15	Pg. 26-30	148, 151 ★, 153 ★
5	May 20/15	Pg. 31-35	
6	May 21/15	Pg. 36-39	
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		REVIEW	
		TEST wednesday, May 27, 2015	

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Your teacher has important instructions for you to write down below.

Trigonometry

STRAND Measurement		DAILY TOPIC	EXAMPLE
4. Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.	4.1	Explain the relationships between similar right triangles and the definitions of the primary trigonometric ratios.	
	4.2	Identify the hypotenuse of a right triangle and the opposite and adjacent sides for a given acute angle in the triangle.	
	4.3	Solve right triangles, with or without technology.	
	4.4	Solve a problem that involves one or more right triangles by applying the primary Trigonometric ratios or the Pythagorean theorem.	
	4.5	Solve a problem that involves indirect and direct measurement, using the trigonometric ratios, the Pythagorean theorem and measurement instruments such as a clinometer or metre stick.	

[C] Communication [PS] Problem Solving, [CN] Connections [R] Reasoning, [ME] Mental Mathematics [T] Technology, and Estimation, [V] Visualization

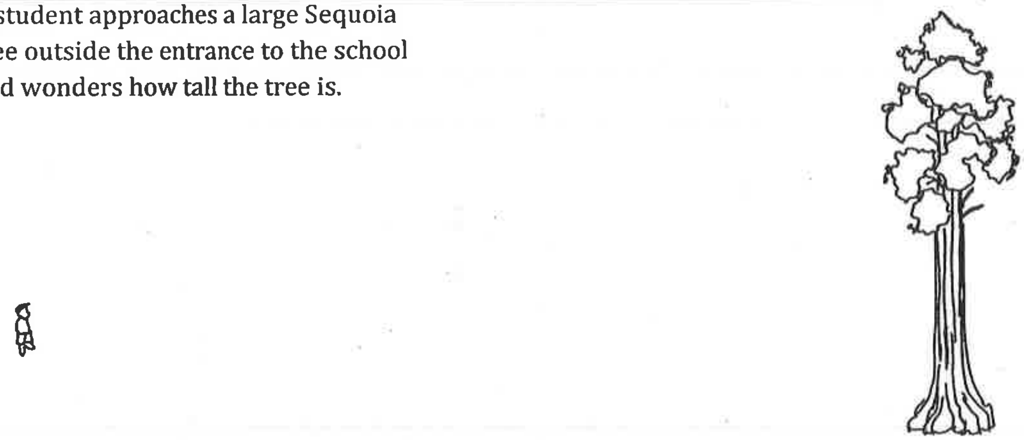
Key Terms

Term	Definition	Example
Triangle		
Similar Triangles		
Theta		
Acute angle		
Obtuse angle		
Right Triangle		
Oblique Triangle		
Legs		
Hypotenuse		
Trigonometry		
Opposite side		
Adjacent side		
Sine ratio		
Cosine ratio		
Tangent ratio		
Theta (θ)		

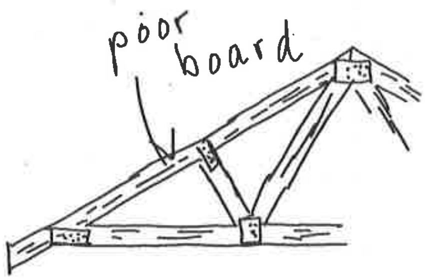
Why Trigonometry?

There is an application of trigonometry that could solve each problem below.

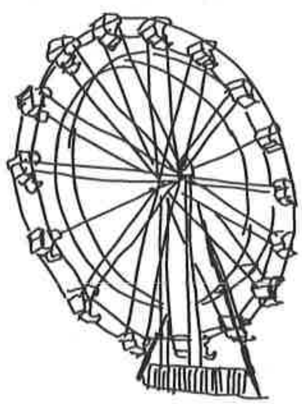
A student approaches a large Sequoia tree outside the entrance to the school and wonders how tall the tree is.



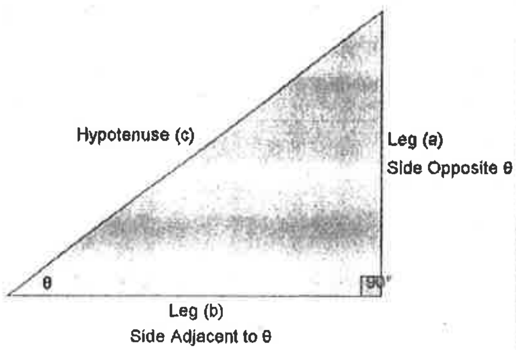
A homeowner wants to cut a new board to replace a decaying roof truss. He can measure the horizontal distance and the angle of inclination but needs to know how long to cut the board.



An engineer is constructing a Ferris wheel for a downtown park. There are 16 passenger carts.



The Right Triangle



A right triangle is a triangle with one right angle (90°).

The side opposite the right angle is called the hypotenuse.

The other two sides are called "legs".

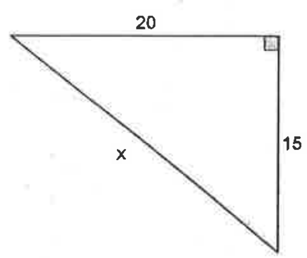
The sides of the right triangle form a Pythagorean Triple. That is, they satisfy the Pythagorean Theorem: $a^2 + b^2 = c^2$.

Some Pythagorean Triples:
(3,4,5), (5,12,13), (7,24,25), (8,15,17), (9,40,41), (11,60,61,), ...

(6,8,10)

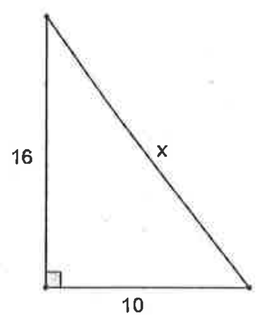
Find the indicated side length (nearest tenth) in the following right triangles.

1.



$$a^2 + b^2 = c^2$$
$$20^2 + 15^2 = c^2$$
$$625 = c^2$$
$$25 = c$$

2.



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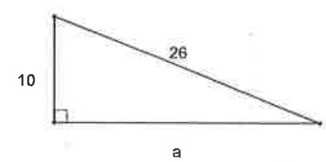

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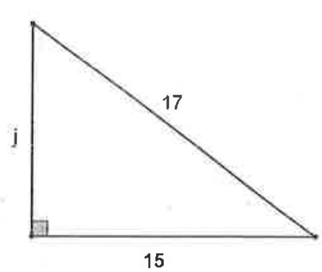

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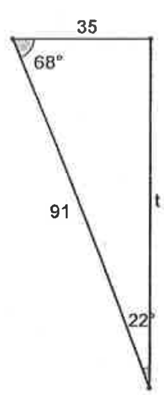

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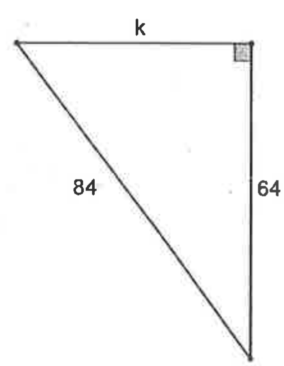
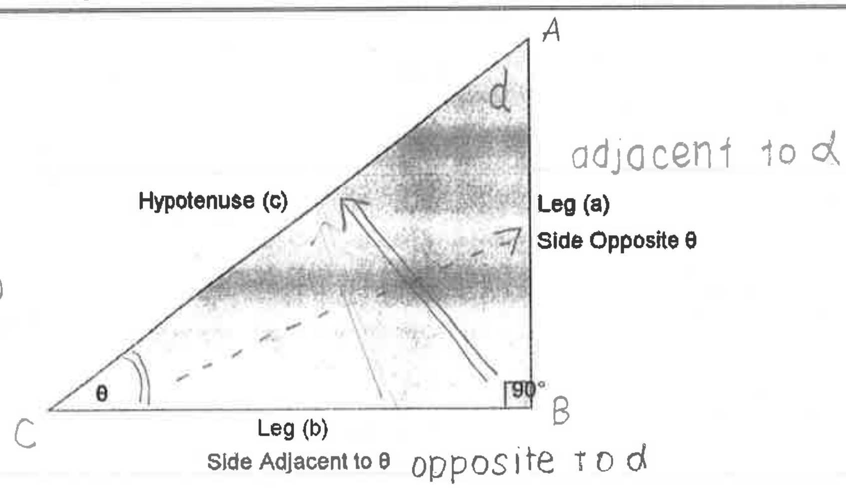


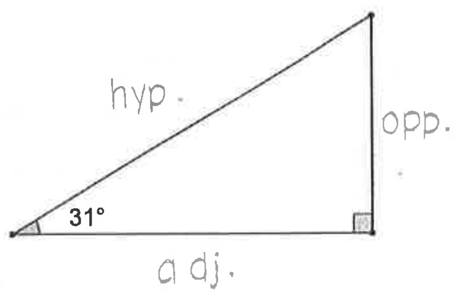
Labeling the Right Triangle for use with Trigonometry.

θ/α/β = theta = (unknown #)



One acute angle is indicated on each of the following triangles. If possible, label each triangle with: opposite, adjacent, and hypotenuse in respect to that angle. Remember, only **right** triangles can be labeled this way.

7.



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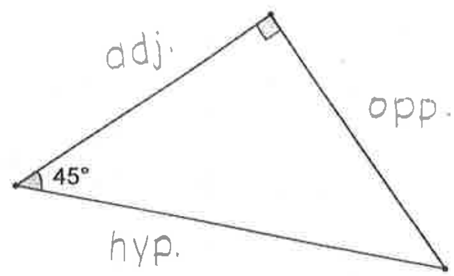

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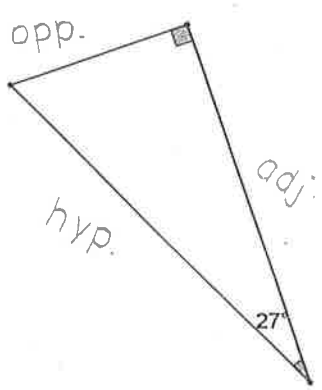

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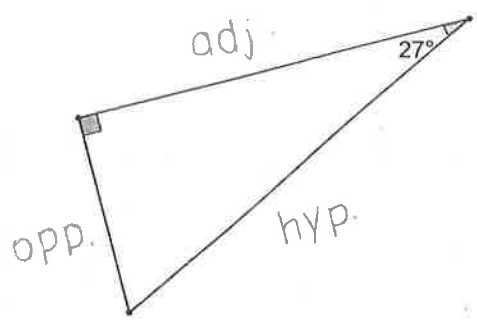


Trigonometry of Right Triangles – THE RATIOS

Since similar right triangles have equivalent ratios for corresponding angles, we can use those ratios to find unknown angles and/or side lengths.

These ratios have been calculated and stored in our calculator for many angles to help us solve problems.

We will use the three **primary trigonometric ratios:**

Tangent ratio Sine ratio Cosine ratio

The Primary Trig. Ratios

For an acute angle in a right triangle:

the ratio of $\boxed{\dfrac{\textit{opposite} \angle\theta}{\textit{hypotenuse}}}$ is called the $\boxed{\text{SINE RATIO.}}$ REMEMBER AS SOH or SO/H

$\sin\theta = \frac{\text{opp.}}{\text{hyp.}} = \frac{O}{H}$

the ratio of $\dfrac{\textit{adjacent} \angle\theta}{\textit{hypotenuse}}$ is called the COSINE RATIO. REMEMBER AS CAH or CA/H

$\cos\theta = \frac{A}{H}$

the ratio of $\dfrac{\textit{opposite} \angle\theta}{\textit{adjacent} \angle\theta}$ is called the TANGENT RATIO. REMEMBER AS TOA or TO/A

$\sin\theta = \frac{O}{H}$ $\cos\theta = \frac{A}{H}$ $\tan\theta = \frac{O}{A}$

SOH	CAH	TOA

Since the ratios of the legs of a right triangle remain constant despite reducing or enlarging the triangle, we can use these ratios to solve proportional problems.

The ratios have been calculated and stored in our calculator for many angles to help us solve problems.

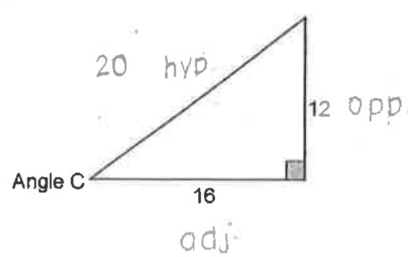
11. Challenge Question

Find the following ratios for angle C in the triangle to the right.

a. $tanC = \frac{12}{16} = \frac{3}{4}$

b. $sinC = \frac{O}{H} = \frac{12}{20} = \frac{3}{5}$

c. $cosC = \frac{A}{H} = \frac{16}{20} = \frac{4}{5}$



$a^2+b^2=c^2$
$12^2+16^2=c^2$
$c=\sqrt{12^2+16^2}$
$c=20$

Writing the Trigonometric Ratios

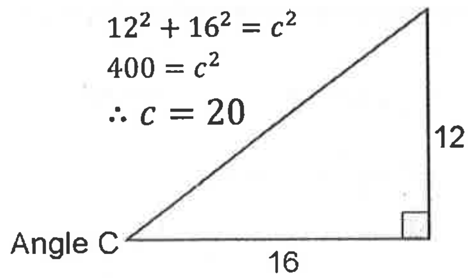
Remember, the ratios can be remembered using SO/H CA/H TO/A.

$a^2 + b^2 = c^2$

$12^2 + 16^2 = c^2$

$400 = c^2$

$\therefore c = 20$



From the diagram we see that...

$tanC = \frac{12}{16}$ or $\frac{3}{4}$

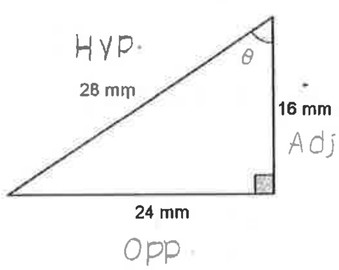
$sinC = \frac{12}{20}$ or $\frac{3}{5}$

$cosC = \frac{16}{20}$ or $\frac{4}{5}$

SOH CAH TOA

Find the three trig. ratios for the indicated angles below. Answer in fraction form.

12.

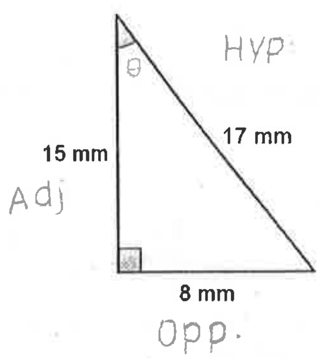


$sin\theta = \frac{24}{28} = \frac{6}{7}$

$cos\theta = \frac{16}{28} = \frac{4}{7}$

$tan\theta = \frac{24}{16} = \frac{3}{2}$

13.

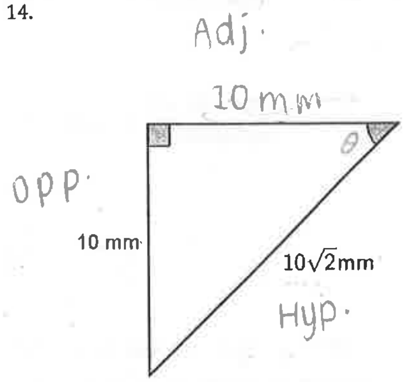


$sin\theta = \frac{8}{17}$

$cos\theta = \frac{15}{17}$

$tan\theta = \frac{8}{15}$

14.



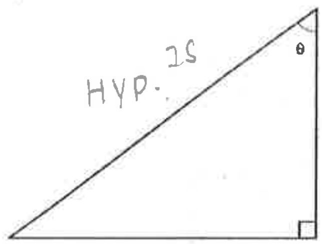
$sin\theta = \frac{10}{10\sqrt{2}} = \frac{1}{\sqrt{2}}$

$cos\theta = \frac{10}{10\sqrt{2}} = \frac{1}{\sqrt{2}}$

$tan\theta = \frac{10}{10} = \frac{1}{1} = 1$

☆ special!? ☆

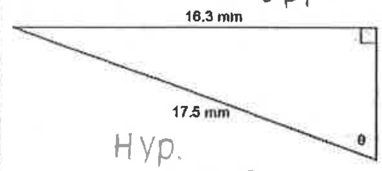
15. Find $sin\theta$.



$9^2+12^2=\sqrt{225}=15.45362...$

$sin\theta=\frac{12}{15}\rightarrow\boxed{\frac{4}{5}}$ $\frac{12}{15}$

16. Find $cos\theta$.



$17.5^2-16.3^2=\sqrt{40.56}$

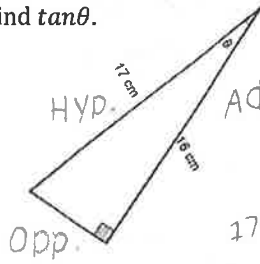
6.368673331

$\Rightarrow \theta$

$\frac{6.369}{17.5}=$

$cos\theta=\boxed{\frac{6.4}{17.5}}$

17. Find $tan\theta$.



$17^2-16^2=\sqrt{33}$

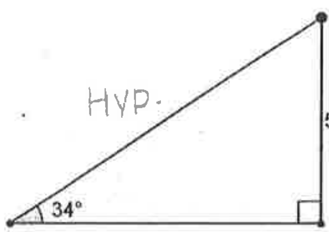
$=5.7$

$tan\theta=\boxed{\frac{5.7}{16}}$

SOH CAH TOA

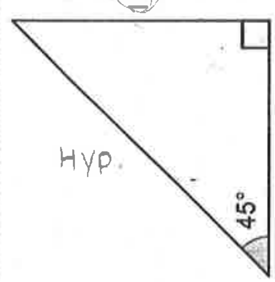
In each of the following diagrams, identify which ratio is represented (Sine, Cosine or Tangent).

18.



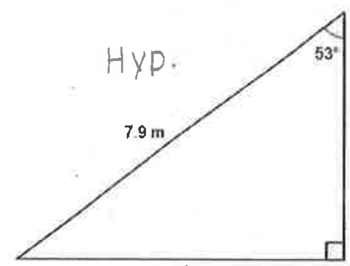
Ratio Tangent

19.



Ratio Tangent

20.



Ratio Sine

★4 decimal points★ ★Degree (DEG) mode★

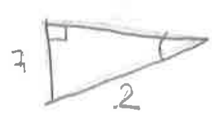
Use a scientific calculator to determine a decimal approximation for each of the following. Round to 4 decimals if necessary.

21. $\sin 30° =$ 0.5000	22. $tan70° =$ 2.7475	23. $cos35° =$ 0.8192
24. $\sin 42° =$ 0.6691	25. $\tan 45° =$ 1.0000	26. $\cos 60° =$ 0.5000

27. Notice that $tan45° = 1$ in the question above. Refer to another question above to help you describe what that means.

$tan=\frac{opp}{adj}$

28. Explain what it means for a right triangle to have a sine ratio equal to $\frac{1}{2}$.



Skill Reminder:

Solve the following equations. Answer to the nearest hundredth if necessary.

29. $\frac{12}{x} = 4$

$\frac{12}{x} = \frac{4}{1}$ set up equivalent fractions

$12 = 4x$ cross-multiply

$3 = x$ isolate the variable

30. $\frac{x}{4} = \frac{6}{1}$

$\boxed{x = 24}$

31. $\frac{x}{4} = \frac{0.55}{1}$

$\boxed{x = 2.2}$

32. $\frac{x}{5} = \frac{10}{2}$

$\frac{2x}{2} = \frac{50}{2}$

$\boxed{x = 25}$

33. $\frac{2}{x} = \frac{1.3}{1}$

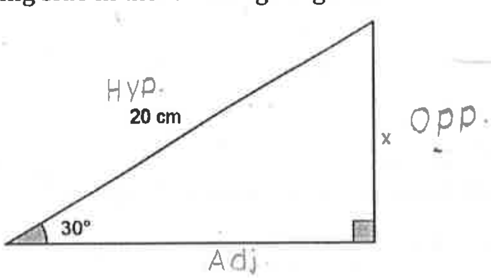
$\frac{1.3x}{1.3} = \frac{2}{1.3}$

$\boxed{x = 1.54}$

34. $\frac{x}{2.5} = \frac{6}{1}$

$\boxed{x = 15}$

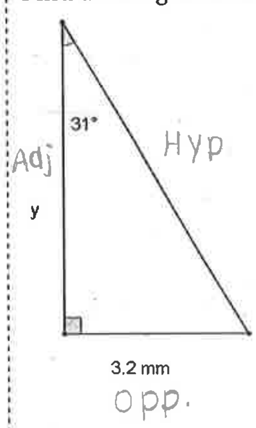
35. **Challenge.**
If we know $\sin 30° = 0.5000$, find the length of the missing side in the following diagram.



$\frac{0.5000}{1} = \frac{x}{20}$

$\boxed{10\text{ cm} = x}$

36. **Challenge.**
Find the length of the indicated side.



$\tan 31° = 0.600860619$

$\frac{0.600860619}{1} = \frac{3.2}{y}$

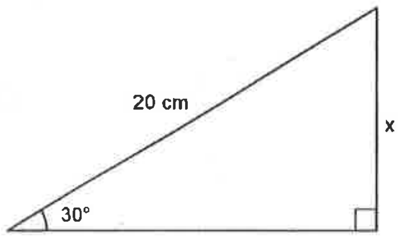
$0.600860619y = 3.2$

$\boxed{y = 5.3\text{ mm}}$

Finding Side Lengths Using Trigonometry

Find the length of the indicated side using an appropriate trigonometric ratio. Answer to tenths.

37.



We know:

$\sin 30° = 0.5000$

We also know: $\sin 30° = \frac{x}{20}$

We can say:

$0.5000 = \frac{x}{20}$

Solve the proportion:

$20(0.5000) = x$

$10.0 = x$

SOH CAH TOA

NOTE

To solve this problem…we can write…

$\sin 30° = \frac{x}{20}$

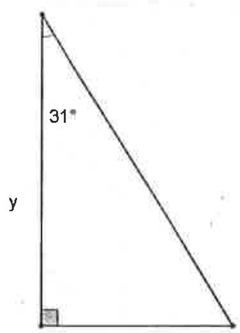
Multiply both sides by 20 to give:

$20\sin 30° = x$

Type $20 \times \sin 30$ into calculator…

$10.0 = x$

38.



$tan\theta = \frac{opp}{adj}$

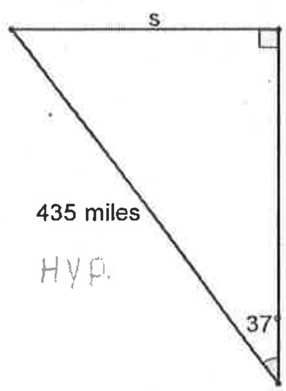
$tan31 = \frac{3.2}{y}$

$ytan31 = 3.2$

$y = \frac{3.2}{tan31} = 5.3$ mm

39.

Opp. =?



Hyp.

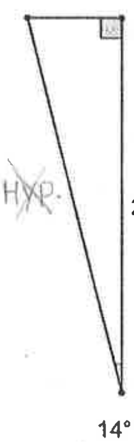
~~Adj.~~

$\frac{\sin 37}{1} = \frac{s}{435}$

$\boxed{s = 435 \sin 37} = \boxed{261.8 \text{ miles}}$

40.

Opp. w



~~Hyp.~~

Adj.

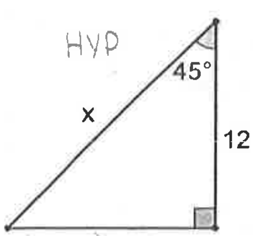
$\tan 14 = \frac{w}{22}$

$w = 22\tan 14$

$\boxed{w = 5.5 \text{ ft}}$

41.

Hyp



Adj.

~~Opp~~

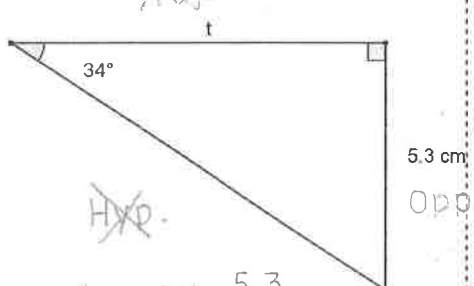
$\frac{\cos 45}{1} = \frac{12}{x}$

$\frac{\cos 45 x}{\cos 45} = \frac{12}{\cos 45}$

$\boxed{x = 17.0}$

42.

Adj.



~~Hyp.~~

Opp.

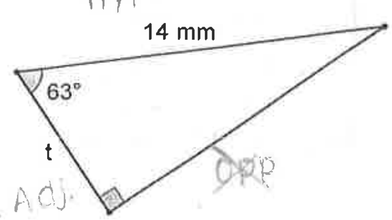
$\frac{\tan 34}{1} = \frac{5.3}{t}$

$\frac{\tan 34 t}{\tan 34} = \frac{5.3}{\tan 34}$

$\boxed{t = 7.9 \text{ cm}}$

43.

Hyp.



Adj.

~~Opp~~

$\frac{\cos 63}{1} = \frac{t}{14}$

$t = 14\cos 63$

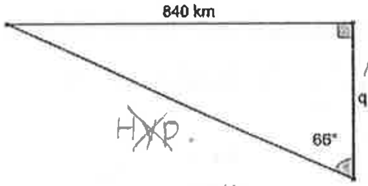
$\boxed{t = 6.4 \text{ mm}}$

✓

SOH CAH TOA

Find the length of the indicated side using an appropriate trigonometric ratio. Answer to tenths.

44.



Opp. Adj. ~~Hyp.~~

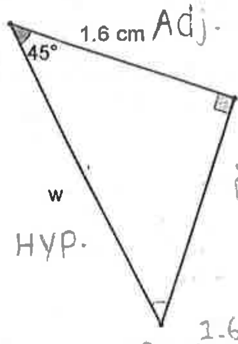
$\tan 66 = \frac{840}{q}$

$\tan 66 q = 840$

$q = \frac{840}{\tan 66}$

$\boxed{q = 374.0\text{ km}}$

45.



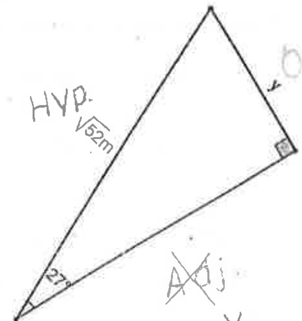
Adj. ~~Opp.~~ Hyp.

$\cos 45 = \frac{1.6}{w}$

$\cos 45 w = 1.6$

$w = \frac{1.6}{\cos 45} \rightarrow \boxed{w = 2.3\text{ cm}}$

46.



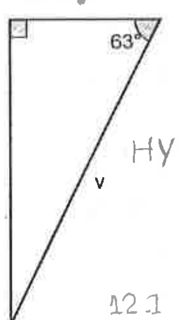
Hyp. Opp. ~~Adj.~~

$\sin 27 = \frac{y}{\sqrt{52}}$

$y = \sqrt{52}\sin 27$

$\boxed{y = 3.3\text{ m}}$

47.



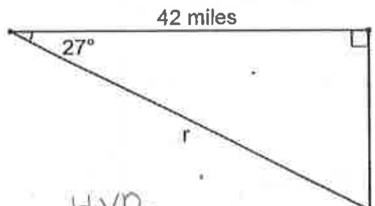
~~Adj.~~ Opp. Hyp.

$\sin 63 = \frac{12.1}{v}$

$v\sin 63 = 12.1$

$\boxed{v = 13.6\text{ m}}$

48.



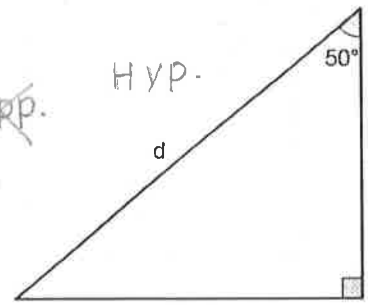
Adj. ~~Opp.~~ Hyp.

$\cos 27 = \frac{42}{r}$

$r\cos 27 = 42$

$\boxed{r = 47.1\text{ miles}}$

49.

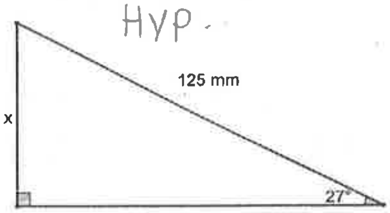


Hyp. ~~Adj.~~ Opp

$\sin 50 = \frac{17}{d}$

$d\sin 50 = 17 \rightarrow \boxed{d = 22.2\text{ cm}}$

50.



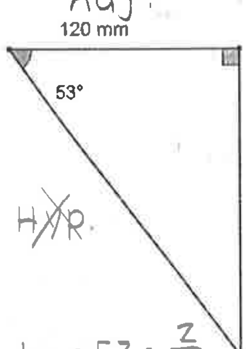
Hyp. Opp. ~~Adj.~~

$\sin 27 = \frac{x}{125}$

$125\sin 27 = x$

$\boxed{x = 56.7\text{ mm}}$

51.



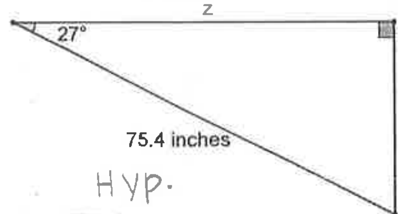
Adj. Opp. ~~Hyp.~~

$\tan 53 = \frac{z}{120}$

$z = 120\tan 53$

$\boxed{z = 159.2\text{ mm}}$

52.



Adj. ~~Opp.~~ Hyp.

$\cos 27 = \frac{z}{75}$

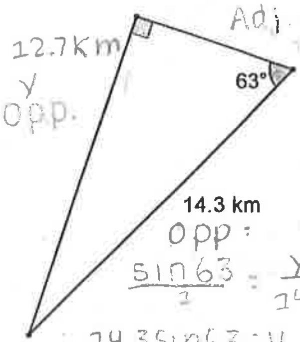
$75\cos 27 = z$

$z = 66.8$

✓

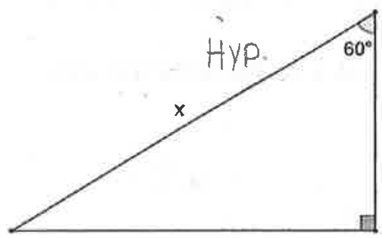
Find the length of the indicated side using an appropriate trigonometric ratio.

53. Find the length of each leg.



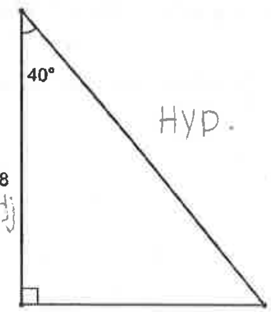
Opp:
$\frac{\sin 63}{1} = \frac{y}{14.3}$
$14.3 \sin 63 = y$
$\boxed{\text{Opp} = 12.7\text{Km}}$
Adj: $\cos 63 = \frac{x}{14.3}$
$14.3 \cos 63 = x \rightarrow \boxed{\text{Adj} = 6.5\text{km}}$

54.



$\frac{\sin 60}{1} = \frac{19.6}{x}$
$\sin 60 x = 19.6$
$\boxed{x = 22.6\text{mm}}$

55.



$\frac{\tan 40}{1} = \frac{2x}{18}$
$18 \tan 40 = 2x \rightarrow \boxed{x = 7.6}$

56. The tangent ratio is a ratio of what two sides in a right triangle?

$\frac{\text{Opposite}}{\text{Adjacent}}$

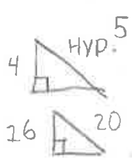
57. Can you use the tangent ratio to find the hypotenuse of a right triangle?

yes you can do
$\text{opp.}^2 + \text{adj.}^2 = \text{hyp.}^2$

58. Can you use the sine ratio to find the hypotenuse of a right triangle?

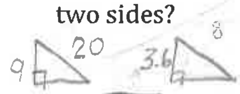
The sine ratio already includes the hyp.

59. The sine ratio of a right triangle is $\frac{4}{5}$. If the hypotenuse is 20 cm long, what are the lengths of the other two sides?



$\sqrt{20^2 - 16^2} = \sqrt{144}$
$\boxed{16\text{cm}, 12\text{cm}}$

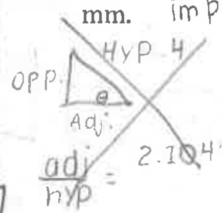
60. The cosine ratio of a right triangle is $\frac{9}{20}$. If the hypotenuse is 8 m long, what are the lengths of the other two sides?



$\sqrt{8^2 - 3.6^2} = \boxed{51.04}$
$\boxed{3.6\text{m}, 7.1\text{m}}$

61. The cosine ratio for a right triangle is 2.1042. Find the opposite side if the hypotenuse is 4 mm.

impossible



$\frac{\text{adj}}{\text{hyp}} =$

62. The sine ratio for a right triangle is 7.1004. Find the hypotenuse if the opposite side is 17 cm.

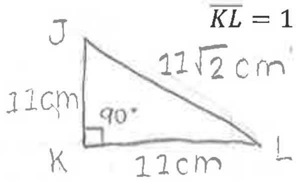
$\frac{\text{opp}}{\text{hyp}}$

impossible

$\frac{7.1004}{1}$

63. Draw a diagram illustrating the cosine ratio for $\angle J$ in ΔJKL. if

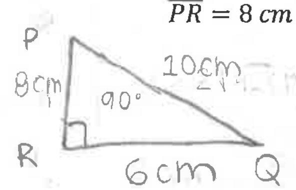
$\angle K = 90°$,
$\overline{JK} = 11\ cm$,
$\overline{KL} = 11\ cm$



$11^2 + 11^2 = 242$
$\sqrt{242} \rightarrow \sqrt{121 \times 2}$
$11\sqrt{2}$ cm

64. Draw a diagram illustrating the tangent ratio for $\angle P$ in ΔPQR if

$\angle R = 90°$,
$\overline{PQ} = 10cm$,
$\overline{PR} = 8\ cm$



$10^2 - 8^2 = 36$
$\sqrt{36} = 6$

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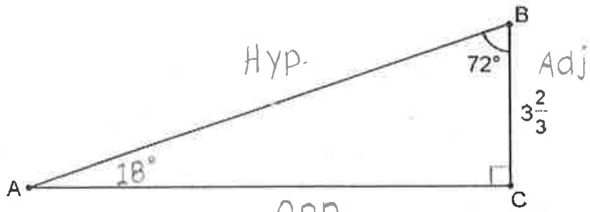
Solving Triangles:

To "solve a triangle" means to find the length of all unknown sides and measure of unknown angles.

65. Explain the steps you would take to solve the following triangle.



$\cos 72 = \frac{3\frac{2}{3}}{Hyp}$

$\cos 72\, Hyp = 3\frac{2}{3} \rightarrow Hyp = 11.9$

solve opp. $\tan 72 = \frac{opp}{3\frac{2}{3}}$

$Opp = 3\frac{2}{3}\tan 72$

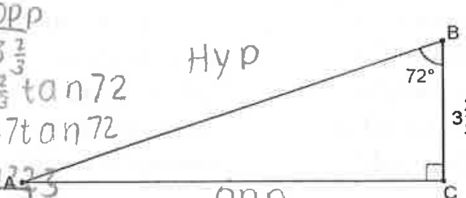
$= 3.6667\tan 72$

$= 11.2849223$

$= 11.3 \approx 11\frac{1}{3}$

Solve the following triangles. Answer to tenths.

66.



$\tan 72 = \frac{Opp}{3\frac{2}{3}}$

$Opp = 3\frac{2}{3}\tan 72$

$= 3.6667\tan 72$

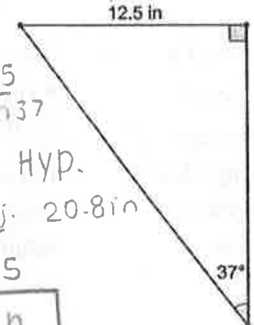
$= 11.2849223$

$\boxed{Opp = 11\frac{2}{3}}$

$\cos 72 = \frac{3\frac{2}{3}}{Hyp}$

$\cos 72\, Hyp = 3\frac{2}{3} \rightarrow \boxed{Hyp = 11.9}$

67.



$\sin 37 = \left(\frac{12.5}{hyp}\right)\frac{hyp}{1}$

$\frac{\sin 37\, hyp}{\sin 37} = \frac{12.5}{\sin 37}$

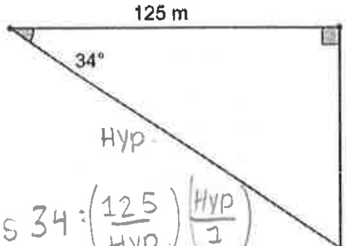
$\boxed{hyp = 20.8in}$

$\tan 37 = \left(\frac{12.5}{Adj}\right)\frac{Adj}{1}$

$\tan 37\, Adj = 12.5$

$\boxed{Adj = 16.6in}$

68.



$\cos 34 = \left(\frac{125}{Hyp}\right)\left(\frac{Hyp}{1}\right)$

$\cos 34\, Hyp = 125$

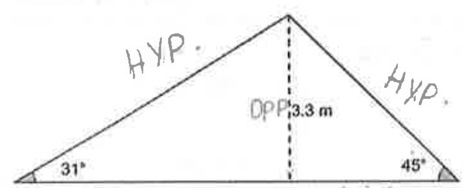
$\boxed{Hyp = 150.8m}$

$\tan 34 = \left(\frac{opp}{125}\right)\frac{125}{1}$

$125\tan 34 = opp$

$\boxed{opp = 84.3m}$

69. The dotted line is an altitude (perpendicular to base).



$\sin 31 = \frac{3.3}{Hyp}$

$Hyp \sin 31 = 3.3$

$\boxed{Hyp = 6.4m}$

$\tan 31 = \frac{3.3}{adj}$

$adj.\tan 31 = 3.3$

$adj = 5.5m$

$Hyp = \sin 45 = \frac{3.3}{Hyp}$

$Hyp \sin 45 = 3.3$

$\boxed{Hyp = 4.7m}$

$\tan 45 = \frac{3.3}{adj}$

$adj.\tan 45 = 3.3$

$adj = 3.3m$

$5.5m + 3.3m = 8.8m$

$\boxed{Adj. = 8.8m}$

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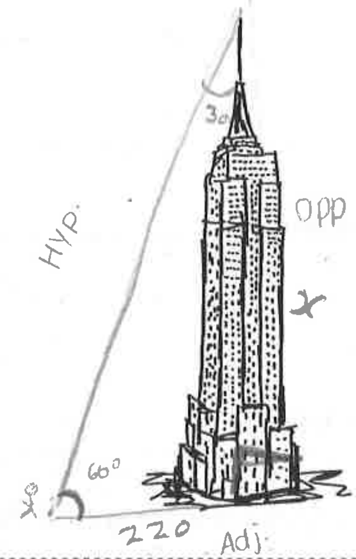
Solve each of the following word problems. Include a diagram in your solution.

70. From a point 220 m from the Empire State Building, a tourist measures the angle of inclination to the top to be 60°. Calculate the height of the building to the nearest metre...

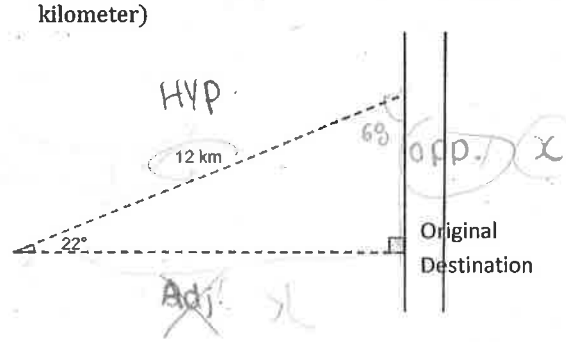
$\tan 60° = \frac{x}{220}$

$220 \tan 60 = x$

$\boxed{opp = 381m}$



71. A hiker loses track of her direction and wanders 22 degrees off course. If she continues to walk for 12 km to the river destination, how far away from her original destination will she be? (nearest tenth of a kilometer)

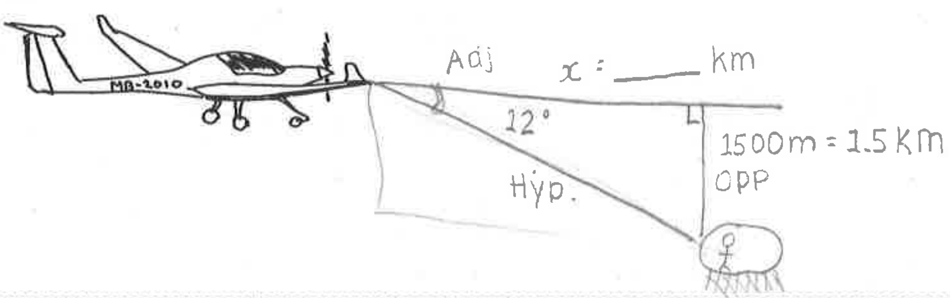


$\sin 22 = \frac{x}{12}$

$12 \sin 22 = x$

$\boxed{opp = 4.5Km}$

72. An airplane approaches a control tower. The angle of depression from the pilot to the tower is 12°. If the plane is flying at an altitude of 1500 m, how far is the plane from being directly above the tower (to the nearest kilometer)?

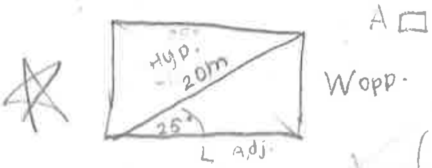


$\tan 12 = \frac{1.5}{x}$

$\tan 12 x = 1.5$

$\boxed{x = 7Km}$

73. Find the area of a rectangle with a diagonal of 20 m if the angle between the diagonal and longer side is 25 degrees. (nearest unit)



A□ = lw

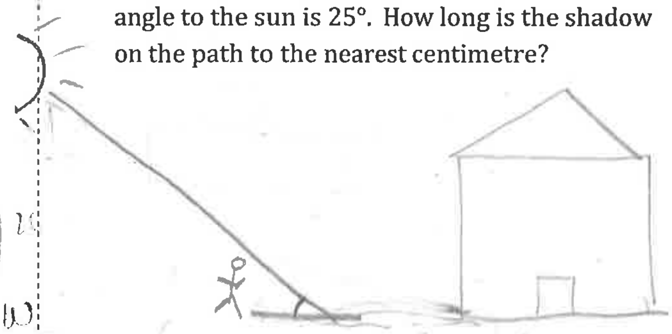
$20 \left(\cos 25 = \frac{L}{20}\right) 20$ $\quad$ $20 \left(\sin 25 = \frac{W}{20}\right) 20$

$20 \cos 25 = L$ $\quad$ $20 \sin 25 = W$

Adj = 18.13m → A $\quad$ opp = 8.45m → B

$\boxed{Area□ = 153m^2}$

74. A student ~~crossing to the west building~~ casts a shadow on the path. She is 165 cm tall and the angle to the sun is 25°. How long is the shadow on the path to the nearest centimetre?



$x\left(\tan 25 = \frac{165}{x}\right)x$

$\frac{\tan 25 x}{\tan 25} = \frac{165}{\tan 25}$

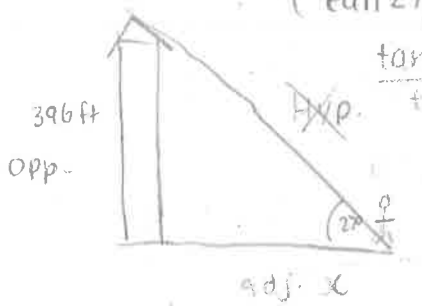
$x = \boxed{354 cm}$

75. A radio tower is 396 feet tall. How far from the base of the tower is a technician if the angle of inclination to the top of the tower is 27°? Answer to the nearest foot.

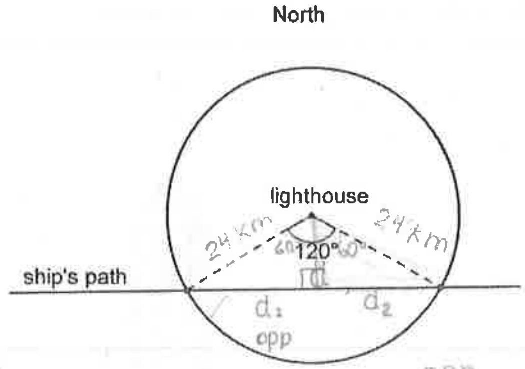
$x\left(\tan 27 = \frac{396}{x}\right)x$

$\frac{\tan 27x}{\tan 27} = \frac{396}{\tan 27}$

$\boxed{x = 777\text{ft}}$



76. A lighthouse attendant has a range of visibility of 24 km. A ship on the horizon passes by the lighthouse. The attendant sees the ship for a total of 120 degrees. For how many kilometers was the ship within the attendant's range of sight? (nearest tenth)



$\sin 60 = \frac{opp}{24}$

$= 20.78460969 \times 2$

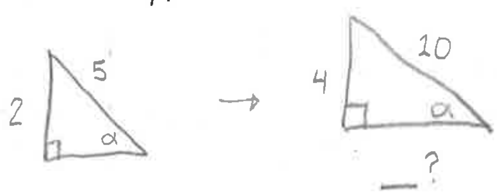
$= \boxed{41.6\text{ km}}$

Draw a scale diagram that would **represent** each of the following.

77. Draw a triangle that has the following:

$sin\alpha = \frac{2}{5}$, hypotenuse is 10 cm long.

$\sin\alpha = \frac{opp}{hyp}$



$a^2 + 4^2 = 10^2$

$a^2 = 10^2 - 4^2$

$a = \sqrt{(10^2 - 4^2)}$

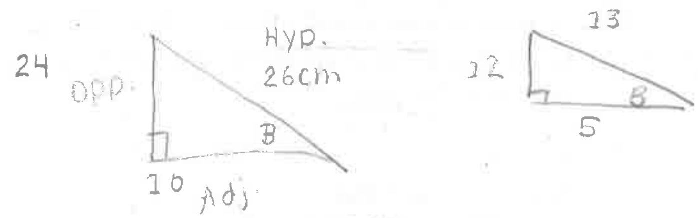
$a = \sqrt{(100-64)}$

$a = \sqrt{84}$

$\boxed{a = 9.2\text{cm}}$

78. Draw a triangle that has a the following:

$tan\beta = \frac{12}{5}$, hypotenuse is 26 cm long.



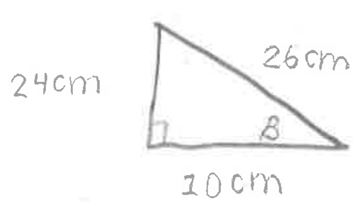
$\tan B = \frac{12}{5} \leftarrow \frac{opp.}{adj.}$

$h^2 = 12^2 + 5^2$

$= 144 + 25$

$= 169$

$h = 13$



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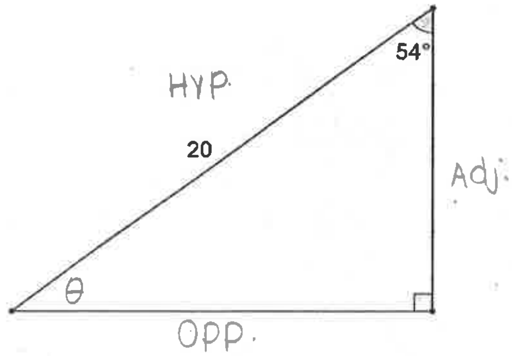
79. Solve the triangle.



$\boxed{\theta = 36°}$

Adj. $= {}^{20}\left(\cos 54 = \frac{adj}{20}\right)\frac{20}{1}$

$20\cos 54 = adj.$

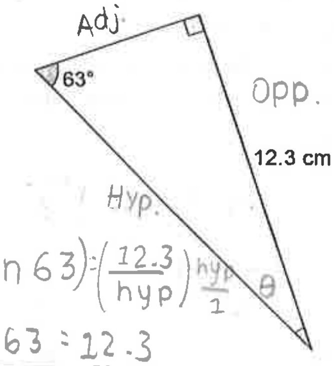
$\boxed{adj = 11.7557}$

opp $= {}^{20}\left(\sin 54\right) = \left(\frac{opp}{20}\right)\frac{20}{1}$

$20\sin 54 = opp$

$\boxed{opp = 16.1803}$

80. Solve the triangle.



$\boxed{\theta = 27°}$

Hyp $= {}^{hyp}\left(\sin 63\right) = \left(\frac{12.3}{hyp}\right)\frac{hyp}{1}$

$\frac{hyp\sin 63}{\sin 63} = \frac{12.3}{\sin 63}$

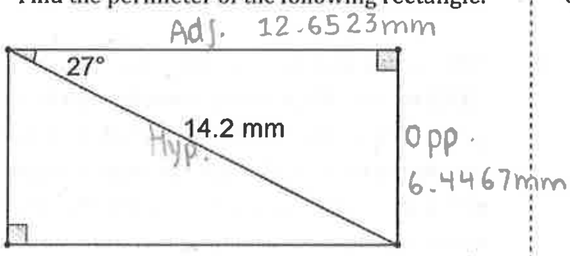
$\boxed{hyp = 13.8046\,cm}$

Adj. $= {}^{adj}\left(\tan 63\right) = \left(\frac{12.3}{adj.}\right)\frac{adj}{1}$

$\frac{adj.\tan 63}{\tan 63} = \frac{12.3}{\tan 63}$

$\boxed{adj. = 6.2672\,cm}$

81. Find the perimeter of the following rectangle.



adj. $= 14.2\left(\frac{\cos 27}{1} = \frac{Adj.}{14.2}\right)\frac{14.2}{1}$

$14.2\cos 27 = adj.$

$adj. = 12.6523\,mm$

opp $= {}^{14.2}\left(\sin 27\right) = \left(\frac{opp}{14.2}\right)14.2$

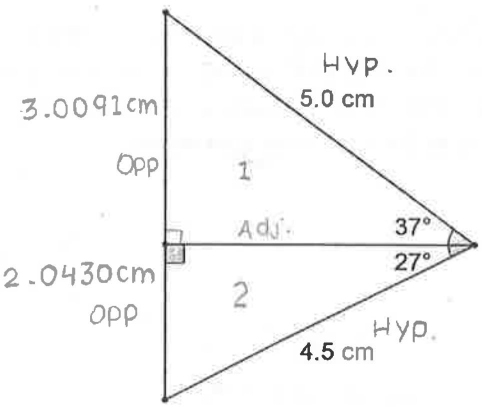
$14.2\sin 27 = opp$

$opp = 6.4467\,mm$

Adj. + Opp. + Adj. + Opp.

$P_{\square} = \boxed{38.1979\,mm}$

82. Find the total area.



Area: $\frac{(opp_1 + opp_2)(adj.)}{2}$

$= \boxed{10.0868\,cm^2}$

$opp_1 = {}^{5}\left(\sin 37\right) = \left(\frac{opp}{5}\right)\frac{5}{1}$

$5\sin 37 = opp$

$opp = 3.0091\,cm \Rightarrow A$

$opp_2 = {}^{4.5}\left(\sin 27\right) = \left(\frac{opp}{4.5}\right)\frac{4.5}{1}$

$4.5\sin 27 = opp$

$opp = 2.0430\,cm \Rightarrow B$

Adj. $= {}^{5}\left(\cos 37\right) = \left(\frac{adj.}{5}\right)\frac{5}{1}$

$5\cos 37 = adj.$

$adj. = 3.9932\,cm \Rightarrow C$

✓

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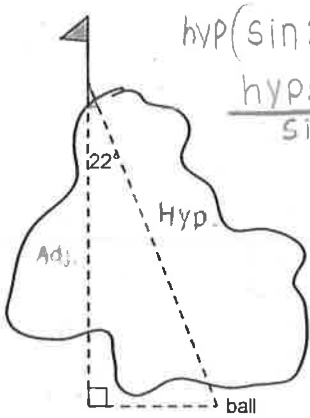
83. While golfing with his father-in-law, Mr. J hits a shot short of a pond. He walks 20 m to his left to a point directly across the pond from the hole. The angle between the two lines of sight is 22°. Find the distance from his ball to the hole to the nearest tenth of a metre.



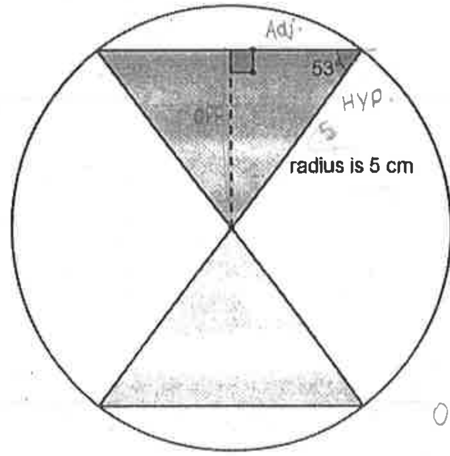
$hyp(\sin 22) = \left(\frac{20}{hyp}\right)\frac{hyp}{1}$

$\frac{hyp \sin 22}{\sin 22} = \frac{20}{\sin 22}$

$hyp. = \frac{20}{\sin 22}$

$\boxed{hyp = 53.4\text{ m}}$

84. Find the area of the circle that is not covered by the shaded triangles. Answer to the nearest tenth.



Area of circle:

$\pi r^2 = \pi(5)^2$

$A = 78.53981634\text{ cm}^2$ ⇒ A

$Adj. \cdot (\cos 53) = \left(\frac{adj}{5}\right)\frac{5}{1}$

$5\cos 53 = adj.$

$adj = 3.009075116\text{ cm}$ ⇒ B

$opp \cdot (\sin 53) = \left(\frac{opp}{5}\right)\frac{5}{1}$

$5\sin 53 = opp$

$opp = 3.99317755\text{ cm}$ ⇒ C

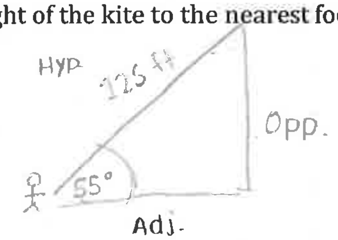
Area of triangles:

$2\left(\frac{(2\,adj.)(opp)}{2}\right)$

$A = 24.0315424\text{ cm}^2$ ⇒ D

$A_o - A_\Delta = \boxed{54.5\text{ cm}^2}$

85. Anya lets out 125 feet of kite string at Clover Point. The wind pulls the kite string tight at an angle of 55° to the ground. Approximate the height of the kite to the nearest foot.



What assumptions did you make?

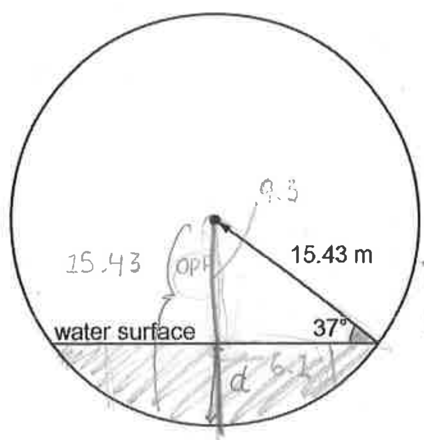
The string was completely straight/not bent.

$125(\sin 55) = \left(\frac{opp}{125}\right)\frac{125}{1}$

$125\sin 55 = opp$

$\boxed{opp = 102\text{ ft}}$

86. The radius of a circular tunnel in Shanghai is 15.43 m. During a flood, a worker in the water at the side of the tunnel measured an angle to the centre to be 37°. Find the depth of the water at its deepest point. (The water surface forms a chord across the tunnel.)



$\sin 37 = \frac{opp}{15.43}$

$opp = 9.286005807$

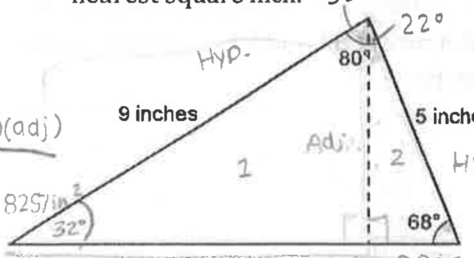
$d = 15.43 - opp$

$\boxed{d = 6.143994193\text{ m}}$

SOH CAH TOA.

Solve the following problems involving triangles.

87. Find the area of the triangle below to the nearest square inch.



Area:

$\frac{(\text{Opp}_1 + \text{Opp}_2)(\text{adj})}{2}$

$= 22.66708257 \text{ in}^2$

$\boxed{23 \text{ in}^2}$

Adj. : $9(\cos 58) = \left(\frac{adj}{9}\right)\frac{9}{1}$

adj : 4.769273378 in ⇒ C

Opp Opp

$\text{opp}_1 \; 9(\sin 58) = \left(\frac{opp}{9}\right)\frac{9}{1}$

$\text{opp}_1 = 7.63243286\text{5 in}$ ⇒ A

$\text{opp}_2 \; 5(\sin 22) = \left(\frac{opp}{5}\right)\frac{5}{1}$

opp_2 : 1.873032967 in ⇒ B

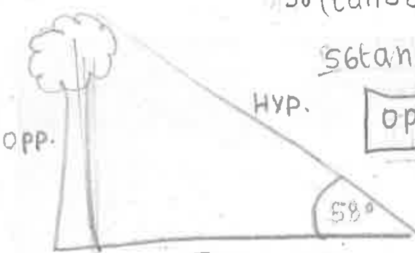
☆ 22 in² ☆

88. ~~At 11:00 in the morning,~~ the angle of elevation to the sun 58°. A tree in the school yard casts a shadow of 56 m. How tall is the tree to the nearest metre?

$56(\tan 58) = \left(\frac{opp}{56}\right)\frac{56}{1}$

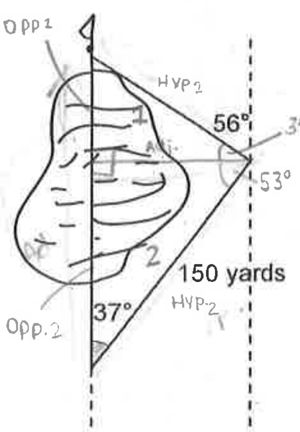
$56 \tan 58 = \text{opp}$

$\boxed{\text{opp} = 90 \text{ m}}$



89. Tucker has two choices to get his ball to the hole at the 17th at Cordova Bay, go around the lake or go over it. He decides to go around the lake as shown on the diagram. How much farther does he have to hit the ball going around the lake instead of going straight over it? Answer to the nearest yard.

☆ (Careful, not a right triangle shown.) ☆



$\text{Opp}_2 : 150(\sin 53) = \left(\frac{opp}{150}\right)\frac{150}{1}$

Opp_2 : 119.795326 5 yd ⇒ A

$\text{Adj.} \; 150(\cos 53) = \left(\frac{adj}{150}\right)\frac{150}{1}$

adj. : 90.27225347 yd ⇒ B

$\text{opp}_1 : 90.2\ldots(\tan 34) = \left(\frac{opp}{90.27225347}\right)\frac{90.2\ldots}{1}$

Opp_1 : 60.8894038 ⇒ C

$\text{Hyp}_1 : \text{hyp}(\cos 34) = \left(\frac{90.27225347}{hyp}\right)\frac{hyp}{1}$

$\frac{\text{hyp}\cos 34}{\cos 34} = \frac{90.2\ldots}{\cos 34}$

hyp : 108.8880124 ⇒ D

Straight: $\text{opp}_1 + \text{opp}_2$ = 180.6847303 yd

Around: $\text{Hyp}_1 + \text{Hyp}_2$: 258.8880124 yd

Around - Straight

$= \boxed{78 \text{ yd}}$

90. Find the area of the circle that is not covered by the shaded rectangle to the nearest square unit.

r: 7.512813949 cm ⇒ C

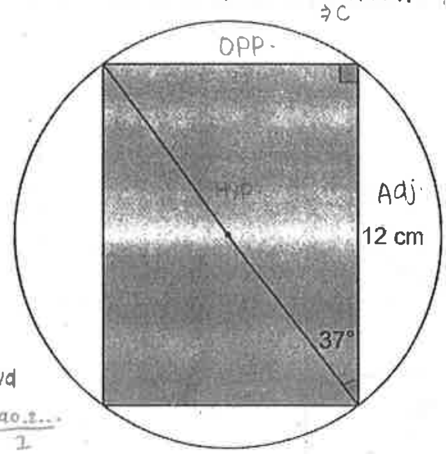
$\text{Hyp}: \text{Hyp}(\cos 37) = \left(\frac{12}{\text{Hyp}}\right)\frac{\text{Hyp}}{1}$

$\frac{\text{Hyp}\cos 37}{\cos 37} = \frac{12}{\cos 37}$

Hyp : 15.0256279 cm ⇒ A

$\text{Opp} : 12(\tan 37) = \left(\frac{opp}{12}\right)\frac{12}{1}$

Opp : 9.042648601 cm ⇒ B



Circle:

$\pi r^2 \rightarrow \pi(7.512813949)^2$

A: 177.3189457 cm ⇒ D

square: opp × adj.

= 108.5117832 cm ⇒ E

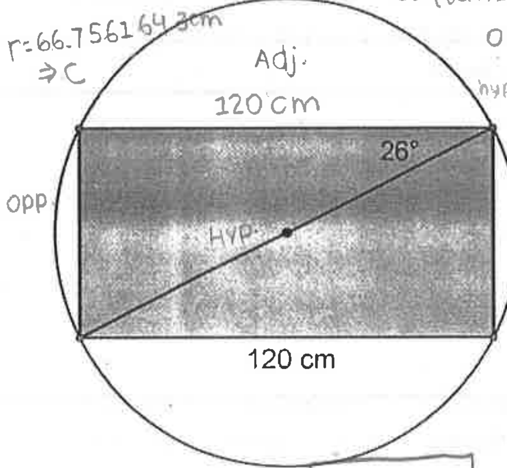
$A_{\bigcirc} - A_{\square} = \boxed{69 \text{ cm}^2}$

✓

91. Find the area of the circle not covered by the shaded **rectangle** to the nearest 100 cm.

$120(\tan 26) = \left(\frac{opp}{120}\right)\frac{120}{1}$

opp: 58.52791063 cm ⇒ A



$\cos 26 = \left(\frac{120}{hyp}\right)\frac{hyp}{1}$

$\frac{hyp\cos 26}{\cos 26} = \frac{120}{\cos 26}$

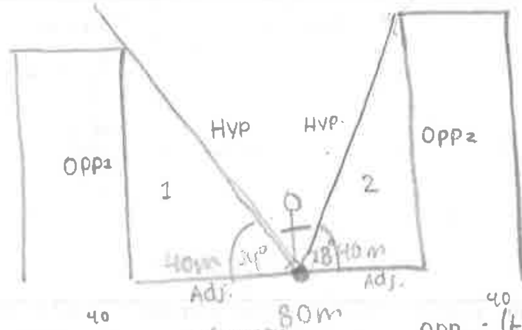
hyp: 133.5122329 cm ⇒ B

Circle:
πr^2
$\pi(66.7561643)^2$
= 14000.12778 cm

Rectangle:
Adj. × Opp.
= 7023.349275

Area: $A_o - A_{\square}$ = $\boxed{7000\ cm^2}$

92. Sandra stands at the midpoint between two buildings and measures the angles of elevation to their tops to be 14° and 18°. If the two buildings are 80 metres apart, what is the difference in their heights? Answer to the nearest metre.



$40(\tan 14) = \left(\frac{opp}{40}\right)\frac{40}{1}$

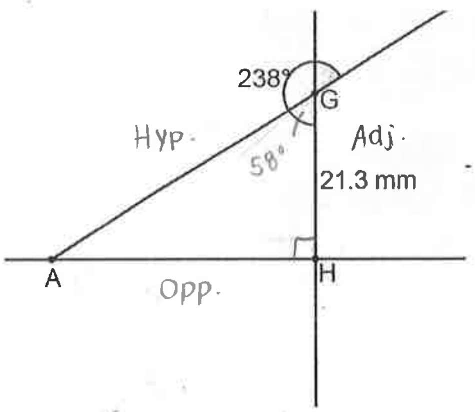
opp1: 9.973120114 m ⇒ A

$40(\tan 18) = \left(\frac{opp}{40}\right)\frac{40}{1}$

opp2: 12.99678785 m ⇒ B

opp2 - opp1 = $\boxed{3m}$

93. Find the length of AG to the nearest tenth of a millimetre.



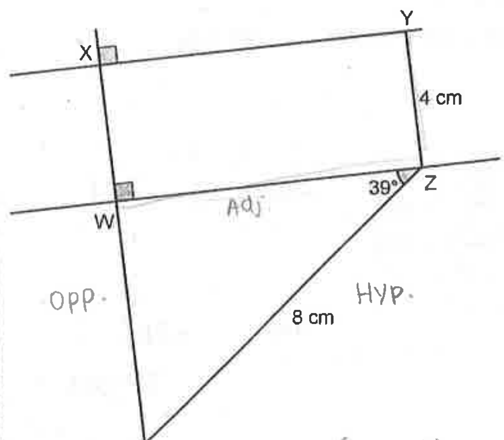
$hyp(\cos 58) = \left(\frac{21.3}{hyp}\right)\frac{hyp}{1}$

$\frac{hyp\cos 58}{\cos 58} = \frac{21.3}{\cos 58}$

hyp: 40.19480219 mm

$\boxed{AG = 40.2mm}$

94. Find the area of rectangle WXYZ to the nearest square unit.



$8(\cos 39) = \left(\frac{adj.}{8}\right)\frac{8}{1}$

adj.: 6.217167692 cm ⇒ A

adj × 4: 24.86867077

$\boxed{A_{\square} = 25 cm^2}$

SOH CAH TOA

Finding Angles Using the Three Ratios

Recall:

The three primary trig. ratios:

Tangent Ratio: $\tan\theta = \dfrac{\textit{length of side opposite } \theta}{\textit{length of side adjacent } \theta}$

Sine Ratio: $sin\theta = \dfrac{\textit{length of side opposite } \theta}{\textit{length of hypotenuse}}$

Cosine Ratio: $cos\theta = \dfrac{\textit{length of side adjacent } \theta}{\textit{length of hypotenuse}}$

Unless otherwise stated, calculate the measure of angles to the nearest tenth of a degree.

Eg. 42.8°

$\tan A = \dfrac{\text{opp length}}{\text{adj Length}}$

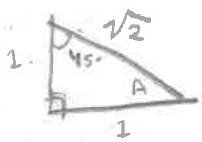
The stored values in your calculator allow you to find angles using the ratios.

The magic of sin^{-1}, cos^{-1}, and tan^{-1}.

The "inverse trigonometric functions". These functions convert the stored ratios in your calculator to the angle.

Challenge

95. Find the measure of angle A in a right triangle if $tanA = 1.000$.



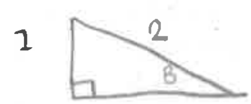
$\tan A = \frac{1}{1}$

$A = \tan^{-1}\left(\frac{1}{1}\right)$

$\boxed{A = 45°}$

Challenge

96. Find the measure angle B in a right triangle if $sinB = \frac{1}{2}$



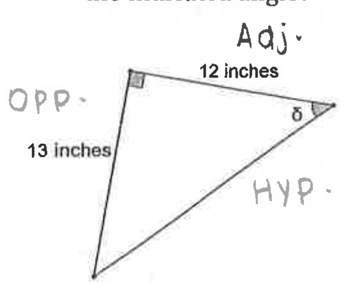
$\sin B = \frac{1}{2}$

$B = \sin^{-1}\left(\frac{1}{2}\right)$

$\boxed{B = 30°}$

Challenge

97. What ratio would you use to find the measure of the indicated angle?



$\boxed{\tan\delta = \frac{13}{12}}$

Find the measure of the indicated angle.

$\tan\delta = \frac{13}{12}$

$\delta = \tan^{-1}\left(\frac{13}{12}\right)$

$\boxed{\delta = 47.2906°}$

✓

Use the Inverse functions to find the indicated angle to the nearest tenth.

98. Find the measure angle A in a right triangle if tanA=1.000.

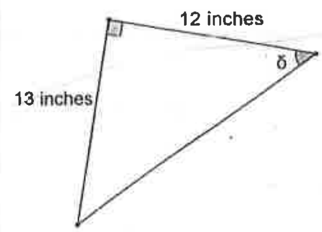
Use the $\tan^{-1}$ button.
Type:
$\tan^{-1}(1.000)=A$
$A = 45°$

99. Find the measure angle B in a right triangle if sinB=0.5000.

Use the $\sin^{-1}$ button.
Type:
$\sin^{-1}(1 \div 2)=B$
$B = 30°$

100. What ratio would you use to find the measure of the indicated angle?

Use the tangent ratio: $\tan\delta = \frac{13}{12}$



Find the measure of the indicated angle.

Type:
$\tan^{-1}(13 \div 12)= \delta$
$\delta = 47.3°$

101. Find angle A, if $\sin A = 0.2654$.

$\angle A = \sin^{-1} .2654$
$\boxed{\angle A = 15.4°}$

102. Find angle B, if $\cos B = \frac{5}{7}$.

$\angle B = \cos^{-1}(5 \div 7)$
$\boxed{\angle B = 44.4°}$

103. Find angle Q, if $\tan Q = \frac{15}{8}$.

$\angle Q = \tan^{-1}(15 \div 8)$
$\boxed{\angle Q = 61.9°}$

104. Find angle T, if $\sin T = \frac{15}{22}$.

$T = \sin^{-1}\left(\frac{15}{22}\right)$
$\boxed{T = 43.0°}$

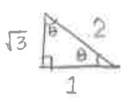
105. Find angle D, if $\cos D = \frac{11}{10}$.

$D = \cos^{-1}\left(\frac{11}{10}\right)$
$\boxed{\text{★ No solution ★}}$

106. Find angle U, if $\tan U = 2.6784$.

$U = \tan^{-1}(2.6784)$
$\boxed{U = 69.5°}$

107. In a right triangle, one acute angle has sine ratio of 0.5. Find the sine ratio of the other acute angle.

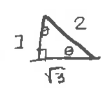


$2^2 - 1^2 = 4 - 1 = 3 \rightarrow \boxed{\sqrt{3}}$
$\boxed{\sin\theta = \frac{\sqrt{3}}{2}}$

108. In a right triangle, one acute angle has cosine ratio of $\frac{1}{\sqrt{2}}$. Find the sine ratio of the other acute angle.

$\sqrt{2}^2 - 1^2 = 2 - 1 = 1$
$\boxed{\cos\theta = \frac{1}{\sqrt{2}}}$

109. In a right triangle, one acute angle has cosine ratio of $\frac{1}{2}$. Find the tangent ratio of the other acute angle.



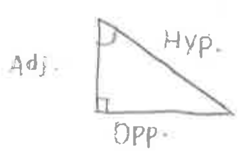
$2^2 - 1^2 = 4 - 1 = \sqrt{3}$
$\boxed{\tan\theta = \frac{1}{\sqrt{3}}}$

110. Which of the three trigonometric ratios (sine, cosine, tangent) can have a value greater than 1 ?

$\boxed{\text{tangent}}$

★ Why 10? ★

111. Draw a right triangle and use it to explain your answer to the previous question.



tangent ratio = $\frac{\text{opposite}}{\text{adjacent}}$

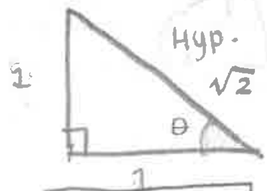
sin + cos ratios = $\frac{\text{opp}}{\text{hyp}}$ or $\frac{\text{adj}}{\text{hyp}}$

hyp. = longest side so sin + cos ratios can never be $\angle 1$.

112. Draw a right triangle with an acute angle that has an adjacent side equal in length to the opposite side. Find the cosine ratio for that angle.

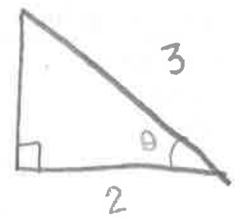
$1^2+1^2=2$

$\sqrt{2}$



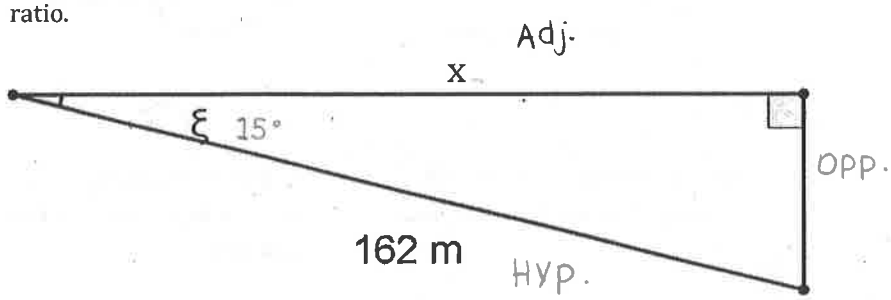
$\boxed{\cos\theta = \frac{1}{\sqrt{2}}}$

113. Draw a right triangle with an acute angle that has a hypotenuse 50% longer than the adjacent side. Find the cosine ratio for that angle.



$\boxed{\cos\theta = \frac{2}{3}}$

114. Use a protractor to measure the indicated angle. Then determine the length of side x using the cosine ratio.

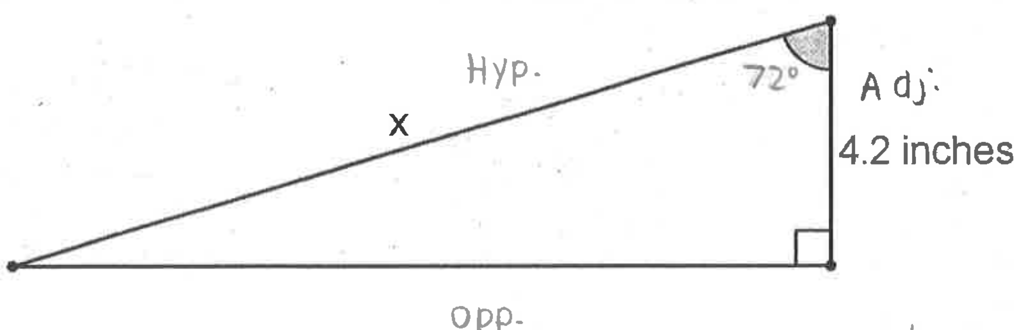


$162 (\cos 15) = \left(\frac{Adj}{\cancel{162}}\right) \frac{\cancel{162}}{1}$

Adj. $= 156.4799839$

$\boxed{15°, 156 m}$

115. Use a protractor to measure the indicated angle. Then determine the length of side x using the cosine ratio.



$hyp.(\cos 72) = \left(\frac{4.2}{\cancel{hyp}}\right) \frac{\cancel{hyp}}{1}$

$\frac{hyp \cos 72}{\cos 72} = \frac{4.2}{\cos 72}$

$hyp = 13.59148551$

$\boxed{72°, 14 in}$

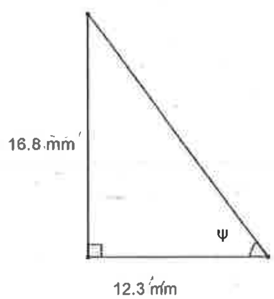
SOH CAH TOA

Working with the ratios to find angles.

Have a plan…

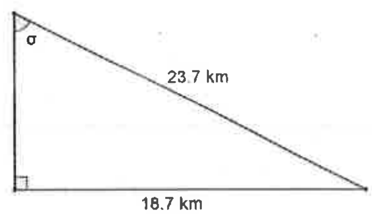
1. Choose the correct ratio {sine, cosine, or tangent}.
2. Fill in the known side lengths into your chosen ratio.
3. Use the "inverse trig. function" to convert ratio → angle.

116. What ratio do the given sides form for the indicated angle?



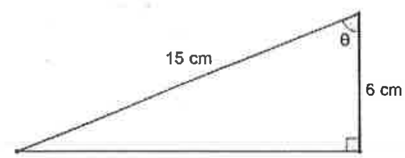
Sine Cosine Tangent

117. What ratio do the given sides form for the indicated angle?



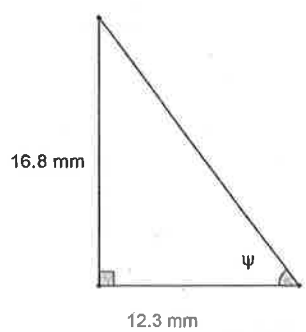
Sine Cosine Tangent

118. What ratio do the given sides form for the indicated angle?



Sine Cosine Tangent

119. Calculate the measure of angle ψ to the nearest tenth of a degree.

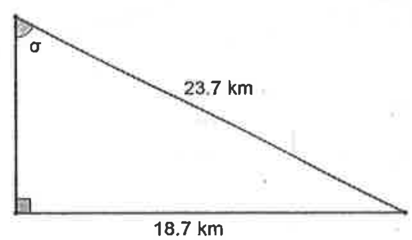


$\tan\psi = \frac{16.8}{12.3}$

$\psi = \tan^{-1}\left(\frac{16.8}{12.3}\right)$

$\boxed{\psi = 53.8^\circ}$

120. Calculate the measure of angle σ to the nearest tenth of a degree.

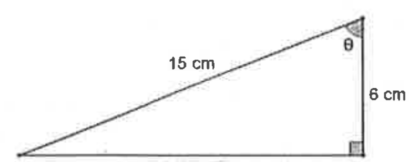


$\sin\sigma = \frac{18.7}{23.7}$

$\sigma = \sin^{-1}\left(\frac{18.7}{23.7}\right)$

$\boxed{\sigma = 52.1^\circ}$

121. Calculate the measure of angle θ to the nearest tenth of a degree.



$\cos\theta = \frac{6}{15}$

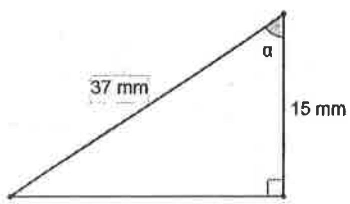
$\theta = \cos^{-1}\left(\frac{6}{15}\right)$

$\boxed{\theta = 66.4^\circ}$

SOH CAH TOA

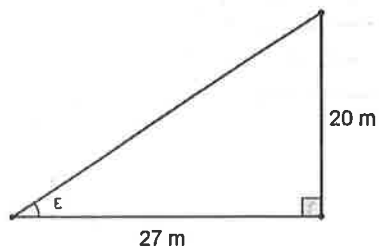
Working with the ratios to find angles.

122. What ratio do the given sides form for the indicated angle?



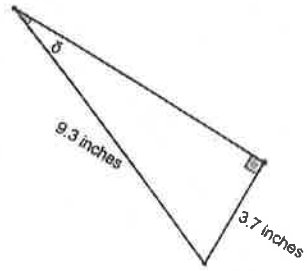
Sine Cosine Tangent

123. What ratio do the given sides form for the indicated angle?



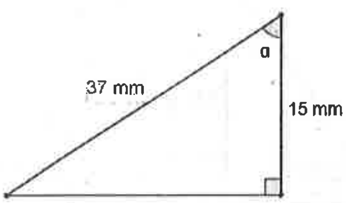
Sine Cosine Tangent

124. What ratio do the given sides form for the indicated angle?



Sine Cosine Tangent

125. Calculate the measure of angle ∝ to the nearest tenth of a degree.

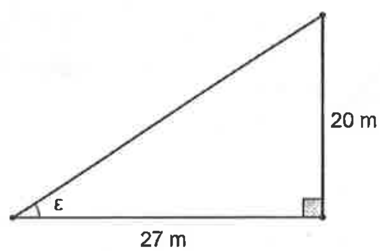


$\cos\alpha = \frac{15}{37}$

$\alpha = \cos^{-1}\left(\frac{15}{37}\right)$

$\boxed{\alpha = 66.1^\circ}$

126. Calculate the measure of angle ε to the nearest tenth of a degree.

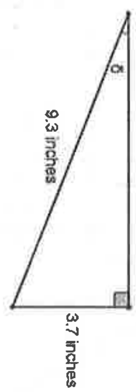


$\tan\varepsilon = \frac{20}{27}$

$\varepsilon = \tan^{-1}\left(\frac{20}{27}\right)$

$\boxed{\varepsilon = 36.5^\circ}$

127. Calculate the measure of angle θ to the nearest tenth of a degree.



$\sin\theta = \frac{3.7}{9.3}$

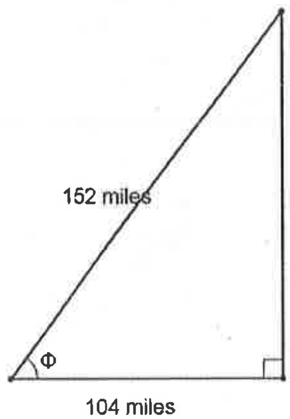
$\theta = \sin^{-1}\left(\frac{3.7}{9.3}\right)$

$\boxed{\theta = 23.4^\circ}$

✓

Find the measure of the indicated angle. Round answers to the nearest tenth of a degree.

128.

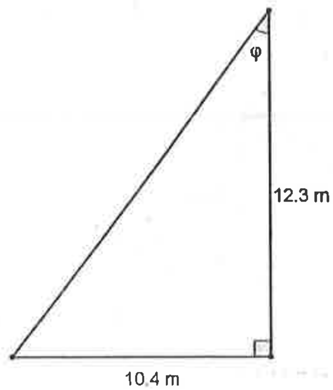


$\cos\Phi = \frac{104}{152}$

$\Phi = \cos^{-1}\left(\frac{104}{152}\right)$

$\boxed{\Phi = 46.8^\circ}$

129.

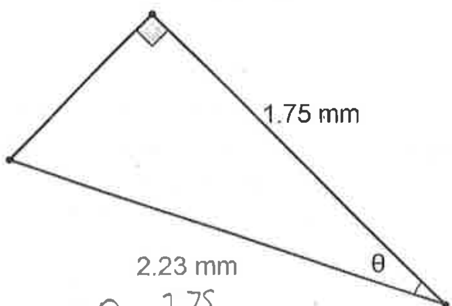


$\tan\psi = \frac{10.4}{12.3}$

$\psi = \tan^{-1}\left(\frac{10.4}{12.3}\right)$

$\boxed{\psi = 40.2^\circ}$

130.

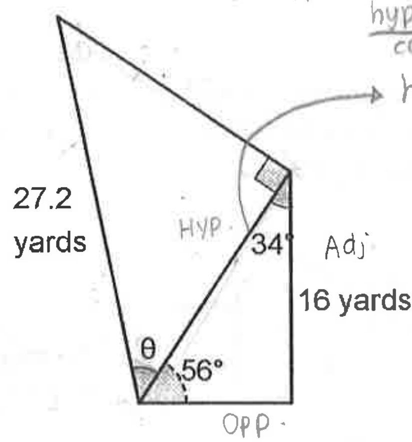


$\cos\theta = \frac{1.75}{2.23}$

$\theta = \cos^{-1}\left(\frac{1.75}{2.23}\right)$

$\boxed{\theta = 38.3^\circ}$

131.



Hyp: $hyp\left(\cos 34 = \frac{16}{hyp}\right) hyp$

$\frac{hyp\cos 34}{\cos 34} = \frac{16}{\cos 34}$

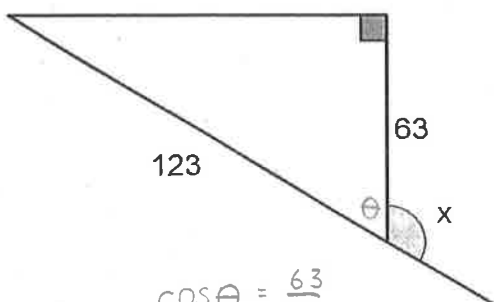
hyp $= 19.29948728$ yd $\Rightarrow A$

$\cos\theta = \frac{19.29948728}{27.2}$

$\theta = \cos^{-1}\left(\frac{19.29948728}{27.2}\right)$

$\boxed{\theta = 44.8^\circ}$

132. Find the measure of angle x to the nearest tenth of a degree..



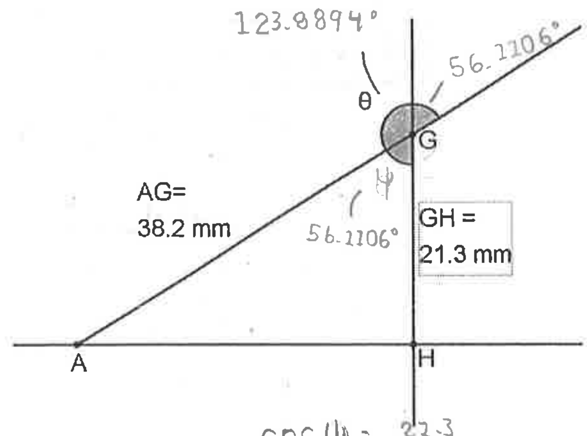
$\cos\theta = \frac{63}{123}$

$\theta = \cos^{-1}\left(\frac{63}{123}\right)$

$\theta = 59.18984336^\circ$

$\boxed{\angle x = 120.8^\circ}$

133. Find the measure of angle θ to the nearest degree.



$\cos\psi = \frac{21.3}{38.2}$

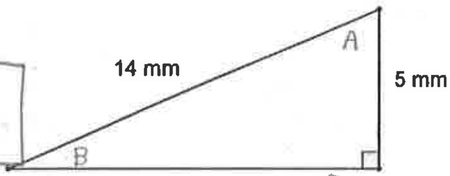
$\psi = \cos^{-1}\left(\frac{21.3}{38.2}\right)$

$\psi = 56.11059448^\circ \Rightarrow A$

$\boxed{\angle\theta = 236^\circ}$

Solve the following triangles. Calculate answers to the nearest tenth.

134.



$\boxed{69.1°, 20.9°, 13.1\text{mm}}$

$14^2 - 5^2 = \sqrt{171} \rightarrow 13.07669683\text{mm}$

$\Rightarrow A$

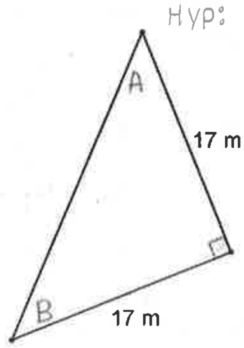
A: $\cos A = \frac{5}{14} \rightarrow A = \cos^{-1}(\frac{5}{14})$

$A = 69.07516757°$

$B = \sin B = \frac{5}{14} \rightarrow B = \sin^{-1}(\frac{5}{14})$

$B = 20.92483243°$

135.



Hyp: $17^2 + 17^2 = \sqrt{578} \rightarrow 24.04163056\text{m}$

$\tan A = \frac{17}{17} \rightarrow A = \tan^{-1}(\frac{17}{17})$

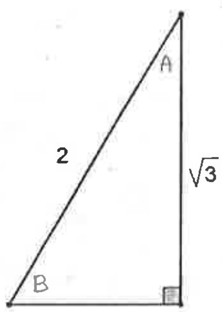
$A = 45°$

$\tan B = \frac{17}{17} \rightarrow B = \tan^{-1}(\frac{17}{17})$

$B = 45°$

$\boxed{45°, 45°, 24.0\text{m}}$

136.



$2^2 - \sqrt{3}^2 = \sqrt{1} \rightarrow 1$

$\sin A = \frac{1}{2}$

$A = \sin^{-1}(\frac{1}{2})$

$A = 30°$

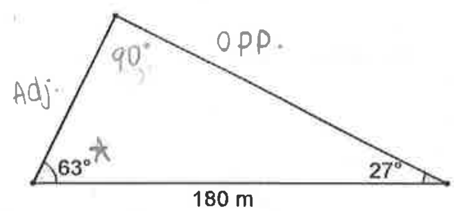
$\cos B = \frac{1}{2}$

$B = \cos^{-1}(\frac{1}{2})$

$B = 60°$

$\boxed{30°, 60°, 1}$

137.



Hyp.

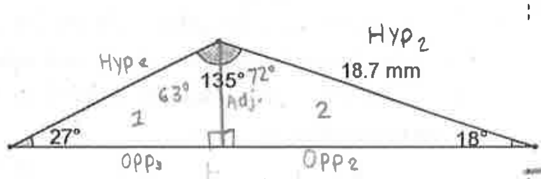
$180(\cos 63) = (\frac{adj}{180})180$

adj. $= 81.71828995\text{m}$

$180(\sin 63) = (\frac{opp}{180})180 \rightarrow$ opp. $= 160.3811744\text{ m}$

$\boxed{90°, 81.7\text{m}, 160.4\text{m}}$

138.



adj. = $18.7(\cos 72) = (\frac{adj}{18.7})18.7 \rightarrow$ adj. $= 5.778617795\text{mm}$

$\Rightarrow A$

$\text{opp}_2 = 18.7(\sin 72) = (\frac{opp}{18.7})18.7 \rightarrow$ opp: 17.78475685mm

$\Rightarrow B$

$\text{Hyp}_1 = \frac{hyp(\cos 63)}{\cos 63} = \frac{(\frac{5.778617795}{hyp})hyp}{\cos 63}$

hyp: 12.72849938 mm

$\Rightarrow C$

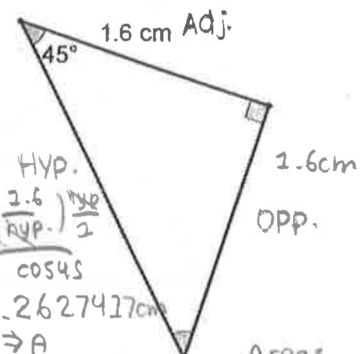
$\text{Opp}_1 = 5.7\ldots(\tan 63) = (\frac{opp}{5.778617795})5.778617795$

$\text{opp}_1 = 11.34117599\text{mm}$

$\boxed{12.7\text{mm}, 29.1\text{mm}}$

139. Challenge.

Find the Area of the following triangle to the nearest tenth of a square unit.



hyp Hyp.

$\frac{(\cos 45)}{\cos 45} = \frac{(\frac{1.6}{hyp})hyp}{\cos 45}$

hyp $= 2.2627417\text{cm}$

$\Rightarrow A$

$1.6(\tan 45) = (\frac{opp}{1.6})1.6$

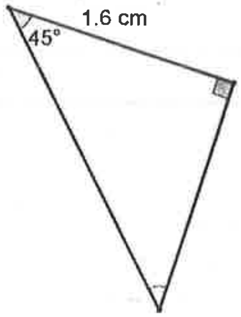
opp: 1.6 cm

Area: $\frac{(1.6)(1.6)}{2}$

$= \boxed{1.3\text{cm}^2}$

Find the area of the following triangles. Units for each question are indicated.

140. Nearest tenth.



$$\text{Area} = \frac{base \times height}{2}$$

Find base:

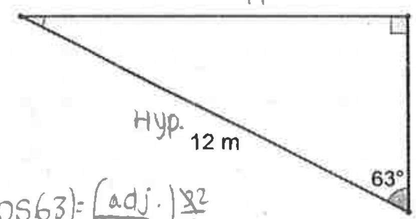
$$tan45 = \frac{opposite}{1.6}$$

$$\therefore base = 1.6\,cm$$

$$\text{Area} = \frac{1.6 \times 1.6}{2}$$

$$\text{Area} = 1.2cm^2$$

141. Nearest square metre.



$12\left(\cos 63 = \left(\frac{adj.}{12}\right)\right)\frac{12}{1}$

adj = 5.447885997m

$12\left(\sin 63 = \left(\frac{opp}{12}\right)\right)\frac{12}{1}$

opp = 10.69207829m

$A\triangle \ \frac{adj \times opp}{2} \rightarrow \boxed{29m^2}$

142. Nearest hundred square centimetres.

$100\left(\cos 45 = \left(\frac{adj}{100}\right)\right)\frac{100}{1}$

adj_1 = 70.71067812cm → A

$100\left(\sin 45 = \left(\frac{opp}{100}\right)\right)\frac{100}{1}$

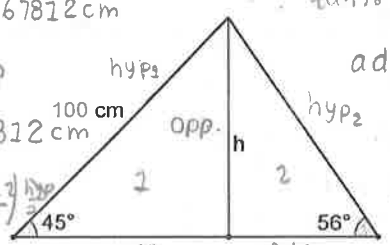
opp = 70.71067812cm → A

$\frac{hyp\left(\sin 56 = \left(\frac{70.71067812}{hyp}\right)\right)\frac{hyp}{1}}{\sin 56 \qquad \sin 56}$

hyp_2 = 85.2924891 → B

$Adj_2 \ \frac{adj\left(\tan 56 = \left(\frac{70.71067812}{adj}\right)\right)\frac{adj}{1}}{\tan 56 \qquad \tan 56}$

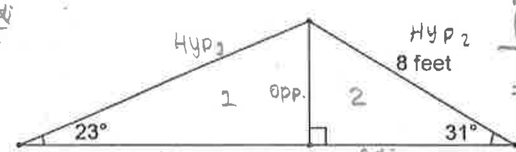
adj.2 = 47.69495462



Area: $\frac{(adj_1 + adj_2)(opp)}{2}$

$= \boxed{4200cm^2}$

143. Nearest square foot.



Area: $\frac{(Adj_1 + Adj_2)(opp)}{2}$

$= \boxed{34ft^2}$

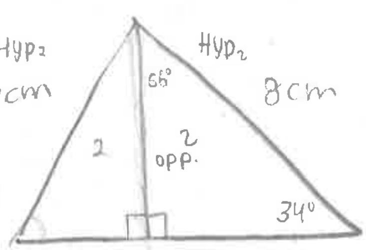
$8\left(\cos 31 = \left(\frac{adj}{8}\right)\right)\frac{8}{1}$ → adj_2: 6.857338406ft → A

$8\left(\sin 31 = \left(\frac{opp}{8}\right)\right)\frac{8}{1}$ → opp: 4.120304599ft → B

$\frac{hyp\left(\sin 23 = \left(\frac{4.12}{hyp}\right)\right)\frac{hyp}{1}}{\sin 23 \qquad \sin 23}$ → hyp: 10.54511478ft → C

$\frac{adj\left(\tan 23 = \left(\frac{4.12}{adj}\right)\right)\frac{adj}{1}}{\tan 23 \qquad \tan 23}$ → adj_1: 9.706829338ft → D

144. A triangle has side lengths of 8 cm, 7cm and 12 cm. Find the area of the triangle if the angle between the 8 cm and 12 cm side is 34°. Answer to the nearest square cm.

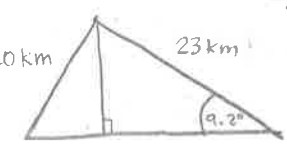


$8\left(\sin 34 = \left(\frac{opp}{8}\right)\right)\frac{8}{1}$

$\frac{opp = 4.473543228cm \times 12}{2}$

$\boxed{A = 27cm^2}$

145. A triangle has side lengths of 10 km, 23 km and 32 km. The angle opposite the 10 km side is 9.2° Find the area of the triangle. Answer to the nearest square km.



$23\left(\sin 9.2 = \left(\frac{opp}{23}\right)\right)\frac{23}{1}$

opp: 3.677267317 → A

$\frac{opp \times 32}{2}$

A = 58.83627707

$A = \boxed{59km^2}$

SOH CAH TOA

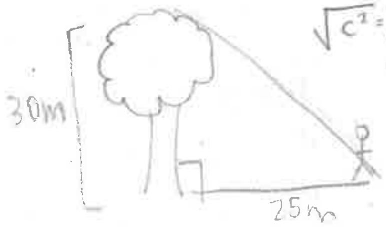
Applications of trigonometry.

146. A kite stuck in a nearby tree. A child standing 25 m from the base of a tree pulls the string tight. If the tree is 30 m tall, approximately how far is the kite from the child to the nearest metre?

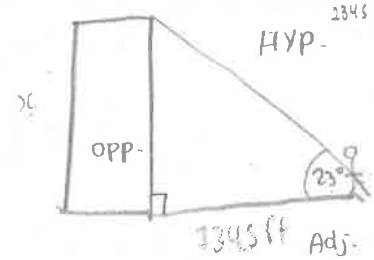
$30^2 + 25^2 = c^2$

$\sqrt{c^2} = \sqrt{1525}$

$c = \boxed{39\text{m}}$



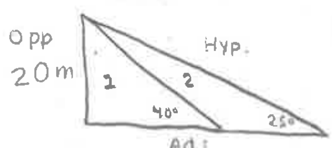
147. A surveyor measures the angle of elevation to the top of a building to be 23°. If the surveyor is 1345 feet from the base of the building, how tall is the building to the nearest foot?



$1345(\tan 23) = \left(\frac{opp}{1345}\right)\frac{1345}{1}$

opp: $\boxed{571\text{ft}}$

148. From the top of a 20 m cliff above a road, the angle of depression to two approaching cars is 25° and 40° respectively. How far apart are the cars to the nearest metre?

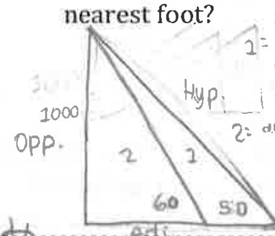


$2 - 1 = \boxed{19\text{m}}$

1: $\frac{adj \cdot (\tan 40)}{\tan 40} = \frac{\left(\frac{20}{adj}\right)\frac{adj}{1}}{\tan 40}$ → adj.: 23.83507185 → A

2: $\frac{adj \cdot (\tan 25)}{\tan 25} = \frac{\left(\frac{20}{adj}\right)\frac{adj}{1}}{\tan 25}$ → adj = 42.89023847 → B

149. Two hot air balloons float above the ocean at a height of 1000 feet. From a sailboat an observer measures the angle of elevation to one balloon is 60° and to the other balloon is 50°. [both balloons are on the same bearing from the observer] How far apart are the balloons to the nearest foot?



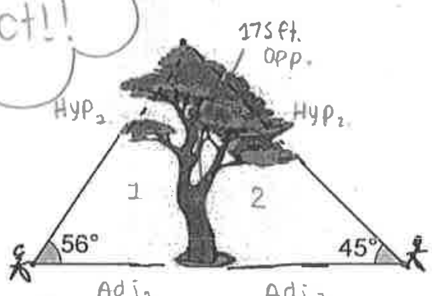
1: $\frac{adj \cdot (\tan 50)}{\tan 50} = \frac{\left(\frac{1000}{adj}\right)\frac{adj}{1}}{\tan 50}$ → adj₁: 839.0996312ft → A

2: $\frac{adj \cdot (\tan 60)}{\tan 60} = \frac{\left(\frac{1000}{adj}\right)\frac{adj}{1}}{\tan 60}$ → adj₂: 577.3502692ft

$adj_1 - adj_2 =$ $\boxed{262\text{ft}}$

150. Two boys on opposite sides of the tree below measure the angle of elevation to the top of the tree. If the tree is 175 feet tall, how many feet apart are the boys?

add, don't subtract!!

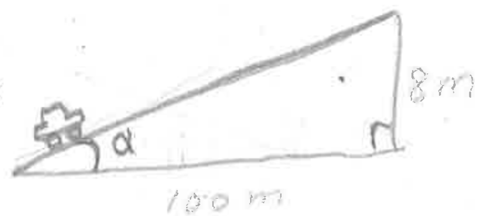


$Adj_1 = \frac{adj \cdot (\tan 56)}{\tan 56} = \frac{\left(\frac{175}{adj}\right)\frac{adj}{1}}{\tan 56}$ → $adj_1 = 118.0389904$ft

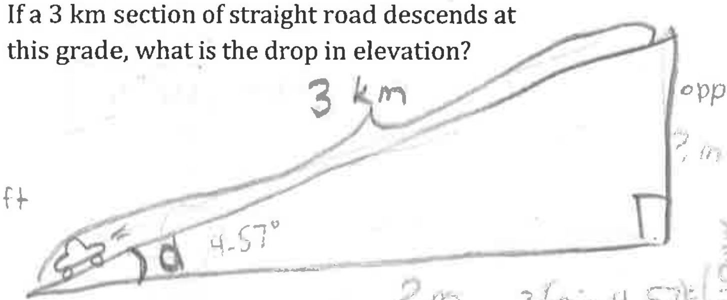
$Adj_2 = \frac{adj \cdot (\tan 45)}{\tan 45} = \frac{\left(\frac{175}{adj}\right)\frac{adj}{1}}{\tan 45}$ → $adj_2 = 175$ft

$adj_2 + adj_1 = \boxed{293\text{ft}}$

151. Highway sign shows that the road descends at a rate of 8%. Draw a diagram that shows what this means.



If a 3 km section of straight road descends at this grade, what is the drop in elevation?



$3(\sin 4.57) = \left(\frac{opp}{3}\right)\frac{3}{1}$

opp = 0.239 km

$\tan\alpha = \frac{8}{100}$

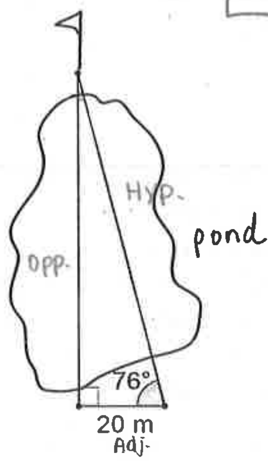
$\alpha = \tan^{-1}\left(\frac{8}{100}\right)$

$\alpha = 4.57°$ → A

152. While golfing with his father-in-law, Mr. J hits a shot short of a pond. The flag (hole) is directly across the pond from his ball. He paces 20 m to the right of his ball and measures the angle back to the hole to be 76°. How far is the ball from the hole to the nearest metre?

$20(\tan 76) = \left(\frac{opp}{\cancel{20}}\right)\frac{\cancel{20}}{1}$

$opp = \boxed{80m}$



153. A hiker leaves base camp travelling due north at 5 km/h. After two hours, she turns and travels east. Three hours later, she sprains her ankle. At what bearing would a rescue team need to travel to reach the injured hiker? How far away is she from base camp? (nearest tenth)

$10^2 + 15^2 = c^2$

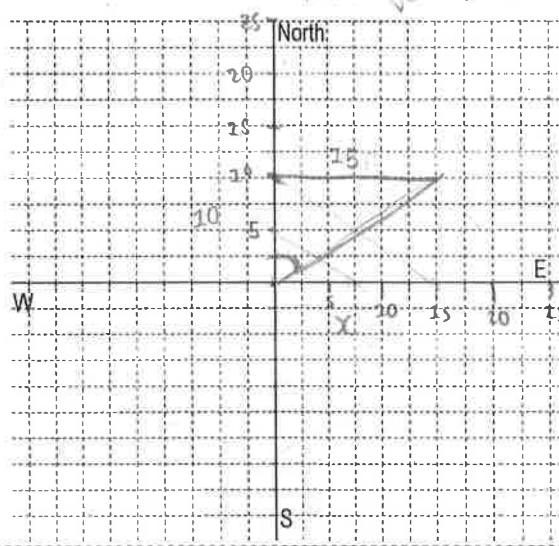
$\sqrt{325} = \sqrt{c^2} \rightarrow 18.0m$

$\tan\theta = \frac{15}{10}$

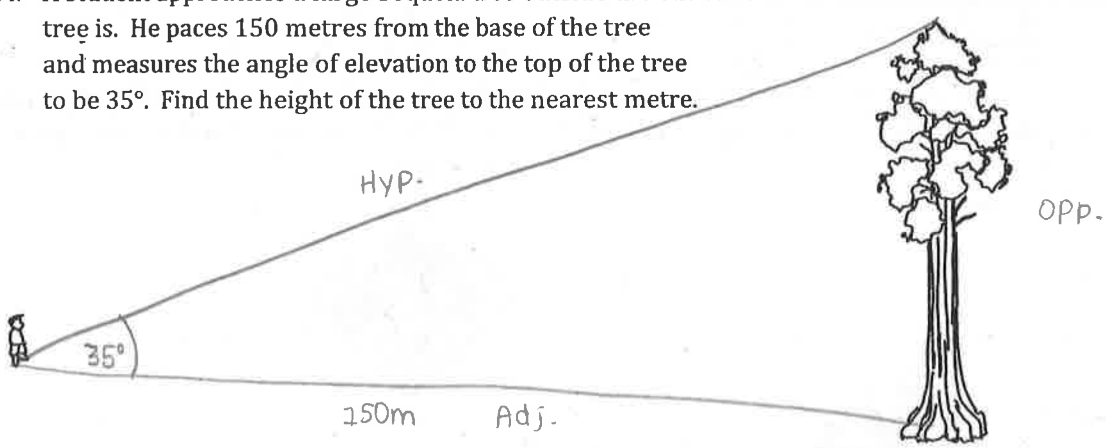
$\theta = \tan^{-1}\left(\frac{15}{10}\right)$

$\theta = 56.3°$

$\boxed{18.0m, 56.3°}$



154. A student approaches a large Sequoia tree outside the entrance to the school and wonders how tall the tree is. He paces 150 metres from the base of the tree and measures the angle of elevation to the top of the tree to be 35°. Find the height of the tree to the nearest metre.



$150(\tan 35) = \left(\frac{opp}{\cancel{150}}\right)\frac{\cancel{150}}{1}$

$opp = \boxed{105m}$

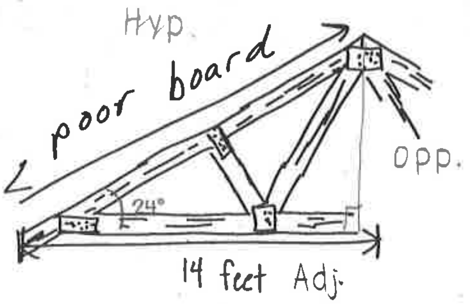
SOH CAH TOA

155. A homeowner wants to cut a new board to replace a decaying roof truss. He can measure the horizontal distance and the angle of inclination but needs to know how long to cut the board. The horizontal distance is 14 feet and the angle of inclination is 24°. Find the distance to the nearest tenth of a foot.

$hyp(\cos 24) = \left(\frac{14}{hyp}\right)\frac{hyp}{1}$

$\frac{hyp \cos 24}{\cos 24} = \frac{14}{\cos 24}$

$\boxed{hyp = 15.3 ft}$



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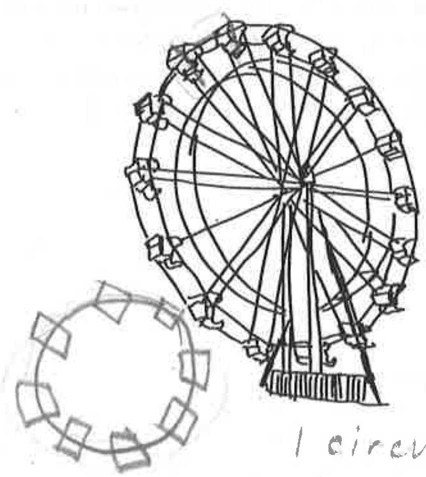
156. An engineer is constructing a Ferris wheel for a downtown park. There are 16 passenger carts and the radius of the wheel is 10 metres. How far apart are the passenger carts to the nearest hundredth of a metre?

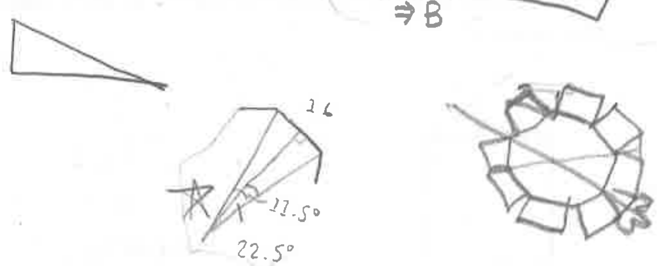
$C = 2\pi r \rightarrow C = 2\pi(10) \rightarrow C = 62.83185307 \div 16 =$

$\Rightarrow A$

$\boxed{3.926990817}$

$\Rightarrow B$





1 circumference = 62 m

each cart = $\frac{circ}{16}$

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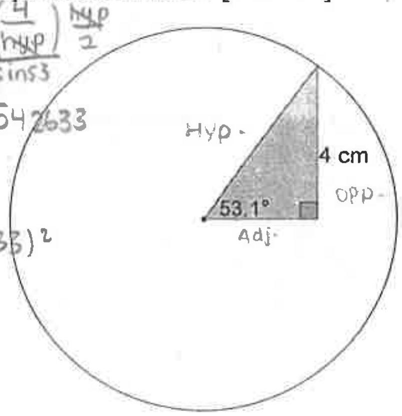
157. Find the area of the circle to the nearest square centimetre. $[A = \pi r^2]$

$\frac{hyp(\sin 53.1)}{\sin 53} \left(\frac{4}{hyp}\right)\frac{hyp}{1}$
$\sin 53$

$hyp = 5.008542633$
$\Rightarrow A$

$\pi r^2 \rightarrow$
$\pi(5.008542633)^2$

$A_o = \boxed{79 cm^2}$



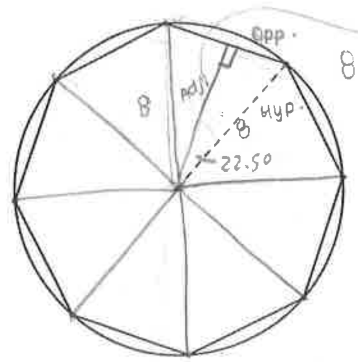
158. Find the perimeter of the octagon inscribed in a circle of radius 8 cm. (Nearest cm)

$8(\sin 22.5) = \left(\frac{opp}{8}\right)\frac{8}{1}$

$opp = 3.061467459$
$\times 2$

$= 6.122934918$
$\times 8 =$

$= 49 cm$



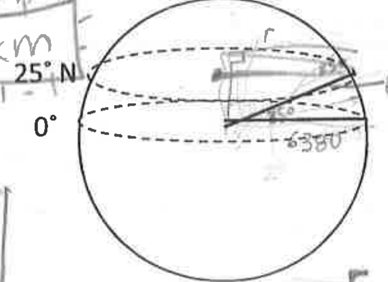
159. Find the length of the 25° line of latitude. The Earth's radius is 6380 km. Answer to the nearest km.

$2\pi r$

$2\pi(5782.24)$

$= \boxed{36331\text{ km}}$

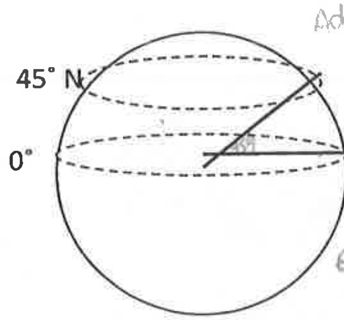
$C = 2\pi r$



$\sin 65° = \frac{r}{6380}$

$\boxed{r = 5782\text{ km}}$

160. Find the length of the 45° line of latitude. The Earth's radius is 6380 km. Answer to the nearest km.



opp. Adj. 6380 Hyp. 45° 45° 6380

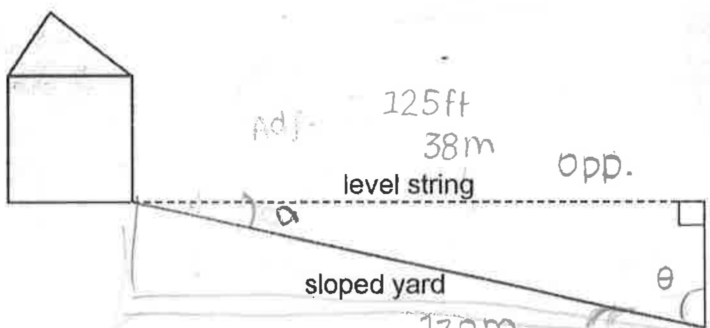
$(\sin 45)\left(\frac{opp}{6380}\right)\frac{6380}{1}$

opp = 4511.34 km

$2\pi r = 2\pi(4511.34)$

$= \boxed{28\,346\text{ km}}$

161. Mr. Teespré 's backyard slopes away from his house towards the beach. The instructions for his new lawnmower state that the mower should not be used if the slope is greater than 15°. Being a trigonometry specialist, he extends a level string 125 feet from the base of his house. From that point, he measures that the distance along the ground back to his house is 130 m. Is his yard too steep for this mower?



Adj. 125ft 38m opp. opp. Adj. 130m Hyp.

$\sin\theta = \frac{38}{130}$

$\theta = \sin^{-1}\left(\frac{38}{130}\right)$

$\theta = 16.99616482$

$\approx 17°$

$\boxed{\text{too steep}}$

38 73 130 38 m

$\cos\alpha = \frac{38}{130}$

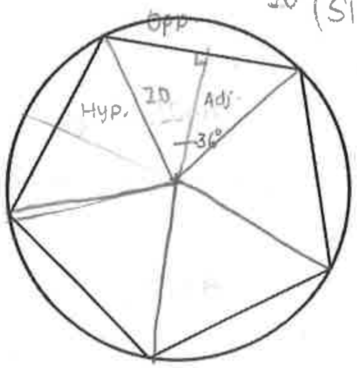
$\alpha = \cos^{-1}\left(\frac{38}{130}\right)$

$\alpha = 73°$

38 m

162. A regular pentagon is inscribed in a circle of radius 10 cm. Calculate the perimeter of the pentagon. Answer to the nearest cm.

$10(\sin 36) = \left(\frac{opp}{10}\right)\frac{10}{1}$



opp = 5.877852523

×2

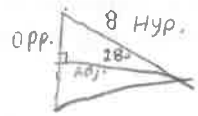
11.75570505

×5

$\boxed{59cm}$

163. A regular decagon (10 sides) is inscribed inside a circle of radius 8 cm. Find the perimeter of the decagon. Answer to the nearest cm.

$8(\sin 18) = \left(\frac{opp}{8}\right)\frac{8}{1}$



2.472135955

×2

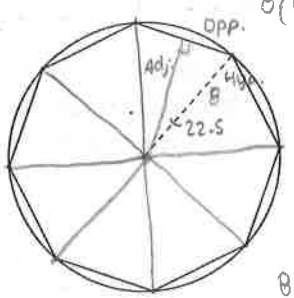
= 4.94427191

×10

= $\boxed{49 cm}$

164. Find the area of the octagon inscribed in a circle of radius 8 cm. Answer to the nearest square cm.

$8(\cos 22.5) = \left(\frac{adj}{8}\right)\frac{8}{1}$



adj. = 7.39103626 ⇒ A

$8(\sin 22.5) = \left(\frac{opp}{8}\right)\frac{8}{1}$

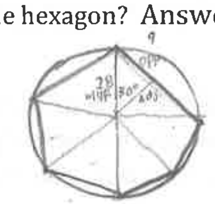
opp = 3.061467459 ⇒ B

8 (adj × opp)

= $\boxed{181cm^2}$

165. A regular hexagon is inscribed in a circle with a radius 18 cm. What would be the side length of the hexagon? Answer to the nearest cm.

$18(\cos 30) = \left(\frac{adj}{18}\right)\frac{18}{1}$



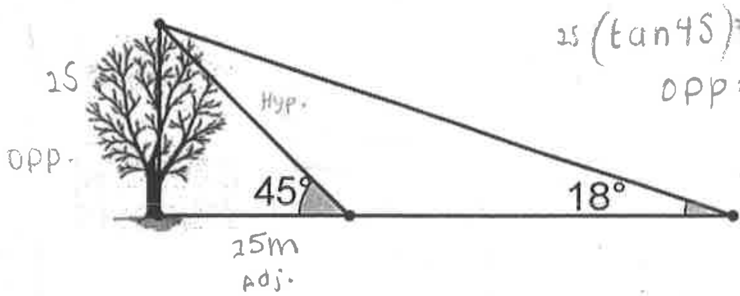
adj. = 15.58845727 ⇒ A

$18(\sin 30) = \left(\frac{opp}{18}\right)\frac{18}{1}$

opp = 9

9 × 2 = $\boxed{18cm}$

166. From a point 15 m from the base of a tree, a woman found the angle of inclination to the top of the tree to be 45°. Her sister found the angle to be 18° from a point farther away from the base of the tree. How far away are the two women away from each other? (nearest tenth of a metre)



$15(\tan 45) = \left(\frac{opp}{15}\right)\frac{15}{1}$

opp = 15

$\frac{adj(\tan 18)}{\tan 18} = \left(\frac{15}{adj}\right)\frac{adj}{1} \div \tan 18$

adj. = 46.16525306

−15

= $\boxed{31.2 m}$

✓

More word problems using right triangles:

Side x is opposite < X

- Draw a diagram.
- Fill in known values.
- Let a variable represent unknown(s)
- Choose an appropriate strategy to solve for the unknown(s).
- Interpret the problem.

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167. Solve the triangle given the following.
ΔXYZ
$x = 9\ cm$
$\angle Y = 90°$
$\angle Z = 36°$

Where's x?

X Y Z

$\boxed{\angle X = 54°}$

$9(\tan 36) = \left(\frac{opp}{9}\right)9$

opp = 6.53888

$\boxed{z = 6.5cm}$

hyp $(\cos 36) = \left(\frac{9}{hyp}\right)$ hyp

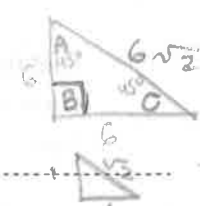
$\overline{\cos 36}$ $\overline{\cos 36}$

hyp = 11.1cm

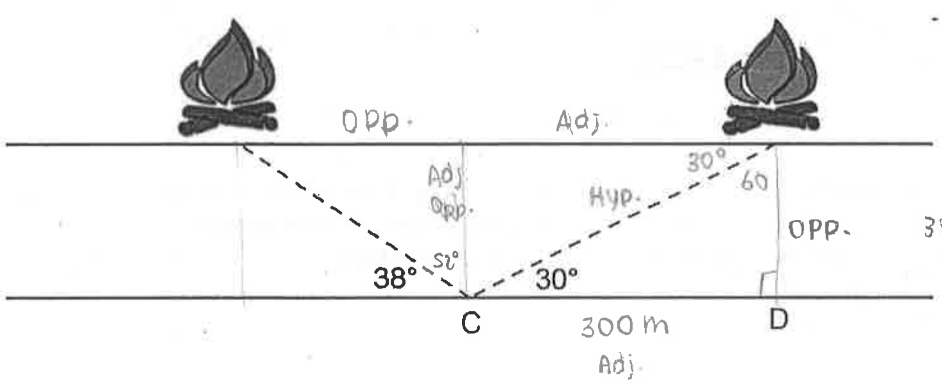
$\boxed{y = 11.1cm}$

ABC, isosceles RAT

c = 6



168. A firefighter is walking along the river at point C when she spots two fires on the opposite river bank. She measures the angles below and paces a distance of 300 m from point C to point D. Point D is directly across the river from one of the fires. How far apart are the fires to the nearest metre?



hyp $(\cos 30) = \left(\frac{300}{hyp}\right)$ hyp

$\overline{\cos 30}$ $\overline{\cos 30}$

hyp = $\frac{300}{\cos 30}$

hyp = 346.4101625

$346(\sin 30) = \left(\frac{opp}{346.4101625}\right)346$

opp = 173.2050808

$173(\tan 52) = \left(\frac{opp}{173.2050808}\right)173$

opp = 221.6923938
+300

= $\boxed{522\ m}$

