Multiplying Polynomials without TILES (also called expanding or distribution)

Multiplying Binomials: *use FOIL

Eg.1. $(x+3)(x+6) = x^2 + 6x + 3x + 18 = x^2 + 9x + 18$ ① ② combine like terms.

FOIL
Firsts - Outsides-Insides-Lasts
(x)(x) + (x)(6) + (3)(x) + (3)(6)

Eg.2. $(2x+1)(x-5) = 2x^2 - 10x + x - 5 = \mathbf{2x^2 - 9x - 5}$

Multiplying a Binomial by a Trinomial:

Eg. $(y-3)(y^2-4y+7) = y^3 - 4y^2 + 7y - 3y^2 + 12y - 21 = \mathbf{y^3 - 7y^2 + 19y - 21}$ ① ② ③ Combine Like Terms.

Multiply each term in the first polynomial by each term in the second.

Multiplying: Binomial × Binomial × Binomial

Eg.
$(x+2)(x-3)(x+4)$
$= (x^2 - 3x + 2x - 6)(x+4)$
$= (x^2 - x - 6)(x+4)$
$= x^3 + 4x^2 - x^2 - 4x - 6x - 24$
$= \mathbf{x^3 + 3x^2 - 10x - 24}$

① Multiply the first two brackets (FOIL) to make a new trinomial.

② Then multiply the new trinomial by the remaining binomial

③ Combine Like Terms.

Multiply the following as illustrated above.

128. $(x+2)(x-5)$

$x^2 - 5x + 2x - 10$

$x^2 - 3x - 10$

129. $(2x+1)(x-3)$

$2x^2 - 6x + x - 3$

$2x^2 - 5x - 3$

130. $(x-3)(x-3)$

$x^2 - 3x - 3x + 9$

$x^2 - 6x + 9$

Multiply the following.

131. $(x+2)(x+2)$

$x^2+2x+2x+4$

x^2+4x+4

132. $(2x+1)(3x-3)$

$6x^2-6x+3x-3$

$6x^2-3x-3$

133. $(2x+1)(2x-1)$

$2x^2-2x+2x-1$

$2x^2-1$

(Bi x Bi x Binomial ① multiply Binomials.)

134. $(x+2)^2$

$(x+2)(x+2)$

$x^2+2x+2x+4$

x^2+4x+4

135. $(2x+5)^2$

$(2x+5)(2x+5)$

$4x^2+10x+10x+25$

$4x^2+20x+25$

136. $(x-1)(x-1)(x+4)$

$(x^2-x-x+1)(x+4)$

$(x^2-2x+1)(x+4)$

$x^3+4x^2-2x^2-8x+x+4$

$\Rightarrow x^3+2x^2-7x+4$

Binomial x Trinomial

137. $(x-5)(x^2-5x+1)$

$x^3-5x^2+x-5x^2+25x-5$

$\Rightarrow x^3-10x^2+26x-5$

138. $(2x-3)(3x^2+2x+1)$

$6x^3+2x^2+2x-9x^2-6x-3$

$6x^3-7x^2-4x-3$

139. $(x+2)^3$

$(x+2)(x+2)(x+2)$

$(x^2+2x+2x+4)$

$(x^2+4x+4)(x+2)$

$x^3+2x^2+4x^2+8x+4x+8$

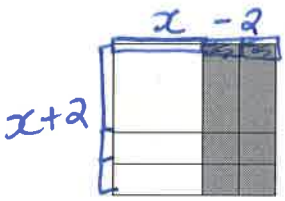
$= x^3+6x^2+12x+8$

Special Products: Follow the patterns

Conjugates: $(a+b)(a-b)$
$= a^2 + ab - ab - b^2$
$= a^2 - b^2$

"difference of squares"
↳means subtraction

140. Write an expression for the following diagram (do not simplify):



$= x^2 + 2x - 2x - 4$

What two binomials are being multiplied above?

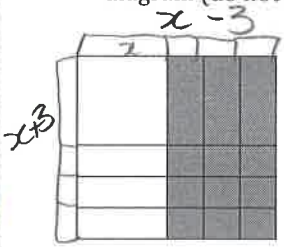
$(x+2)(x-2)$

Write an equation using the binomials above and the simplified product.

$x^2 - 2x + 2x - 4$

$\Rightarrow \boxed{x^2 - 4}$

141. Write an expression for the following diagram (do not simplify):



What two binomials are being multiplied above?

$(x+3)(x-3)$

Write an equation using the binomials above and the simplified product.

$x^2 - 3x + 3x - 9$

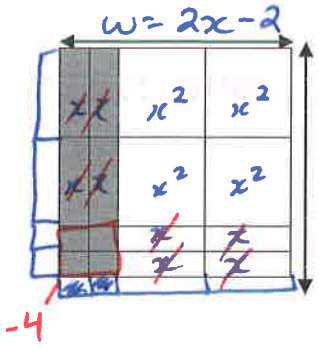
$\boxed{x^2 - 9}$

QUESTION... Describe any patterns you observe in the two questions above.

when the constants of x (variable) have opposite signs +/-, the answer is a binomial.

Remember this pattern...it will be important when we factor "A Difference of Squares" later in this booklet.

142. Write an expression (polynomial) for the following diagram (do not simplify):



What two binomials are being multiplied above?

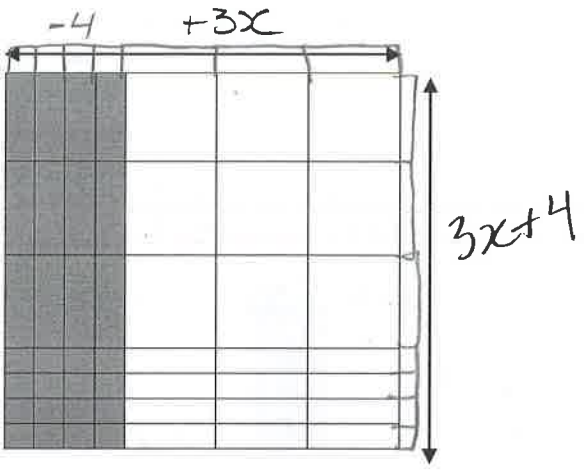
$(2x+2)(2x-2)$

Write an equation using the binomials above and the simplified product.

$4x^2 - 4x + 4x - 4$

$= 4x^2 - 4$

143. Write an expression for the following diagram (do not simplify):



What two binomials are being multiplied above?

$(3x-4)(3x+4)$

Write an equation using the binomials above and the simplified product.

$9x^2 + 12x - 12x - 16$

$= 9x^2 - 16$

Simplify the following.

144. $(x+3)(x-3)$

$x^2 - 3x + 3x - 9$

$x^2 - 9$

145. $(2x+3)(2x-3)$

$2x^2 - 6x + 6x - 9$

$2x^2 - 9$

146. $(3x-1)(3x+1)$

$9x^2 + 3x - 3x - 1$

$9x^2 - 1$

147. $(x+\sqrt{2y})(x-\sqrt{2y})$

$x^2 - x\sqrt{2y} + x\sqrt{2y} - (\sqrt{2y})^2$

$x^2 - (\sqrt{2y})^2 \leftarrow *\ \sqrt{x}^2 = x$

$\therefore x^2 - 2y$

$x(a+b)(a-b)$

$= xa^2 - xb^2$

* FOIL

Simplify the following.

148. $3(b-7)(b+7)$

$(3b-21)(b+7)$

$3b^2 + 21b - 21b - 147$

$3b^2 - 147$

149. $-2(c-5)(c+5)$

$(-2c+10)(c+5)$

$-2c^2 - 10c + 10c + 50$

$-2c + 50$

$(-2)a^2 - (-2)(5^2)$

$-2a^2 - (-2)(25)$

$-2a^2 + 50$

150. $(x+6)(x+4) + (x+2)(x+3)$

$(x^2 + 4x + 6x + 24) + (x^2 + 3x + 2x + 6)$

$(x^2 + 10x + 24) + (x^2 + 5x + 6)$

*collect like terms (+)

$2x^2 + 15x + 30$

151. $3(x-4)(x+3) - 2(4x+1)$

$(3x-12)(x+3) - (8x+2)$

$3x^2 + 9x - 12x - 36 - 8x - 2$

$3x^2 - 11x - 38$

152. $5(3t-4)(2t-1) - (6t-5)$

$(15t-20)(2t-1) - (6t-5)$

$(30t^2 - 15t - 40t + 20) - (6t-5)$

*drop brackets and collect like terms.

$30t^2 - 61t + 25$

153. $10 - 2(2y+1)(2y+1) - (2y+3)(2y+3)$

$10 - ((4y+2)(2y+1)) - (4y^2 + 6y + 6y + 9)$

$10 - (8y^2 + 4y + 4y + 2) - (4y^2 + 12y + 9)$

$10 - 8y^2 - 8y - 2 - 4y^2 - 12y - 9$

$-12y^2 - 20y - 1$

Some key points to master about the Distributive Property...

FOIL

$(a+b)(a-b)$

$a^2 - ab + ab - b^2$

$(a^2 - b^2)$

* difference of * squares

$(a+b)^2$

$(a+b)(a+b)$

$a^2 + ab + ab + b^2$

$a^2 + 2ab + b^2$

$(a+b)^3$

$(a+b)(a+b)(a+b)$

$(a^2 + 2ab + b^2)(a+b)$

$a^3 + ab + 2a^2b + 2ab^2 + ab^2 + b^3$

$a^3 + 3a^2b + 3ab^2 + b^3$

should use "·" instead of "x" } To show multiplication.

Factoring:

When a number is written as a product of two other numbers, we say it is factored.

"Factor Fully" means to write as a product of **prime factors**.

Eg.1.
Write 15 as a product of its prime factors.

$15 = 5 \times 3$

5 and 3 are the prime factors.

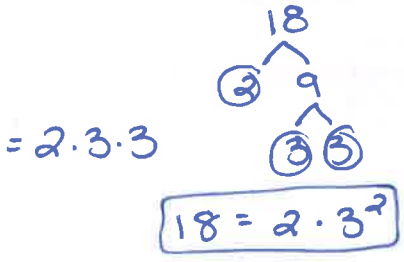
Eg.2.
Write 48 as a product of its prime factors.

$48 = 8 \times 6$
$48 = 2 \times 2 \times 2 \times 3 \times 2$
$48 = 2^4 \times 3$

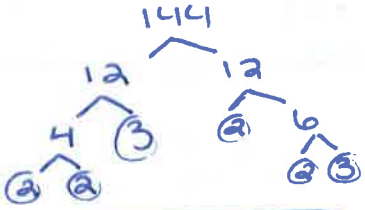
Eg.3.
Write 120 as a product of its prime factors.

$120 = 10 \times 12$
$120 = 2 \times 5 \times 2 \times 2 \times 3$
$120 = 2^3 \times 3 \times 5$

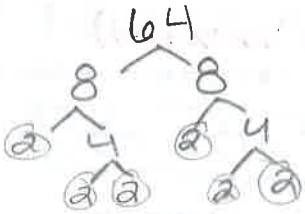
154. Write 18 as a product of its prime factors.



155. Write 144 as a product of its prime factors.



156. Write 64 as a product of its prime factors.



157. Find the greatest common factor (GCF) of 48 and 120.

Look at each factored form.

$48 = 2^4 \times 3$
$120 = 2^3 \times 3 \times 5$

Both contain $2 \times 2 \times 2 \times 3$, therefore this is the GCF,

GCF is 24.

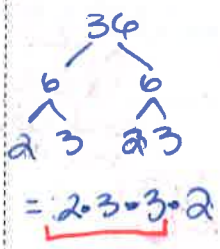
158. Find the greatest common factor (GCF) of 144 and 64.

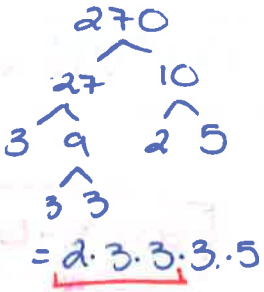
144: 2·2·2·2 3·3
64: 2·2·2·2 2·2

GCF = 2^4

GCF = 16

159. Find the greatest common factor (GCF) of 36 and 270.





common: 2·3·3
= 2·3²

GCF = 18