Factoring a Binomial Common Factor: Hint: There are brackets with identical terms.

(common factor)

The common factor **IS** the term in the brackets!

Eg.1. Factor. $4x(w+1)+5y(w+1)$

$$4x(w+1)+5y(w+1)$$
$$=(w+1)(4x)+(w+1)(5y)$$
$$=(w+1)(4x+5y)$$

$d=(w+1)$

Eg.2. Factor. $3x(a+7)-(a+7)$

$$3x(a+7)-(a+7)$$
$$=(a+7)(3x)-(a+7)(1)$$
$$=(a+7)(3x-1)$$

Sometimes it is easier to understand if we substitute a letter, such as d where the common binomial is.

Consider Eg.1.

$4x(w+1)+5y(w+1)$ — Substitute d for $(w+1)$.

$4xd+5yd$

$d(4x+5y)$ — Now replace $(w+1)$.

$=(w+1)(4x+5y)$

Factor the following, if possible.

177. $5x(a+b)+3(a+b)$ $(a+b)=$"d" $5x(d)+3(d)$ $d(5x+3)$ $\boxed{(a+b)(5x+3)}$ now we factor out "d"	178. $3m(x-1)+5(x-1)$ $3md+5d$ $d(3m+5)$ $\boxed{(x-1)(3m+5)}$	179. $3t(x-y)+(x+y)$ not equal $\therefore$ not factorable.
180. $4t(m+7)+(m+7)$ $4td+d$ $d(4t+1)$ $=\boxed{(m+7)(4t+1)}$	181. $3t(x-y)+(y-x)$ $3td+d$ $d(3t+1)$ $=\boxed{(x-y)(3t+1)}$	182. $4y(p+q)-x(p+q)$ $4yd-xd$ $d(4y-x)$ $\boxed{(p+q)(4y-x)}$

Challenge Question:

Factor the expression $ac+bd+ad+bc$.

rearrange.

$ac+ad+bd+bc$

$a(c+d)+b(d+c)$

$a(c+d)+b(c+d)$ ← factor out $(c+d)$

$=(a+b)(c+d)$

explained over the page.

Factoring: ***Factor by Grouping.*** Hint: 4 terms!

Sometimes a polynomial with 4 terms but no common factor can be arranged so that grouping the terms into two pairs allows you to factor.

You will use the concept covered above...common binomial factor.

Eg.1. Factor $ac + bd + ad + bc$

$ac + bc \quad + ad + bd$ — * Group terms that have a common factor. (rearrange expression)

$c(a + b) + d(a + b)$ — Notice the newly created binomial factor $\boxed{(a + b)}$. "pull out" (a+b)

$= (\boldsymbol{a} + \boldsymbol{b})(\boldsymbol{c} + \boldsymbol{d})$ — Factor out the binomial factor.

Eg.2. Factor $5m^2t - 10m^2 + t^2 - 2t$

$5m^2t - 10m^2 \quad - t^2 + 2t$ — Group.

$5m^2(t - 2) - t(t - 2)$ — *Notice that I factored out a $-t$ in the second group. This made the binomials into common factors, $(t - 2)$.

$= (\boldsymbol{t} - \boldsymbol{2})(\boldsymbol{5m^2} - \boldsymbol{t})$

183. $wx + wy + xz + yz$

$w(x+y) + z(x+y)$

$\boxed{= (w+z)(x+y)}$

184. $x^2 + x - xy - y$

$x(x+1) - y(x+1)$

$= \boxed{(x-y)(x+1}$

185. $xy + 12 + 4x + 3y$

$xy + 3y + 12 + 4x$

$y(x+3) + 4(3+x)$

$\boxed{(y+4)(3+x)}$

186. $2x^2 + 6y + 4x + 3xy$

$2x^2 + 4x + 6y + 3xy$

$2x(x+2) + 3y(2+x)$

$= \boxed{(2x+3y)(x+2)}$

187. $m^2 - 4n + 4m - mn$

$m^2 + 4m - 4n - mn$

$m(m+4) - n(4+m)$

$\boxed{(m-n)(m+4)}$

188. $3a^2 + 6b^2 - 9a - 2ab^2$

$3a^2 - 9a + 6b^2 - 2ab^2$

$3a(a-3) + 2b^2(3-a)$

$3a(a-3) + 2b^2(-a+3)$

$3a(a-3) - 2b^2(a-3)$

$\boxed{(3a-2b^2)(a-3)}$

• 3 terms • no common factor
• a=1

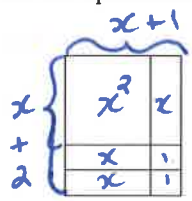
Factoring: $ax^2 + bx + c$ (where a=1) with tiles.

Hint: 3 terms, no common factor, leading coefficient is 1.

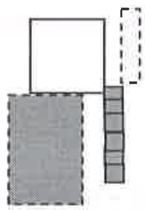
Eg.1. Consider $x^2 + 3x + 2$. The trinomial can be represented by the rectangle below.

Recall, the side lengths will give us the "factors".

$\therefore\ x^2 + 3x + 2 = (x + 1)(x + 2)$



Eg.2. Factor $x^2 - 5x - 6$



Start by placing the "x² tile" and the six "-1 tiles" at the corner. Then you can fill in the "x tiles". You'll need one x tile and six –x tiles.

$\therefore\ x^2 - 5x - 6 = (x + 1)(x - 6)$

Factor the following trinomials using algebra tiles.

189. $x^2 + 6x + 8$

(x+4)

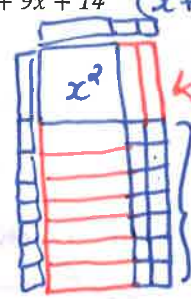
① Start by placing x², then 8 tile diagonal to it

② Fill in other lines....

③ Build L•W based on tiles

(x+2)(x+4)

190. $x^2 + 9x + 14$ (x+2)



(x+7)

① x² and 14

② Fill in rest.

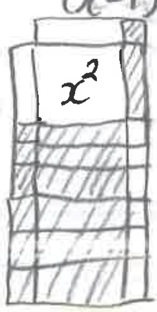
③ Determine sides

(x+7)(x+2)

191. $x^2 - 7x + 6$

(x-1)

(x-6)

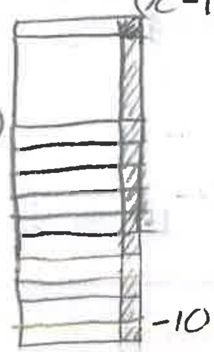


(x-1)(x-6)

when ⊖ fill all "ones" down in a straight line. (not side by side as before)

192. $x^2 + 9x - 10$ (x-1)

(x+10)



(x+10)(x-1)

Factoring: $ax^2 + bx + c$ (where a=1) without tiles.

Did you see the pattern with the tiles?

If a trinomial in the form $x^2 + bx + c$ can be factored, it will end up as $(x + \_\_)(x + \_\_)$.

The trick is to find the numbers that fill the spaces in the brackets.

The Method...
If the trinomial is in the form: $x^2 + bx + c$, look for two numbers that multiply to c, and add to b.

add to

Eg.1.
Factor. $x^2 + 6x + 8$ multiply to

$(x + \_\_)(x + \_\_)$ What two numbers multiply to +8 but add to +6? 2 and 4

$= (x + 2)(x + 4)$ The numbers 2 and 4 fill the spaces inside the brackets.

Eg.2. Factor. $x^2 - 11x + 18$

$(x + \_\_)(x + \_\_)$ What two numbers multiply to +18 but add to -11? -2 and -9

$= (x - 2)(x - 9)$ The numbers -2 and -9 fill the spaces inside the brackets.

Eg.3. Factor. $x^2 - 7xy - 60y^2$ The y's can be ignored temporarily to find the two numbers. Just write them in at the end of each bracket.

$(x + \_\_y)(x + \_\_y)$ What two numbers multiply to -60 but add to -7? -12 and +5

$= (x - 12y)(x + 5y)$ The numbers -12 and +5 fill the spaces in front of the y's.

Factor the trinomials if possible.

193. $a^2 + 6a + 5$

- multiply to 5
- add to 6

$5 \cdot 1 = 5$
$5 + 1 = 6$

$(a + \underline{1})(a + \underline{5})$

194. $n^2 + 7n + 10$

$(n+2)(n+5)$

195. $x^2 - x - 30$

- multiply to -30
- add to -1

$5 \cdot -6$
$5 + (-6)$

$(x + \underline{5})(x + \underline{-6})$

or $(x+5)(x-6)$

Factor the trinomials if possible.

196. $q^2 + 2q - 15$

$5+-3=2$ $\quad$ $5\cdot-3=-15$

$(q-3)(q+5)$

197. $k^2 + k - 56$

$(k-7)(k+8)$

198. $t^2 + 11t + 24$

$(t+3)(t+8)$

199. $y^2 - 7y - 30$

$(y-10)(y+3)$

200. $g^2 - 11g + 10$

$(g-10)(g-1)$

201. $s^2 - 2s - 80$

$(s-10)(s+8)$

202. $m^2 - 12m + 27$

$(m-9)(m-3)$

203. $x^2 - 27 - 6x$

$x^2 - 6x - 27$

• multiply to -27 $\quad$ $3\cdot-9$
• add to -6 $\quad$ $3-9$

$(x+\underline{3})(x+\underline{-9})$

204. $p^2 + 3p - 54$

$(p+6)(p+9)$

205. $2a^2 - 16a + 32$

$2(a^2 - 8a + 16)$

• multiply to 16 $\quad$ $-4\cdot-4=16$
• add to -8 $\quad$ $-4+-4=8$

$2(a-4)(a-4)$

or

$2(a-4)^2$

206. $a^2 - 14a + 45$

$(a-9)(a-5)$

207. $6x + 2x^2 - 20$

$2x^2 + 6x - 20$

$2(x^2 + 3x - 10)$

$2(x+5)(x-2)$

note $x^2 \cdot x^2 = x^4$

$w^3 \cdot w^3 = w^6$

Factor the trinomials if possible.

208. $x^4 - 3x^2 - 10$

$(x^2 + 2)(x^2 + -5)$

• add to -3
• multiply -10 } 2, -5

209. $w^6 + 7w^3 + 12$

$(w^3 + \underline{4})(w^3 + \underline{3})$

• add to 7
• multiply to 12 } 4, 3

210. $p^8 - 4p^4 - 21$

$(p^4 - 7)(p^4 + 3)$

Factor out $-x$

211. $56x + x^2 - x^3$

$-x^3 + x^2 + 56x$

$-x(x^2 - x - 56)$

• multiply to -56
• add to -1 } 7, -8

$-x(x+7)(x-8)$

212. $x^4 + 11x^2 - 80$

$(x^2 + 16)(x^2 - 5)$

213. $x^2 - 3x + 7$

• multiply to 7
• add to -3

* not factorable

214. $x^2 - 6xy + 5y^2$

$x^2 + 5y^2 - 6xy$

$(x + \underline{-5}y)(x + \underline{-1}y)$

$\boxed{(x-5y)(x-y)}$

215. $x^2 + 5xy - 36y^2$

$\boxed{(x+9y)(x-4y)}$

216. $a^2b^2 - 5ab + 6$

ab
x

• multiply to 6
• add to -5 } -2, -3

$(ab + \underline{-2})(ab + \underline{-3})$

$\boxed{(ab-2)(ab-3)}$

Challenge Question
Factor $2x^2 + 7x + 6$.

$2x^2 + 4x + 3x + 6$

$2x(x+2) + 3(x+2)$

$\boxed{(2x+3)(x+2)}$