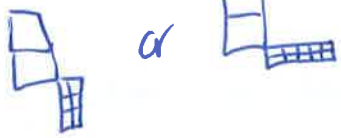
Factoring $ax^2 + bx + c$ where $a \neq 1$

When the trinomial has an x^2 term with a coefficient other than 1 on the x^2term, you cannot use the same method as you did when the coefficient is 1.

We will discuss 3 other methods:
1. Trial & Error 2. Decomposition 3. Algebra Tiles

start like:



Trial & Error:

Eg.1. Factor $2x^2 + 5x + 3$.

$2x^2 + 5x + 3 = (\quad)(\quad)$ We know the first terms in the brackets have product of $2x^2$

$2x^2 + 5x + 3 = (2x \quad)(x \quad)$ $2x$ *and* x have a product of $2x^2$, place them at front of brackets.

The product of the second terms is 3. (1, 3 or -1, -3).
These will fill in the second part of the binomials.

can no longer say then to add = middle term

multiply to 3

List the possible combinations of factors.

$(2x + 1)(x + 3)$
$(2x + 3)(x + 1)$
$(2x - 1)(x - 3)$
$(2x - 3)(x - 1)$

IF $2x^2 + 5x + 3$ is factorable, one of these must be the solution.

Expand each until you find the right one.

*guess and check

$(2x + 3)(x + 1) = 2x^2 + 2x + 3x + 3 = 2x^2 + 5x + 3.$ This is the factored form.

Decomposition:

Using this method, you will break apart the middle term in the trinomial, then factor by grouping.

To factor $ax^2 + bx + c$, look for two numbers with a product of ac and a sum of b.

Eg.1. Factor. $3x^2 - 10x + 8$ (a = 3, b = −10, c = 8)

1. We see that $ac = 3 \times 8 = 24$; *and* $b = -10$
We need two numbers with a product of 24, but add to -10...
-6 and -4.

Factor

$3x^2 - 6x - 4x + 8$
$3x(x - 2) - 4(x - 2)$

2. Break apart the middle term.
3. Factor by grouping.

$= (x - 2)(3x - 4)$

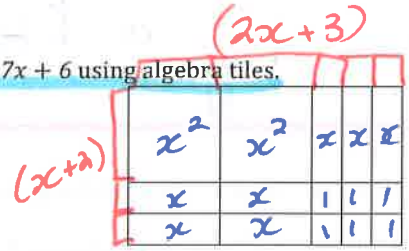
Eg.2. Factor. $3a^2 - 22a + 7$ We need numbers that multiply to 21, but add to -22...
-21 and **-1**

$3a^2 - \mathbf{21}a - \mathbf{1}a + 7$ Decompose middle term.

$3a(a-7) - 1(a-7)$ Factor by grouping.

$= (a-7)(3a-1)$

Eg.3. Factor $2x^2 + 7x + 6$ using algebra tiles.



Arrange the tiles into a rectangle (notice the "ones" are again grouped together at the corner of the x^2 tiles)

Side lengths are $(2x+3)$ and $(x+2)$ $\therefore 2x^2 + 7x + 6 = (2x+3)(x+2)$

Your notes here...

Look to see if a "quick guess" will work, if not... then I use decomposition.

Tiles work, but are time consuming.

Factor the following if possible. (a b c)

217. $2a^2 + 11a + 12$

$a \times c = 2 \times 12 = 24$
• multiply to 24
• add to 11

$2a^2+8a+3a+12$

3, 8

$2a(a+4)+3(a+4)$

$(2a+3)(a+4)$

218. $5a^2 - 7a + 2$

• multiply to 10
• add to -7

-2, -5

$5a^2-5a-2a+2$

$5a(a-1)-2(a-1)$

$(5a-2)(a-1)$

219. $3x^2 - 11x + 6$

multiply to 18
add to -11

$3x^2-9x-2x+6$

$3x(x-3)-2(x-3)$

$(3x-2)(x-3)$

Factor the following if possible.

220. $2y^2 + 9y + 9$

$(2y + 3)(y + 3)$

221. $5y^2 - 14y - 3$

$5y^2 - 15y + 1y - 3$

$5y(y-3) + 1(y-3)$

$(5y+1)(y-3)$

222. $10x^2 - 17x + 3$

think ahead + -15 (multiply to 30), -2 (add to -17)

group #'s so you can factor

$10x^2 - 2x - 15x + 3$

$2x(5x-1) - 3(5x-1)$

$\therefore (5x-1)(2x-3)$

223. $2x^2 + 3x + 1$

$2x^2 + 2x + 1x + 1$

$2x(x+1) + 1(x+1)$

$(2x+1)(x+1)$

224. $6k^2 - 5k - 4$

$6k^2 + 3k - 8k - 4$

$3k(2k+1) - 4(2k+1)$

$(3k-4)(2k+1)$

225. $6y^2 + 11y + 3$ → 6 × 3

$y^2 + 11y + 18$

$(y+9)(y+2)$ (÷3, ÷2)

$3 \div 6_2$; $6_3 \div 2$

$(2y+3)(3y+1)$

diff method

226. $\frac{3x^2 - 16x - 12}{3}$

diff method

$x^2 - 16x - 36$

$(x+2)(x-18)^6$

3 ; 3,1

$(3x+2)(x-6)$

227. $3x^3 - 5x^2 - 2x$

Factor out x

$x^3 - 5x^2 - 6x$

$x(x^2 - 5x - 6)$

$x(x-6)(x+1)$ (÷3, ÷3)

$x(x-2)(3x+1)$

228. $9x^2 + 15x + 4$

$x^2 + 15x + 36$

$(x+12)(x+3)$ (÷3, ÷3)

$(3x+4)(3x+1)$

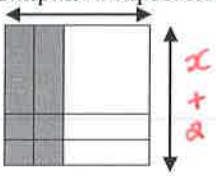
Factor the following if possible.

229. $21x^2 + 37x + 12$	230. $6x^3 - 15x - x^2$	231. $4t + 10t^2 - 6$
232. $3x^2 - 22xy + 7y^2$	233. $4c^2 - 4cd + d^2$	234. $2x^4 + 7x^2 + 6$

Challenge Question

Write a simplified expression for the following diagram of algebra tiles.

$-2 + x$



$(x-2)(x+2)$

$\therefore (x^2-4)$

Recall:
$a^2 - b^2$
$(a-b)(a+b)$

What two binomials are being multiplied in the diagram above?

$(x-2)(x+2)$

Write an equation using the binomials above and the simplified product.

$(x^2 - 4)$