Factoring $ax^2 + bx + c$ where $a \neq 1$

When the trinomial has an $x^2$ term with a coefficient other than 1 on the $x^2$ term, you cannot use the same method as you did when the coefficient is 1.

We will discuss 3 other methods:
1. Trial & Error  
2. Decomposition  
3. Algebra Tiles

**Trial & Error:**

Eg. 1. Factor $2x^2 + 5x + 3$.

$2x^2 + 5x + 3 = (\underline{\hspace{2cm}})(\underline{\hspace{2cm}})$  

$2x^2 + 5x + 3 = (2x)(x)$  

*We know the first terms in the brackets have product of $2x^2$.*

We need to find two numbers whose product is $2 \cdot 3 = 6$.  

*The product of the second terms is 3.* (1, 3 or -1, -3).

These will fill in the second part of the binomials.

List the possible combinations of factors.

- $(2x + 1)(x + 3)$
- $(2x + 3)(x + 1)$
- $(2x - 1)(x - 3)$
- $(2x - 3)(x - 1)$

*Guess and check*

IF $2x^2 + 5x + 3$ is factorable, one of these must be the solution.

Expand each until you find the right one.

$(2x + 3)(x + 1) = 2x^2 + 2x + 3x + 3 = 2x^2 + 5x + 3$.

This is the factorized form.

*Decomposition:*

Using this method, you will break apart the middle term in the trinomial, then factor by grouping.

To factor $ax^2 + bx + c$, look for two numbers with a product of $ac$ and a sum of $b$.

Eg. 1. Factor $3x^2 - 10x + 8$.

1. We see that $ac = 3 \cdot 8 = 24$; and $b = -10$  
   We need two numbers with a product of 24, but add to -10...
   -6 and -4.

2. Break apart the middle term.

3. Factor by grouping.

$3x^2 - 6x - 4x + 8$  

$3x(x - 2) - 4(x - 2)$

$= (x - 2)(3x - 4)$
Eg. 2. Factor. \(3a^2 - 22a + 7\)
- We need numbers that multiply to 21, but add to -22...
- -21 and -1
- Decompose middle term.
- Factor by grouping.

\[3a^2 - 21a - 1a + 7\]
\[3a(a - 7) - 1(a - 7)\]
\[(a - 7)(3a - 1)\]

Eg. 3. Factor \(2x^2 + 7x + 6\) using algebra tiles.

Arrange the tiles into a rectangle (notice the "ones" are again grouped together at the corner of the \(x^2\) tiles)

Side lengths are \((2x + 3)\) and \((x + 2)\)

\[\therefore 2x^2 + 7x + 6 = (2x + 3)(x + 2)\]

Your notes here...

Look to see if a "quick guess" will work, if not... then I use decomposition.
Tiles work, but are time consuming.

Factor the following if possible.

217. \(2a^2 + 11a + 12\)
- \(2a^2 + 8a + 3a + 12\)
- \(2a(a + 4) + 3(a + 4)\)
- \((2a + 3)(a + 4)\)

218. \(5a^2 - 7a + 2\)
- \(5a^2 - 5a - 2a + 2\)
- \(5a(a - 1) - 2(a - 1)\)
- \((5a - 2)(a - 1)\)

219. \(3x^2 - 11x + 6\)
- Multiply to 18
- Add to -11
- \(3x^2 - 9x - 2x + 6\)
- \(3x(x - 3) - 2(x - 3)\)
- \((3x - 2)(x - 3)\)
Factor the following if possible.

220. \(2y^2 + 9y + 9\)

\[
(2y + 3)(y + 3)
\]

221. \(5y^2 - 14y - 3\)

\[
5y^2 - 15y + 1y - 3 \quad \rightarrow \quad 5y(y - 3) + 1(y - 3) \quad \rightarrow \quad (5y + 1)(y - 3)
\]

222. \(10x^2 - 17x + 3\)

\[
\text{Multiply to 30} \quad \rightarrow \quad \text{Group} \quad \rightarrow \quad \text{Factor} \quad \rightarrow \quad 10x^2 - 2x - 15x + 3 \quad \rightarrow \quad 2x(5x - 1) - 3(5x - 1) \quad \rightarrow \quad (5x - 1)(2x - 3)
\]

223. \(2x^3 + 3x + 1\)

\[
2x^2 + 2x + x + 1 \quad \rightarrow \quad 2x(x + 1) + 1(x + 1) \quad \rightarrow \quad (2x + 1)(x + 1)
\]

224. \(6k^3 - 5k - 4\)

\[
6k^3 + 3k - 8k - 4 \quad \rightarrow \quad 3k(2k + 1) - 4(2k + 1) \quad \rightarrow \quad (3k - 4)(2k + 1)
\]

225. \(4x^3 + 11y + 3\)

\[
\text{Multiply to 60} \quad \rightarrow \quad \text{Group} \quad \rightarrow \quad \text{Factor} \quad \rightarrow \quad y^3 + 11y + 18 \quad \rightarrow \quad (y + 3)(y + 2)^2 \quad \rightarrow \quad (2y + 3)(3y + 1)
\]

226. \(3x^2 - 16x - 12\)

\[
3 \cdot (x^2 - 16x - 36) \quad \rightarrow \quad 3 (x + 3)(x - 12) \quad \rightarrow \quad (3x + 3)(x - 12)
\]

227. \(3x^3 - 5x^2 - 6x\)

\[
\text{Factor} \quad \rightarrow \quad x^3 - 5x^2 - 6x \quad \rightarrow \quad x(x^2 - 5x - 6) \quad \rightarrow \quad x(x - 3)(x + 2) \quad \rightarrow \quad x(x - 3)(x + 1)
\]

228. \(9x^2 - 15x + 4\)

\[
\frac{x^3 - 5x^2 - 6x}{9} \quad \rightarrow \quad \frac{(x + 2)(x^2 - 3x - 2)}{9} \quad \rightarrow \quad \frac{(x + 2)(x - 3)(x + 1)}{9}
\]

\[
\frac{(3x + 1)}{3} \quad \rightarrow \quad \frac{(3x + 4)(3x + 1)}{3}
\]

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Factor the following if possible.

<table>
<thead>
<tr>
<th>Expression</th>
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</thead>
<tbody>
<tr>
<td>$229.21x^2 + 37x + 12$</td>
<td>$230.6x^3 - 15x - x^2$</td>
<td>$231.4t + 10t^2 - 6$</td>
</tr>
<tr>
<td>$232.3x^2 - 22xy + 7y^2$</td>
<td>$233.4c^2 - 4cd + d^2$</td>
<td>$234.2x^4 + 7x^3 + 6$</td>
</tr>
</tbody>
</table>

Challenge Question

Write a simplified expression for the following diagram of algebra tiles.

![Diagram of algebra tiles]

Recall:

- $(a^2 - b^2)$
- $(a-b)(a+b)$
- $(x^2 - 4)$

What two binomials are being multiplied in the diagram above?

$(x-a)(x+a)$

Write an equation using the binomials above and the simplified product.

$(x^2 - 4)$