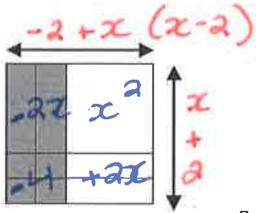
A Difference of Squares

235. Write a simplified expression for the following diagram.



Solution: $x^2 - 2x + 2x - 4$

What two binomials are being multiplied in the diagram above?

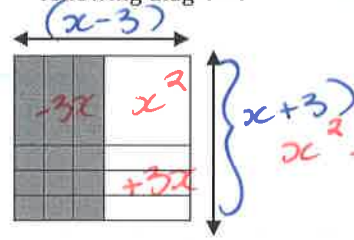
$(x-2)(x+2)$

Write an equation using the binomials above and the simplified product.

$x^2 - 4 = (x-2)(x+2)$

Factored Form

236. Write a simplified expression for the following diagram.



$x^2 - 3x + 3x - 9$

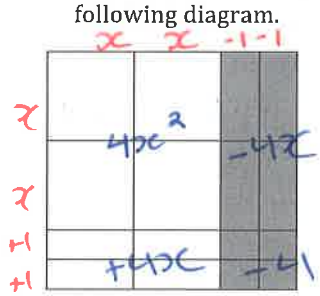
What two binomials are being multiplied above?

$(x-3)(x+3)$

Write an equation using the binomials above and the simplified product.

$x^2 - 9 = \underbrace{(x-3)(x+3)}_{\text{factored.}}$

237. Write a simplified expression for the following diagram.



$4x^2 - 4x + 4x - 4$

What two binomials are being multiplied above?

$(2x+1)(2x-1)$

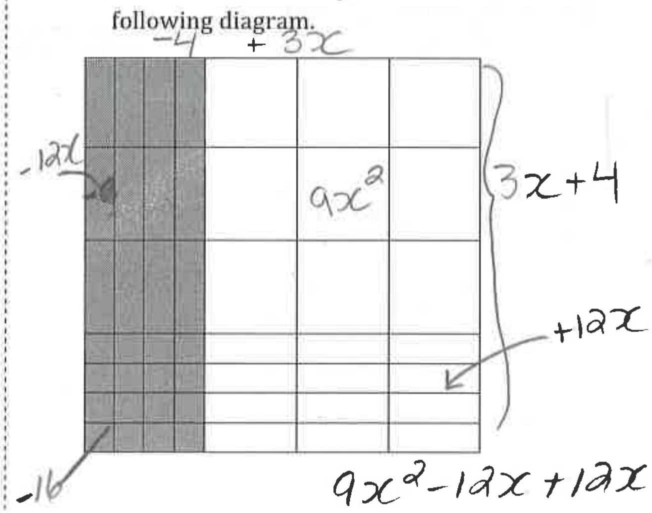
Factor the polynomial represented above by writing the binomials as a product (multiplication).

$4x^2 - 4 = (2x+1)(2x-1)$

$\to 4(x^2-1)$

$4(x-1)(x+1) \Rightarrow$ Factored Fully

238. Write a simplified expression for the following diagram.



$9x^2 - 12x + 12x - 16$

What two binomials are being multiplied above?

$(3x+4)(3x-4)$

Factor the polynomial represented above by writing the binomials as a product (multiplication).

$9x^2 - 16 = (3x+4)(3x-4)$

Factoring a Difference of Squares: $a^2 - b^2$

Conjugates: Sum of two terms and a difference of two terms.

Learn the pattern that exists for multiplying conjugates.

$\boxed{(x+2)(x-2)} = x^2 - 2x + 2x - 4 = \boxed{x^2 - 4}$ The two middle terms cancel each other out.

difference of squares (learnt previously)

We can use this knowledge to quickly factor polynomials that look like $x^2 - 4$.

Eg.1. Factor $x^2 - 9$.

$= (x+3)(x-3)$ 1 Square root each term, place them in 2 2 brackets with 3 opposite signs (+ and -). } 3 requirements

Eg.2. Factor $100a^2 - 81b^2$

$= (10a + 9b)(10a - 9b)$ 1 Square root each term, place them in 2 2 brackets with 3 opposite signs (+ and -).

Factor the following completely.

239. $a^2 - 25$	240. $\sqrt{x^2} - \sqrt{144}$	241. $1 - c^2$
$(a-5)(a+5)$	$= x$ $= 12$ Take square root. $\therefore (x-12)(x+12)$	$(1+c)(1-c)$

I recognize a polynomial is a difference of squares because $a^2 - b^2$; where 'a' and 'b' are both perfect squares. *always a subtraction (−) sign.

The leading coefficient $\neq 1$? That's ok!

Factor the following completely.

242. $4x^2 - 36$

• both are perfect squares.

$\sqrt{4x^2} = 2x$

$\sqrt{36} = 6$

$(2x+6)(2x-6)$

$2(x+3)2(x-3)$

Factored FULLY $\Rightarrow 4(x+3)(x-3)$

243. $9x^2 - y^2$

$\sqrt{9x^2} = 3x$

$\sqrt{y^2} = y$

$(3x+y)(3x-y)$

244. $25a^4 - 36$

$\sqrt{25a^4} = 5a^2 \Rightarrow (5a^2 \cdot 5a^2) = 25a^4$

$\sqrt{36} = 6$

$(5a^2-6)(5a^2+6)$

245. $49t^2 - 36u^2$

$(7t-6u)(7t+6u)$

246. $7x^2 - 28y^2$

factor out 7

$7(x^2-4y^2)$

$7(x-2y)(x+2y)$

247. $-18a^2 + 2b^2$

$2b^2 - 18a^2$

$2(b^2-9a^2)$

$\boxed{2(b-3a)(b+3a)}$

248. $-9 + d^4$

$d^4 - 9$

$(d^2-3)(d^2+3)$

249. $\frac{a^2}{9} - \frac{b^2}{16}$

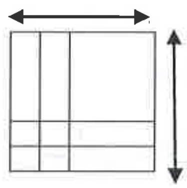
$\left(\frac{a}{3}-\frac{b}{4}\right)\left(\frac{a}{3}+\frac{b}{4}\right)$

250. $\frac{x^2y^2}{49} - 1$

$\left(\frac{xy}{7}-1\right)\left(\frac{xy}{7}+1\right)$

Factoring a Perfect Square Trinomial

251. Write an expression for the following diagram (do not simplify):



$x^2 + 2x + 2x + 4$

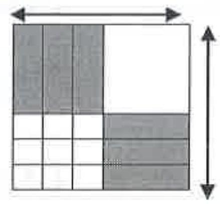
What two binomials are being multiplied above?

$(x+2)(x+2)$

Write an equation using the binomials above and the simplified product.

$x^2 + 4x + 4 = (x+2)(x+2)$
$= (x+2)^2$

252. Write an expression for the following diagram (do not simplify):



$x^2 - 3x - 3x + 9$

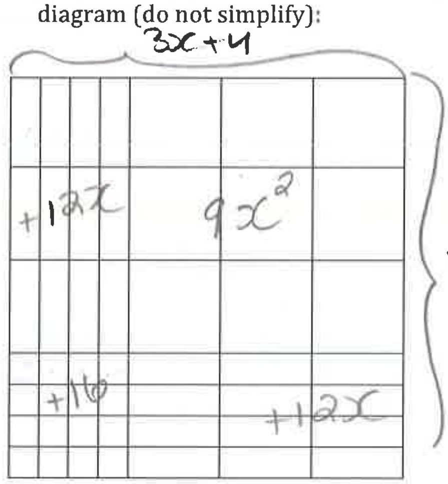
What two binomials are being multiplied above?

$(x-3)(x-3)$

Write an equation using the binomials above and the simplified product.

$x^2 - 6x + 9 = (x-3)(x-3)$
$(x-3)^2$

253. Write an expression for the following diagram (do not simplify):



$9x^2 + 24x + 16$

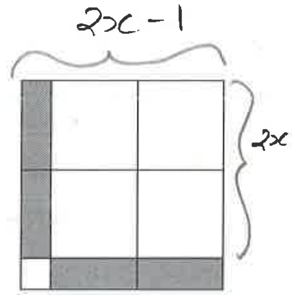
What two binomials are being multiplied above?

$(3x+4)(3x+4)$

Write an equation using the binomials above and the simplified product.

$9x + 24x + 16 = (3x+4)^2$

254. Write an expression for the following diagram (do not simplify):



$x^2 - 2x - 2x + 1$

What two binomials are being multiplied above?

$(2x-1)(2x-1)$

Write an equation using the binomials above and the simplified product.

$x^2 - 4x + 1 = (2x-1)^2$

PERFECT SQUARE TRINOMIALS

You may use the methods for factoring trinomials to factor ***trinomial squares*** but recognizing them could make factoring them quicker and easier.

Eg.1. Factor.
$x^2 + 6x + 9$ — Recognize that the first and last terms are both perfect squares.

$(x + 3)^2$ — Guess by taking the square root of the first and last terms and put them in two sets of brackets.

Check: Does $2(x)(3) = 6x$
Yes! Trinomial Square!

In a trinomial square, the middle term will be double the product of the square root of first and last terms. Wow, that's a mouthful!

$(x + 3)^2$ — Answer in simplest form.

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$

Remember this pattern

Eg.2. Factor.
$121m^2 - 22m + 1$

$(11m - 1)^2$ — Guess & Check. $2(11m\times -1) = -22m$.
Since the middle term is negative, binomial answer will be a subtraction.

Recognizing the pattern WILL save time + effort!!

Factor the following.

255. $x^2 + 14x + 49$ $\sqrt{x^2} = x$ $\quad \sqrt{49} = 7$ $+$ $\therefore (x+7)^2$	256. $4x^2 - 4x + 1$ $\sqrt{4x^2} = 2x$ $\quad \sqrt{1} = 1$ $-$ $\therefore (2x-1)^2$	257. $9b^2 - 24b + 16$ $(3b-4)^2$
258. $64m^2 - 32m + 4$ Factored $(8m-2)^2$ $(8m-2)(8m-2)$ $2(4m-1)2(4m-1)$ $\boxed{4(4m-1)^2}$ Factored Fully!	259. $81n^2 + 90n + 25$ $(9n+5)^2$	260. $81x^2 - 144xy + 64y^2$ $(9x-8y)^2$

Create a Factoring Flowchart.

Start with the first thing you should do....collect like terms.

Start

Is there a GCF?

No → How many Terms?

Yes → Factor out the GCF → How many Terms?

2 Terms

Is there a difference of squares?

$(a^2-b^2)=(a-b)(a+b)$

ex. $x^2-25=(x-5)(x+5)$

$9x^2-64=(3x-8)(3x+8)$

3 Terms

Factor into Two Binomials

ax^2+bx+c

$(x\quad)(x\quad)$

ex. $x^2-4x-21$

$=(x-7)(x+3)$

$10x^2+11x+3$

$=(2x+1)(5x+3)$

use:
- box method
- decomposition
- guess + check
- algebra tiles.

remember: *perfect square?

$a=1$: $\otimes=c$, $\oplus=b$

$a\neq1$: $\otimes=a\cdot c$, $\oplus=b$

4 Terms

Factor by Grouping

$ac+bc+ad+bd$

$c(a+b)+d(a+b)$

$(c+d)(a+b)$

ex. $6ab+2b+9a+3$

$2b(3a+1)+3(3a+1)$

$\boxed{(2b+3)(3a+1)}$

if none of these are possible, the polynomial may not be factorable.

Combined Factoring. Factor the following completely.

261. $3a^2 - 3b^2$

$3(a^2 - b^2)$

$3(a-b)(a+b)$

262. $4x^2 + 28x + 48$

$4(x^2 + 7x + 12)$

$\otimes = 12$

$\oplus = 7$

$4(x+3)(x+4)$

263. $x^4 - 16$

$\sqrt{x^4} = x^2$ $\sqrt{16} = 4$

$(x^2+4)\underbrace{(x^2-4)}$

$(x^2+4)(x+2)(x-2)$

264. $2y^2 - 2y - 24$

$2(y^2 - y - 12)$

$2(y-4)(y+3)$

265. $16 - 28x + 20x^2$

$20x^2 - 28x + 16$

$4\underbrace{(5x^2 - 7x + 4)}$

cannot factor further

266. $m^4 - 5m^2 - 36$

$\otimes = -36$, $\oplus = -5$ } -9, $+4$

$(m^2+4)\underbrace{(m^2-9)}$

$(m^2+4)(m-3)(m+3)$

267. $x^8 - 1$

$(x^4+1)\underbrace{(x^4-1)}$

$(x^2+1)\underbrace{(x^2-1)}$

$(x+1)(x-1)$

$(x^4+1)(x^2+1)(x+1)(x-1)$

268. $x^3 - xy^2$

$x(x^2 - y^2)$

$x(x+y)(x-y)$

269. $x^4 - 5x^2 + 4$

$\otimes = 4$, $\oplus = -5$ } -4, -1

$\underbrace{(x^2-1)}\underbrace{(x^2-4)}$

$(x-1)(x+1)(x-4)(x+4)$

HIGHER DIFFICULTY…

For some of the following questions, you may try substituting a variable in the place of the brackets to factor first, and then replace brackets.

270. $(a+b)^2 - c^2$

let $(a+b) = x$

$x^2 - c^2$

$(x-c)(x+c)$

$(a+b-c)(a+b+c)$

271. $(c-d)^2 - (c+d)^2$

let $(c-d) = x$

let $(c+d) = y$

$x^2 - y^2$

$(x+y)(x-y)$

$((c-\cancel{d})+(c+\cancel{d}))((\cancel{c}-d)-(\cancel{c}+d))$

$(2c)(-2d)$

$\boxed{= -4cd}$

272. $(m+7)^2 + 7(m+7) + 12$

let $(m+7) = a$

$a^2 + 7a + 12$

$(a+3)(a+4)$ $\otimes = 12$, $\oplus = 7$ } $+3$, $+4$

$(m+7+3)(m+7+4)$

$(m+10)(m+11)$

273. Factor. $a^2 - b^2$
$(x+2)^2 - (x-3)^2$

$[(x+2)+(x-3)][(x+2)-(x-3)]$
$(2x-1)(5)$
$5(2x-1)$

274. Find all the values of k so that $x^2 + kx - 12$ can be factored.

$\hookrightarrow \times = -12$

$-1 \cdot 12$, $1 \cdot -12$ $\} \pm 11$
$-2 \cdot 6$, $2 \cdot -6$ $\} \pm 4$
$-3 \cdot 4$, $3 \cdot -4$ $\} \pm 1$

$\therefore K = \pm 1, \pm 4$ or ± 11

275. For which integral values of k can $3x^2 + kx - 3$ be factored.

$\times = 3 \cdot -3 = -9$
$+ = ?\ k$

$1 \cdot -9$, $-1 \cdot 9$ $\} \pm 8$
$-3 \cdot 3$, ~~$3 \cdot -3$~~ $\} 0$

$\therefore K = \pm 8$

276. What value of k would make $kx^2 + 24xy + 16y^2$ a perfect square trinomial?

perfect squares

$(\_x + 4y)^2$
multiply $= 12xy$
$\therefore 3x$
So, $K = 9$

277. What value of k would make $2kx^2 - 24xy + 9y^2$ a perfect square trinomial?

$\div 2$
$-12xy$
$(\underline{4}x - 3y)^2$
multiply $= 12xy$
$(4x-3y)(4x-3y)$
$\underbrace{16x^2}_{\div 2 = 8} - 24xy - 9y^2$
$\therefore K = 8$

278. For which integral values of k can $6x^2 + kx + 1$ be factored.

a. 5,7

b. $\pm 5, \pm 7$

c. $-5, -7$

d. all integers from 5 to 7.

$\times = 6$
$-3 \cdot -2$, $3 \cdot 2$ $\} \pm 5$
$-1 \cdot -6$, $1 \cdot 6$ $\} \pm 7$

279. Expand and simplify.
$-2(3m+4)^2$

$-2(3m+4)(3m+4)$
$-2(9m^2 + 12m + 12m + 16)$
$-2(9m^2 + 24m + 16)$
$\boxed{-18m^2 - 48m - 32}$

280. If $a = 2x + 3$, write $a^2 - 5a + 3$ in terms of x.

$(2x+3)^2 - 5(2x+3) + 3$
$(4x^2 + 9) - 10x - 15 + 3$
$\boxed{4x^2 - 10x - 3}$

281. Lindsay was helping Anya with her math homework. She spotted an error in Anya's multiplication below. Find and correct any errors.

Multiply:
5x(2x+1)+2(2x+1)
=10x+1+4x+2
=14x+3

$5x \cdot 1 = 5x$

$5x \cdot 2x = 10x^2$

$10x^2 + 5x + 4x + 2$

$5x(2x+1) + 2(2x+1)$

$(5x+2)(2x+1)$

282. When asked to factor the following polynomial, Timmy was a little unsure where to start.

Factor: 10x + 5 + 2xy + y

$5(2x+1) + y(2x+1)$

What type of factoring could you tell him to perform to help him along?

Factor by Graping

283. Find and correct any errors in the following factoring.

$2 \cdot 12 = 24$

$\otimes = 24$
$\oplus = -5$
$-3, -8$

$2x^2 - 5x - 12$

$= 2x^2 - 12x + 2x - 12$

$2x^2 - 8x - 3x - 12$

$\cancel{= 2x(x-6) + 2(x-6)}$

$2x(x-4) + 3(x-4)$

$\cancel{= (2x+2)(x-6)}$

$(2x+3)(x-4)$

284. Explain why

$3x^2 - 17x + 10 \neq (3x + 1)(x + 10)$

$3x^2 - 17x + 10 \neq 3x^2 + 30x + x + 10$

$\neq 3x^2 + 31x + 10$

285. Find and correct any errors in the following multiplication.

$(x^2 + 2)^2$

$= x^4 + 4$

$(x^2+2)(x^2+2)$

$x^4 + 2x^2 + 2x^2 + 4$

$\boxed{x^4 + 4x^2 + 4}$

286. Explain why it is uncommon to use algebra tiles to multiply the following

$(x + 1)^3$

b/c it is cubed not squared. ie: multiply 3 thing.

287. Multiply the expression above.

$(x+1)(x+1)(x+1)$

$x^2 + x + x + 1\ (x+1)$

$x^2 + 2x + 1\ (x+1)$

$x^3 + x^2 + 2x^2 + 2x + x + 1$

$\boxed{x^3 + 3x^2 + 3x + 1}$

ADDITIONAL MATERIAL

Solving Quadratic Equations:

One of two methods will be used depending on the equation.

Isolating the variable in one place:

Solve. $x^2 - 25 = 0$
$x^2 = 25$
$x = 5 \text{ or } -5$

Solve. $3x^2 - 12 = 0$
$3x^2 = 12$
$x^2 = 4$
$x = 2 \text{ or } -2$

We can only isolate the variable when there are not x terms as well as x^2 terms.

> ZERO PRODUCT RULE
>
> For two terms to have a product equal to zero, one or both must be equal to zero.

Solve by factoring with the zero product rule:
With quadratic equations like $x^2 + 7x + 12 = 0$, we cannot isolate the variable because x and x^2 cannot be combined.

We must factor the polynomial.

$x^2 + 7x + 12 = 0$
$(x+3)(x+4) = 0$

Factor.
Think... what would make the left side equal to 0.
Use the *zero product rule.*
If $x = -3$ or $x = -4$, the entire left side would equal 0.

$x = -3 \text{ or } -4$

Solve. $2x^2 + 7x + 6 = 0$

$(2x+3)(x+2) = 0$

$x = -2 \text{ or } -\frac{3}{2}$