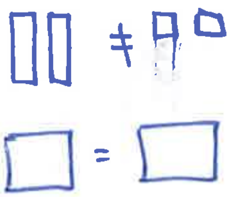
Like Terms

39. When considering algebra tiles, what makes two tiles "alike"?

same size + shape

eg.



40. What do you think makes two algebraic terms alike? (Remember, tiles are used to represent the parts of an expression.)

If the algebraic terms have the same variable and same degree/exponent => "Like Terms"

Collecting Like Terms without tiles:

Exactly the same variable & exponents.

You have previously been taught to combine like terms in algebraic expressions.

Terms that have the same variable factors, such as $7x$ and $5x$, are called ***like terms***. here the are both x

Simplify any expression containing like terms by adding their coefficients.

Eg.1. Simplify:

$$7x + 3y + 5x - 2y$$
$$7x + 5x + 3y - 2y$$
$$= 12x + y$$

Eg.2. Simplify

$$3x^2 + 4xy - 6xy + 8x^2 - \boxed{3yx}$$
$$3x^2 + 8x^2 + 4xy - 6xy - \boxed{3xy}$$
$$= 11x^2 - 5xy$$

Remember ... $3yx$ is the same

$3xy = 3yx$

substitute

Simplify by collecting like terms. Then evaluate each expression for $x = 3, y = -2$.

41. $\boxed{3x} + 7y \boxed{- 12x} + 2y$

collect ① like Terms

$3x - 12x + 7y + 2y$

$\boxed{-9x + 9y}$

② Simplify

③ Substitution

$-9x + 9y$

$-9(3) + 9(-2)$

$-27 - 18$

$= -45$

42. $2x^2 + 3x^3 - 7x^2 - 6$

$2x^2 - 7x^2 + 3x^3 - 6$

$3x^3 - 5x^2 - 6$

$3(3)^3 - 5(3)^2 - 6$

$3(27) - 5(9) - 6$

$81 - 45 - 6$

$= 30$

43. $5x^2y^3 - 5 + 6x^2y^3$

$11x^2y^3 - 5$

$11(3^2)(-2^3) - 5$

$11 \cdot (9) \cdot (-8) - 5$

$-792 - 5$

$\boxed{= -797}$

Adding & Subtracting Polynomials without TILES.

ADDITION
To add polynomials, collect like terms.

Eg.1. $(x^2+4x-2)+(2x^2-6x+9)$

Horizontal Method:

$=x^2+4x-2+2x^2-6x+9$

$=x^2+2x^2+4x-6x-2+9$ re-arrange to group like terms.

$=3x^2-2x+7$

Vertical Method:

x^2+4x-2
$2x^2-6x+9$

$=3x^2-2x+7$

"Line up" Like terms.

SUBTRACTION

"apply the ⊖ to the bracket"

It is important to remember that the subtraction refers to all terms in the bracket immediately after it.

To subtract a polynomial, determine the opposite and add.

Eg.2. $(4x^2-2x+3)-(3x^2+5x-2)$

Multiplying each term by -1 will remove the brackets from the **second** polynomial.

This question means the same as:

$(4x^2-2x+3)-\mathbf{1}(3x^2+5x-2)$ ⊖⊖ = ⊕
$=4x^2-2x+3-3x^2-5x+2$
$=4x^2-3x^2-2x-5x+3+2$

$=x^2-7x+5$

alternate:
(We could have used vertical addition once the opposite was determined if we chose.)

Add or subtract the following polynomials as indicated.

44. $(4x+8)+(2x+9)$

$\underbrace{4x+2x}+\underbrace{8+9}$

$\boxed{6x+17}$

45. $(3a+7b)+(9a-3b)$

$3a+7b$
$+9a-3b$

$\boxed{12a+4b}$

46. $(7x+9)-(3x+5)$

apply the ⊖ to everything inside the brackets

$-3x-5$

$\underbrace{7x-3x}+\underbrace{9-5}$

$\boxed{4x+4}$

47. Add.
$(4a-2b)$
$+(3a+2b)$

$7a+0b$

$\boxed{=7a}$

48. Subtract.
$(7x-3y)$
$-(-5x+2y)$

$7x+5x-3y-2y$

$\boxed{=12x-5y}$

49. Subtract.
$(12a-5b)$
$-(-7a-2b)$

$12a+7a-5b+2b$

$\boxed{19a-3b}$

Add or subtract the following polynomials as indicated.

50. $(5x^2 - 4x - 2) + (8x^2 + 3x - 3)$

$$\begin{array}{r} 5x^2 - 4x - 2 \\ +\ 8x^2 + 3x - 3 \\ \hline \end{array}$$

$\boxed{13x^2 - x - 5}$

51. $(3m^2n + mn - 7n) - (5m^2n + 3mn - 8n)$

$-3mn \quad +8n$

$\underbrace{3m^2n - 5m^2n}_{-2m^2n} \underbrace{+ mn - 3mn}_{-2mn} \underbrace{- 7n + 8n}_{+n}$

$\boxed{-2m^2n - 2mn + n}$

52. $(8y^2 + 5y - 7) - (9y^2 + 3y - 3)$

$+3$

$-3y$

$\underbrace{8y^2 - 9y^2}\ \underbrace{+ 5y - 3y}\ \underbrace{- 7 + 3}$

$\boxed{-y^2 + 2y - 4}$

53. $(2x^2 - 6xy + 9) + (8x^2 + 3x - 3)$

$\underbrace{2x^2 + 8x^2}\ \underbrace{- 6xy + 3x - 3}$

NO other "Like Terms"

$\boxed{10x^2 - 6xy + 3x - 3}$

Your notes here...

★ Remember ★

Signs!

$\ominus \cdot \oplus = \ominus$

$\ominus \cdot \ominus = \oplus$

Brackets!

$\ominus(\ \oplus\ ) = \ominus$

$\ominus(\ \ominus\ ) = \oplus$

Multiplication and the Area Model

Sometimes it is convenient to use a tool from one aspect of mathematics to study another.

To find the product of two numbers, we can consider the numbers as side lengths of a rectangle.

How are side lengths, rectangles, and products related? The Area Model

The product of the two sides is the area of a rectangle. $\boxed{A = lw}$

Consider:

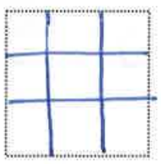
$A = 4 \times 6 = 24$

Length = 4 Width = 6

---

54. Show why $3 \times 3 = 9$ using the area model.

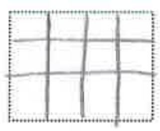
Solution:



= 9

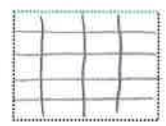
55. Show why $3 \times 4 = 12$ using the area model.

4

3  = 12

56. Calculate 5×4 using the area model.

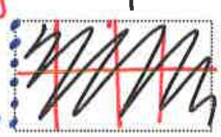
4

5  = 20

57. How might we show $-2 \times 4 = -8$ using the area model?

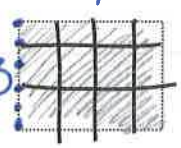
shade-in ⊖ (like the algebra tiles)

4

-2  = -8

58. Calculate -3×4 using the area model.

4

-3  = -12

59. Calculate -5×4 using the area model.

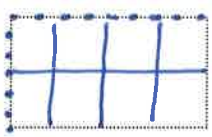
4

-5  = -20

dots to show which # is ⊖

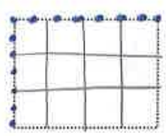
60. How might we show $-2 \times -4 = 8$ using the area model?

-4

-2  = 8

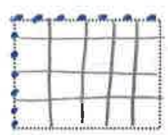
61. Calculate -3×-4 using the area model.

-4

-3  = 12

62. Calculate -5×-4 using the area model.

-5

-4  = 20

* ⊖ + ⊖ = ⊕

There are some limitations when using the area model to show multiplication. The properties of multiplying integers (+,+), (+,-), (-,-) need to be interpreted by the reader.

★ also a good method for the non-calc. section of tests.

63. Show how you could break apart the following numbers to find the product.

$21 \times 12 =$

$= (20 + 1) \times (10 + 2)$
$= 200 + 40 + 10 + 2$
$= 252$

64. Show how you could break apart the following numbers to find the product.

$32 \times 14 =$

$(30 + 2) \times (10 + 4)$
$300 + 120 + 20 + 8$
$= 448$

65. Show how you could break apart the following numbers to find the product.

$17 \times 24 =$

$(10 + 7) \times (20 + 4)$
$200 + 40 + 140 + 28$
$= 408$

66. Draw a rectangle with side lengths of 21 units and 12 units. Model the multiplication above using the rectangle.

	10	2
20	200	40
1	10	2

$= 252$ (units^2)

67. Draw a rectangle with side lengths of 32 units and 14 units. Model the multiplication above using the rectangle.

	20 +	4
30 +	30 × 20 = 300	120
2	40	8

$30 \times 4 = 120$
$2 \times 20 = 40$
$2 \times 4 = 8$

300
120
40
+ 8
$= 468$ units^2

68. Draw a rectangle with side lengths of 17 units and 24 units. Model the multiplication above using the rectangle.

	10 +	7
20 +	200	140
4	40	28

$200 + 140 + 40 + 28$
$= 408$ units^2

69. Use an area model to multiply the following without using a calculator.
23×15

	10	5
20	200	100
3	30	15

$200 + 100 + 30 + 15$
$\boxed{= 345 \text{ units}^2}$

70. Use an area model to multiply the following without using a calculator.
52×48

	40 +	8
50	2000	400
2	80	16

$2000 + 400 + 80 + 16$
$\boxed{= 2496 \text{ units}^2}$

71. Use an area model to multiply the following without using a calculator.
73×73

	70	3
70	4900	210
3	210	9

4900
210
210
9
$= 5329$ units^2

Algebra tiles and the area model: Multiplication/Division of algebraic expressions.

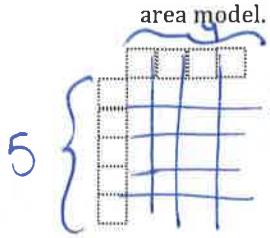
First we must agree that the following shapes will have the indicated meaning.

1 -1 x -x x^2 $-x^2$

We must also remember the result when we multiply:

- Two positives = Positive
- Two negatives = Positive
- One positive and one negative = Negative

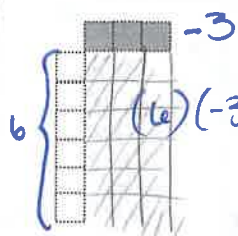
72. Write an equation represented by the diagram below and then multiply the two monomials using the area model.



5×4

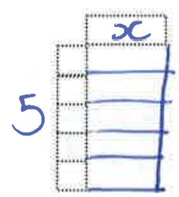
$= 20$

73. Write an equation represented by the diagram below and then multiply the two monomials using the area model.



$(6)(-3) = 18$

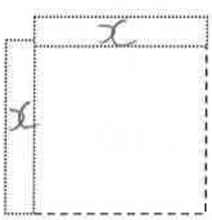
74. Write an equation represented by the diagram below and then multiply the two monomials using the area model.



$5 \times x$

$\boxed{= 5x}$

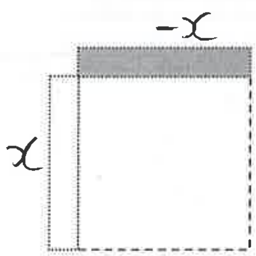
75. Write an equation represented by the diagram below and then multiply the two monomials using the area model.



$(x)(x)$

$= x^2$

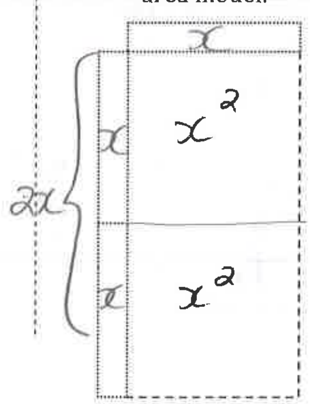
76. If the shaded rectangle represents a negative value, find the product of the two monomials.



$(x)(-x)$

$\boxed{= -x^2}$

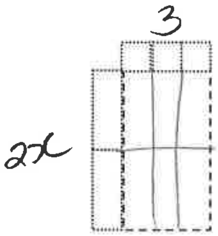
77. Write an equation represented by the diagram below and then multiply the two monomials using the area model.



$(2x)(x)$

$\boxed{= 2x^2}$

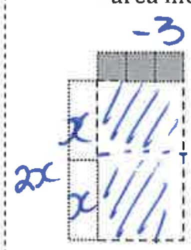
78. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.



$(2x)(3)$

$\boxed{=6x}$

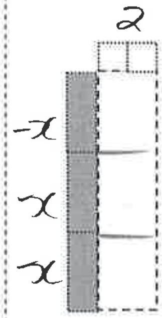
79. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.



$(2x)(-3)$

$\boxed{=-6x}$

80. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.

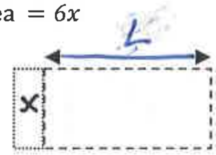


$(-3x)(2)$

$\boxed{=-6x}$

81. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

Area $= 6x$



Length: 6

$A = L \cdot w$

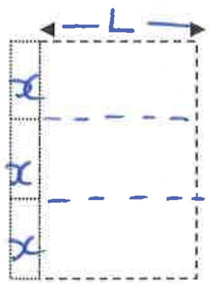
$\frac{6x}{x} = \frac{L \cdot \cancel{x}}{\cancel{x}}$

$\frac{6\cancel{x}}{\cancel{x}} = L$

$\boxed{\therefore L = 6}$

82. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

Area $= 6x^2$



Length: $2x$

$A = L \cdot w$

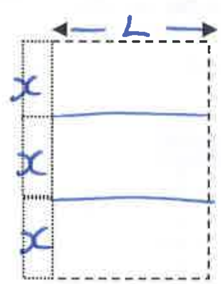
$\frac{6x^2}{3x} = \frac{L \cdot 3x}{3x}$

$\div 3 \ \frac{\cancel{6}x^{\cancel{2}}}{\cancel{3}\cancel{x}} = L$

$\boxed{2x = L}$

83. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

Area $= -6x^2$



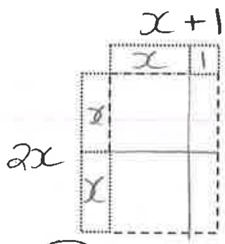
Length: $-2x$

$A = L \cdot w$

$\frac{-\cancel{6}^{2}x^{\cancel{2}}}{\cancel{3}\cancel{x}} = \frac{L \cdot \cancel{3x}}{\cancel{3x}}$

$\boxed{-2x = L}$

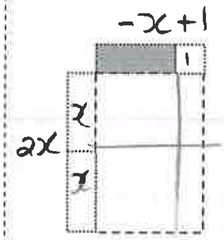
84. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.



$2x(x+1)$

$\boxed{2x^2 + 2x}$

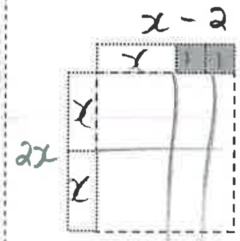
85. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.



$2x(-x+1)$

$\boxed{-2x^2 + 2x}$

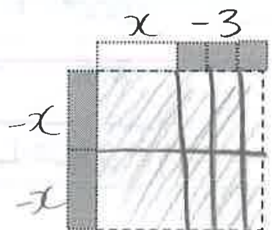
86. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.



$2x(x-2)$

$\boxed{2x^2 - 4x}$

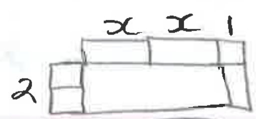
87. Write an equation represented by the diagram below and then multiply the two expressions using the area model.



$-2x(x-3)$

$\boxed{-2x^2 + 6x}$

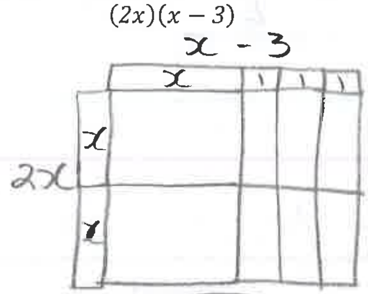
88. Draw and use an area model to find the product:
$(2)(2x+1)$



$(2)(2x+1)$

$\boxed{4x + 4}$

89. Draw and use an area model to find the product:
$(2x)(x-3)$



$(2x)(x-3)$

$\boxed{2x^2 - 6x}$