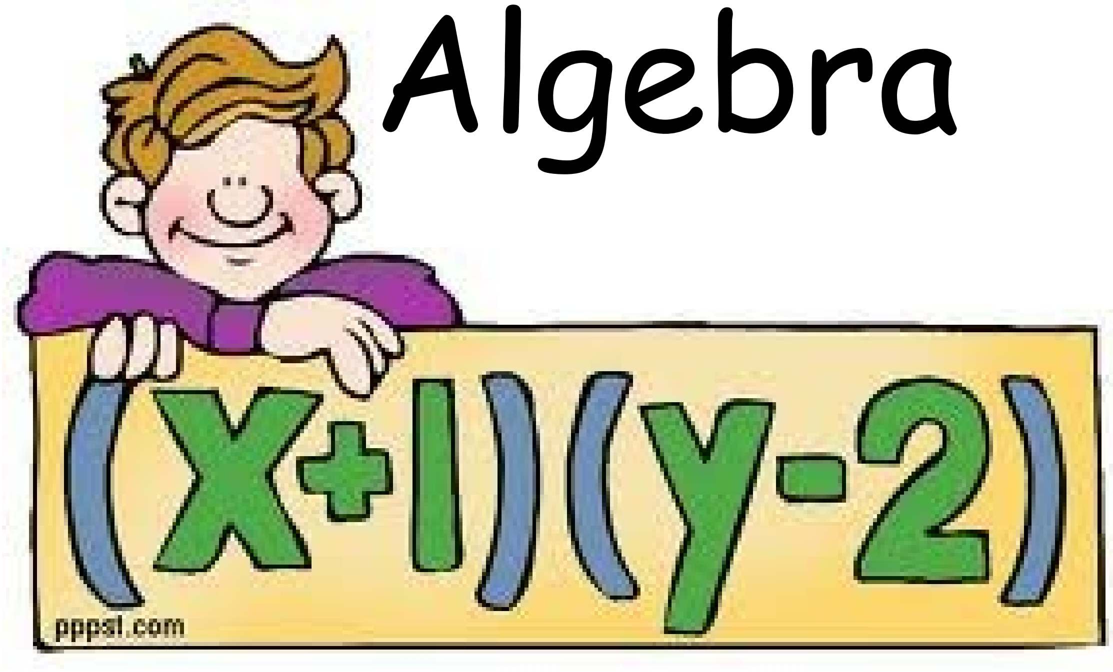
4.1 The Language of Algebra



Algebra Terms

A **variable** is a letter that can represent any number

For example, the formula for the area of a rectangle is:

Area of a rectangle = length × width

If A represents the area of the rectangle, l represents the length of the rectangle and w represents the width of the rectangle, then we can write the formula for the area of the rectangle as follows:

$A = l \times w$ *In this formula, the letters A, l and w are called* ***pronumerals.***

Example: *x could represent the number of goals a soccer player scored in a game*

Algebra Terms

The sum/total is the answer when you add

the sum of a and b, is a + b

The difference is the answer when you subtract the smaller number form the larger

the difference of a and b, is a - b

Algebra Terms

A product is the answer when you multiply

$a \times b$ is written ab

the **product** of a and b, is a × b

A quotient is the answer when you divide

$a \div b$ is written $\frac{a}{b}$

The quotient of a and b, is a ÷ b

Algebra Terms

Double: multiply by 2

ex. Double 16 is 16 × 2 = 32

Halve: divide by 2

ex. Half of 16 is 16 ÷ 2 =8

Triple: multiply by 3

ex. Triple 9 is 9 × 3 = 27

Square: multiply a number by itself

ex. Square 7 is 7 × 7 = 49

Consecutive: describes the numbers that follow directly after each other

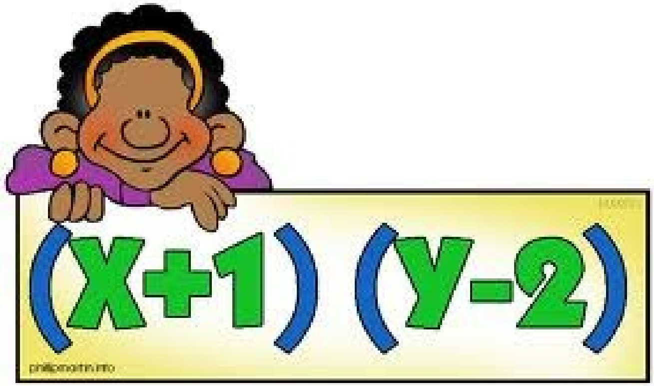
ex. 7, 8, 9, 10 are consecutive numbers; 11, 13, 15 are consecutive **odd** numbers, 11, 14, 17 are **not consecutive numbers**.

Algebra Terms

A term may have one or more pronumerals (variables) or may be just a number.

Ex. 5a, 7q, 9g/5, w,400,abc

A term is part of an expression



Algebra Terms

A coefficient is the number in front of a variable.

- If the term is being subtracted, the coefficient is a negative number
- If there is no number in front, the coefficient is 1

Example: 9ay 4a w-16zy….the coefficients are 9, 4, 1 and -16

Algebra Terms

An algebraic expression is a combination of numbers and variables together with mathematical operations

ex. $3x + 2zy$

ex. $8 \div (3a - 2b) + 41$

Expressions are made by adding, subtracting, multiplying or dividing terms

Algebra Terms

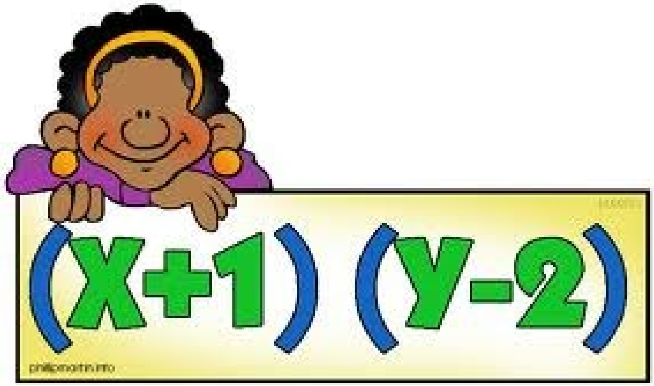
A polynomial is an algebraic expression with **1 or more terms.**

2 or more terms are *separated by addition or subtraction*

ex. $x^2 + 3x$

ex. $(-2x^2) + 5x - 4$

Polynomials are used in math to solve algebraic problems.



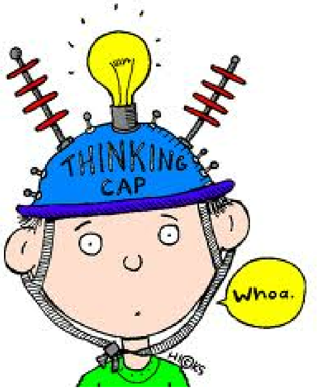
Algebra Terms

An equation always has an equals sign =

ex. $r = 5a + 7y$ $2(v-6) = 12$

A constant is a number whose value doesn't change, it always remains the same

ex. 2001 -3 73715 -8



TRY THIS!

Language of Algebra

$4a + b - 12c + 5$

1. List the individual terms in the expression

2. In the expression, state the coefficients of a, b, c and d

3. What is the constant term?

4. State the coefficient of b in the expression

$3a + 4ab + 5b^2 + 7b$

SOLUTION

$4a + b - 12c + 5$

1. List the individual terms in the expression

Each part of an expression is a term. Terms get added (or subtracted) to make an expression. So, there are four terms: 4a, b, 12c and 5

2. In the expression, state the coefficients of a, b, c and d

The coefficient is the number in front of a variable. So,

The coefficient of a is 4. the coefficient of b is 1 because b is the same as 1 × b. the coefficient of c is -12 because this term is being subtracted. And the coefficinet of d is 0 because there are no terms with d.

SOLUTION

$4a + b - 12c + 5$

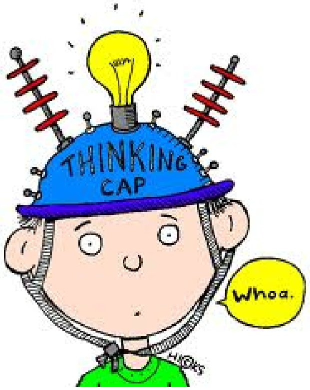
3. What is the constant term?

A constant term is any term that does not have a variable (letter) in front of it. The constant is 5

4. State the coefficient of b in the expression

$3a + 4ab + 5b^2 + 7b$

Although there is a 4 in front of ab and a 5 in front of b^2, neither of these are terms containing **just "b"**, so they are ignored. We are looking for **only 'b' by itself**. So, the coefficient is 7, for 7b.

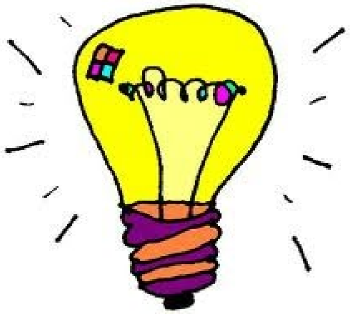


TRY THIS!

Write an expression for this sentence:

Start with a number, multiply it by three then add five

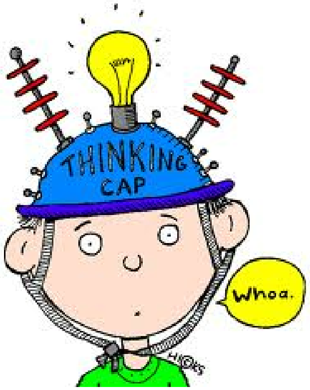
Let the starting number be “y”



1. Start with a number “y”
2. Multiply by 3 y × 3
3. Then add 5 y × 3 + 5

So the algebraic expression is:

$$3y + 5$$



TRY THIS!

Write an **expression** for each of the following

1. The sum of 3 and k
2. The product of m and 7
3. 5 is added to one half of k
4. The sum of a and b is doubled

Solutions

1. The sum of 3 and k

The word sum means '+' so, 3+k

2. The product of m and 7

The word 'product' means to × so, m × 7 or 7m

3. 5 is added to one half of k

One half of k can be written $\frac{1}{2}$ × k (because 'of' means ×) or k/2 because k is begin divided by two

4. The sum of a and b is doubled

The values of a and b are being added and the result is multiplied by 2. brackets are required to multiply the qhole result by two and not just the value of b

$(a + b) \times 2$ or $2(a + b)$

What is a polynomial? an expression that can have constants (like 4), variables (like x or y), and exponents (like the 2 in y^2), that can be combined using addition, subtraction multiplication or division.

BUT:
- no dividing by a variable $\cancel{\frac{2}{x}}$
- variable exponents must be whole numbers (0,1,2,3....)
- can not have an infinite numberal terms. (ie: must end)

Algebra Tiles & Visual Representation

Red tiles represent positive 1

Positive 1 -tile

White tiles represent negative 1

Negative 1 - tile

Green tiles this shape represent positive x

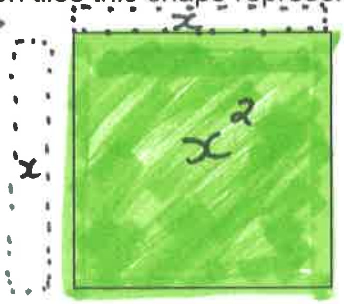
x tile

White tiles this shape represent negative x

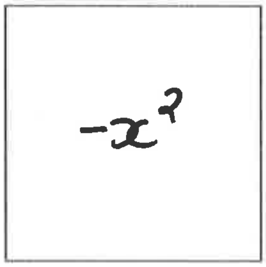
- x tile

Green tiles this shape represent positive x^2

imagine this is $x \cdot x = x^2$



White tiles this shape represent negative x^2



______ $-x^2$ ______ x^2

(sorry... this printed weird)

copy on next page.

USING THE 2 PAGES YOUR TEACHER HAS PROVIDED, MAKE YOURSELF 1 SET OF POSITIVE ALGEBRA TILES AND 1 SET OF NEGATIVE ALGEBRA TILES

- There are ziplock bags on my desk for students to store tiles.
- This is Homework must be cut + coloured by next class!

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

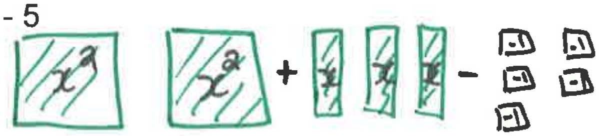
x^2	x^2	x^2	x	x	x	x
x^2	x^2	x^2	x	x	x	x
x^2	x^2	x^2	x	x	x	x
x^2	x^2	x^2	x	x	x	x

model with YOUR algebra tiles, then draw!

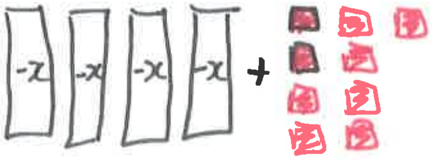
PRACTICE

Example #1: Use algebra tiles to model each expression below.

a) $2x^2 + 3x - 5$

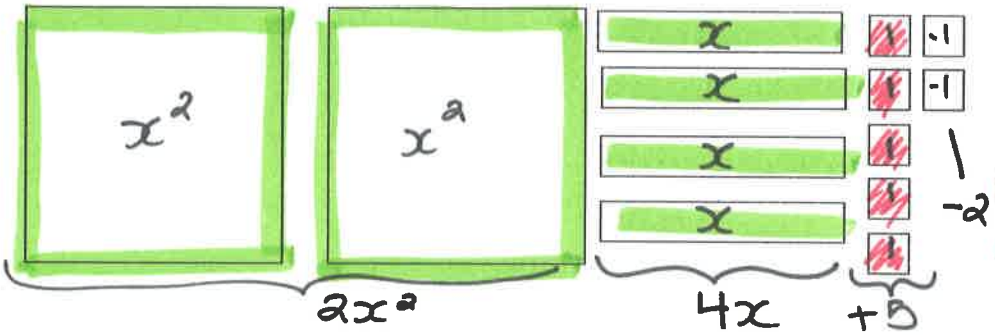


b) $-4x + 9$



Example #2: Write the expression represented by the algebra tiles below. (color some so they aren't all "negative")

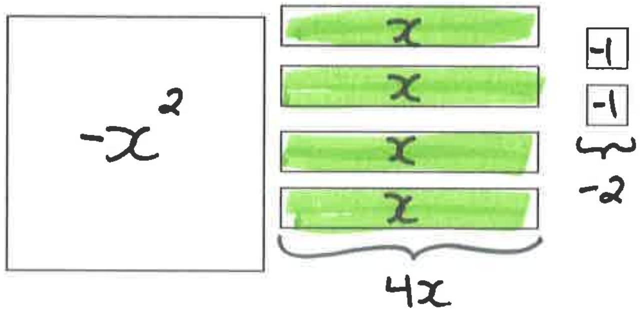
a)



$= 2x^2 + 4x + \underbrace{5 - 2}$

$\boxed{= 2x^2 + 4x + 3}$

b)



$= -x^2 + 4x - 2$

A polynomial is one term or the sum/difference of terms whose variables have whole number exponents.

Terms are numbers, variables, or the product of a number and a variable (ex. 6, x, or $3x^2y$)

The expression is **NOT A POLYNOMIAL** when:

- There is a ***negative exponent*** ex. 2^{-3}, x^{-4}
- The ***variable cannot be in the denominator*** of a fraction ex. $\frac{2}{x}$
- The ***variable cannot be inside a radical*** ex. $\sqrt{x}$, $\sqrt[3]{y}$

square or cube root sign

Example #3: Which of the following are polynomials? Explain your reasoning.

a) $-2x + 6$ ✓

yes, polynomial

b) $-10x^2 + \cancel{\sqrt{x}}$

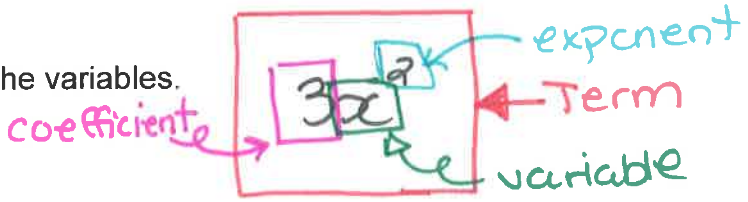
No, variables cannot be inside radicals

c) $\cancel{\frac{1}{x}} - 3x + 2$

No! variables cannot be the denominator

Vocabulary

Coefficients are the numbers in front of the variables.



The term with the greatest sum of exponents (from the variables only) determines the **degree** of the polynomial.

eg. $5w^2$ degree = 2 ; $3x^{\boxed{2}} + 9xy + y^{\boxed{3}}$ degree = 3 b/c 3>2

The **constant** term is the one without the variable (its value does not change/vary when the value of x change, it remains constant)

eg. $3x^2 + 9xy + y^3 + \boxed{7}$ ← constant. a number "by itself"

Example #4: For each polynomial below, determine the coefficients, the degree and the constant.

The sign "belongs" to the term.

Polynomial	Coefficients	Degree	Variable	Constant	# of Terms
$5x^2 - 8x + 2$ ✓ ✓ ✓ 3 terms	5, -8	2	x^2, x	2	3
imaginary 1 $-6x - 7$ ✓ ✓ 2 terms	-6	1	x	-7	2
$-10x^2 + 3x$ ✓ ✓ 2 terms	-10, 3	2	x^2, x	(none)	2
$4z^3 + 5y^2z^2 + 2yz$	4, 5, 2	$y^2z^2 = 2+2=4$ 4	different! z^3, y^2z^2, yz	(none)	3

To get the degree of a term with 1+ variables, add exponents

We classify polynomials by the number of terms.

A **monomial** has one term. eg. $\underbrace{-3x}_{1}$; y^2

A **binomial** has two terms eg. $\underbrace{-3x}_{1}+\underbrace{y^2}_{2}$

A **trinomial** has three terms eg. $\underbrace{-3x}_{1}+\underbrace{y^2}_{2}-\underbrace{8}_{3}$

A polynomial is generally written in descending order. This means we order the terms with the highest degree term first, all the way down to the constant term of degree zero.

eg. $8x^3+2x^{\boxed{5}}-3x^4+3+2x$ is INCORRECT!!

5, 4, 3, 2, 1 .... constant!

order exponents: $2x^{\boxed{5}}-3x^{\boxed{4}}+8x^{\boxed{3}}+2x^{\boxed{1}}+3 \leftarrow$ constant always last!

Evaluating Algebraic Expressions

We can use algebraic expressions to solve problems and solve for things like cost.
The following algebraic expression is used to determine the cost of a school field trip.

C = \$300 + \$10t + \$7.50s

where C is the cost, t is the number of teacher supervisors on the trip and s is the number of students on the trip.

If a school field trip had $\overbrace{\text{4 teacher supervisors}}^{t=4}$ and $\overbrace{\text{100 students}}^{s=100}$ in attendance what would the total cost of the field trip be?

C = \$300 + \$10t + \$7.50s (substitute in values + solve!)

$C = 300 + 10(4) + 7.50(100)$

$C = 300 + 40 + 750 \; \boxed{= \$1\,090}$

Homework ASSIGNMENT #1 Section 4.1 pg 112-115	Required questions	Extra practice	Extension
	2, 3abcd, 4, 5abcd, 6, 7, 11, 13, 14, 15, 16	3ef, 5ef, 9, 10, 12, 17, 18,	21, 22

↑ all students must complete

↑ early finishers or needing extra practice

↑ Bonus Challenge Questions

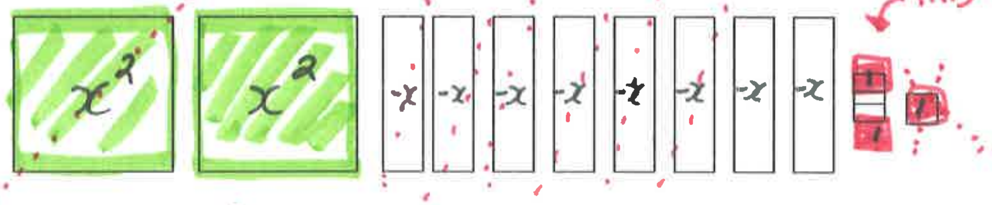
4.2 Adding and Subtracting Polynomials

Investigation: Model each polynomial using algebra tiles.
USE YOUR OWN ALGEBRA TILES TO MODEL ON YOUR DESK!

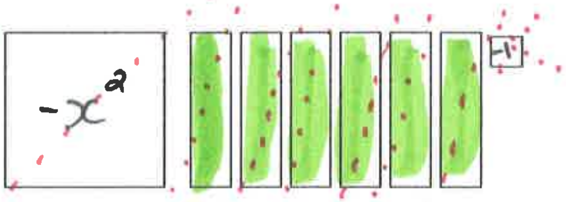
(cancel out...step below)

Fix this!

a) $2x^2 - 8x + 3$



b) $-x^2 + 6x - 1$



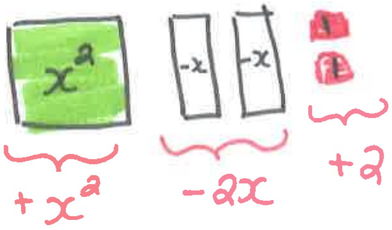
Consider the model for the polynomial $2x^2 - 8x + 3 - x^2 + 6x - 1$.

We organize the tiles by grouping the same sizes together and simplify by removing the opposite pairs.

These opposite pairs are sometimes referred to as **zero pairs** as they are equivalent to zero.
For example: +1 and -1 ; +x and -x , +x² and -x² are ***zero pairs***

The opposite pairs cancel out and we are left with:

Simplified expression: $x^2 - 2x + 2$



A polynomial is in **simplified form** when:

→ Its algebra tile model uses the fewest tiles possible (cancel out all zero pairs)

→ Its symbolic form contains only one term of each degree and no terms with a zero coefficient.

LIKE TERMS are:

→ Terms that can be represented by algebra tiles with the same shape AND size.

→ Terms with the same variable AND same exponent.

→ Constants may be different. *For example: $3x^2$ and $5x^2$ are still "like terms" because they are both "x^2"* ← variable + exponent are the same.

Example #1:

a) List three terms that are like terms with $5x^2$, x^2 , $2x^2$, $-x^2$

b) List three terms that are unlike terms with $5x^2$, x , x^2y , $5x$

Group the like terms in the following expression:

I like to put shapes around to group terms + signs!

Then I re-write below in order

$\boxed{2x}\boxed{+5}\boxed{+x^2}\boxed{+7}\boxed{+36x}\boxed{+3x^2}$

$$\boxed{= 4x^2 + 38x + 12}$$

$$\underbrace{x^2+3x^2}_{4x^2} + \underbrace{2x+36x}_{38x} + \underbrace{5+7}_{12}$$

PRACTICE (give students a few mins to practice, then review answers as a whole class)

Group the like terms in the following expressions:

1) $-6k + 7k$ $= k$

2) $12r \underbrace{-8-12}$

$= 12r - 20$

3) $\boxed{n} - 10 \boxed{+9n} - 3$

$\underbrace{n+9n}\ \underbrace{-10-3}$

$10n - 13$

4) $-4x - 10x$

$-14x$

5) $-r - 10r$

$-11r$

6) $\boxed{-2x} + 11 \boxed{+6x}$

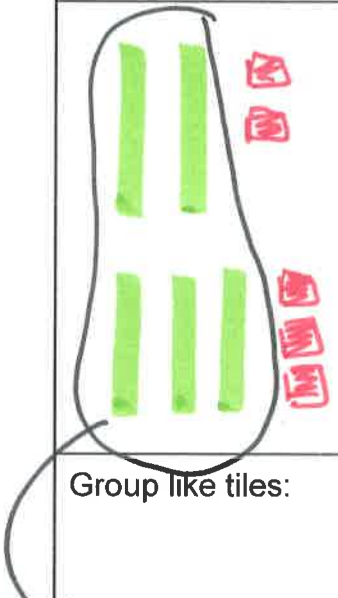
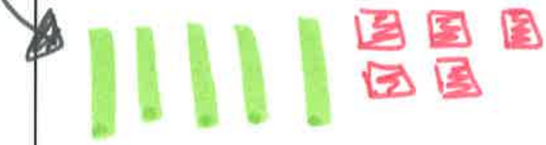
$\underbrace{-2x+6x} + 11$

$4x + 11$

Adding Polynomials

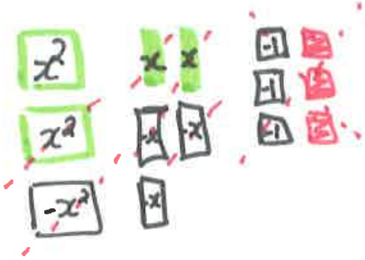
Example #2: What is the sum of 2x + 2 and 3x + 3?

Simplify the polynomial visually using algebra tiles and symbolically with algebra.

Visually	**Symbolically**
	$\boxed{2x}+\boxed{2}$ $\boxed{3x}+\boxed{3}$
Group like tiles: 	Group Like Terms: $\boxed{2x+3x}+\boxed{2+3}$
Remove Any Zero Pairs: There are no zeropairs b/c there are <u>no</u> negative tiles.	Combine Like Terms: $\underbrace{2x+3x}+\underbrace{2+3}$ $\boxed{5x+5}$

Example #3: What is the sum of $2x^2 + 2x - 3$ and $- x^2 - 3x + 3$?

Simplify the polynomial visually using algebra tiles and symbolically with algebra.

Visually	**Symbolically**
 $2x^2+2x-3$ $-x^2-3x+3$	shown here
Group like tiles: 	Group Like Terms: $\boxed{2x^2-x^2}+\boxed{2x+3x}\boxed{-3+3}$
Remove Any Zero Pairs: (cross out above) + re-draw 	Combine Like Terms: $\underbrace{2x^2-x^2}_{\boxed{x^2-x}}+\underbrace{2x-3x}\underbrace{-3+3}_{\varnothing}$

Example #4: $(2x + 3) + (4x - 3)$

$\underbrace{2x+4x}_{\boxed{6x}}\underbrace{+3-3}_{\varnothing}$

Remove the brackets + box like terms

Rearrange so like terms are together

Combine like terms

always show your work!

Example #5: $(2x^2 - 4x - 1) + (3x^2 + 2x + 5)$

if you don't have colours, use diff. shapes! ◡

Remove the brackets

Rearrange so like terms are together

Combine like terms

$\underbrace{2x^2+3x^2}\underbrace{-4x+2x}_{(-4+2=-2)}\underbrace{-1+5}_{(-1+5=4)}$

$\boxed{5x^2-2x+4}$

PRACTICE

DO THE ADDITION QUESTIONS ONLY *(COME BACK TO SUBTRACTION NEXT LESSON)*

Simplify each expression.

1) $(5p^2-3)+(2p^2-3p^3)$

~~2)~~ $(a^3-2a^2)-(3a^2-4a^3)$

(answers on next page) =>

3) $(4+2n^3)+(5n^3+2)$

~~4)~~ $(4n-3n^3)-(3n^3+4n)$

~~5)~~ $(3a^2+1)-(4+2a^2)$

~~6)~~ $(4r^3+3r^4)-(r^4-5r^3)$

~~7)~~ $(5a+4)-(5a+3)$

~~8)~~ $(3x^4-3x)-(3x-3x^4)$

9) $(-4k^4+14+3k^2)+(-3k^4-14k^2-8)$

~~10)~~ $(3-6n^5-8n^4)-(-6n^4-3n-8n^5)$

~~11)~~ $(12a^5-6a-10a^3)-(10a-2a^5-14a^4)$

~~12)~~ $(8n-3n^4+10n^2)-(3n^2+11n^4-7)$

13) $(-x^4+13x^5+6x^3)+(6x^3+5x^5+7x^4)$

14) $(9r^3+5r^2+11r)+(-2r^3+9r-8r^2)$

15) $(13n^2+11n-2n^4)+(-13n^2-3n-6n^4)$

16) $(-7x^5+14-2x)+(10x^4+7x+5x^5)$

Kuta Software - Infinite Algebra 1

Name__________________________________

Adding and Subtracting Polynomials

Date________________ Period____

Simplify each expression.

1) $(5p^2 - 3) + (2p^2 - 3p^3)$

$-3p^3 + 7p^2 - 3$

2) $(a^3 - 2a^2) - (3a^2 - 4a^3)$

$5a^3 - 5a^2$

3) $(4 + 2n^3) + (5n^3 + 2)$

$7n^3 + 6$

4) $(4n - 3n^3) - (3n^3 + 4n)$

$-6n^3$

5) $(3a^2 + 1) - (4 + 2a^2)$

$a^2 - 3$

6) $(4r^3 + 3r^4) - (r^4 - 5r^3)$

$2r^4 + 9r^3$

7) $(5a + 4) - (5a + 3)$

1

8) $(3x^4 - 3x) - (3x - 3x^4)$

$6x^4 - 6x$

9) $(-4k^4 + 14 + 3k^2) + (-3k^4 - 14k^2 - 8)$

$-7k^4 - 11k^2 + 6$

10) $(3 - 6n^5 - 8n^4) - (-6n^4 - 3n - 8n^5)$

$2n^5 - 2n^4 + 3n + 3$

11) $(12a^5 - 6a - 10a^3) - (10a - 2a^5 - 14a^4)$

$14a^5 + 14a^4 - 10a^3 - 16a$

12) $(8n - 3n^4 + 10n^2) - (3n^2 + 11n^4 - 7)$

$-14n^4 + 7n^2 + 8n + 7$

13) $(-x^4 + 13x^5 + 6x^3) + (6x^3 + 5x^5 + 7x^4)$

$18x^5 + 6x^4 + 12x^3$

14) $(9r^3 + 5r^2 + 11r) + (-2r^3 + 9r - 8r^2)$

$7r^3 - 3r^2 + 20r$

15) $(13n^2 + 11n - 2n^4) + (-13n^2 - 3n - 6n^4)$

$-8n^4 + 8n$

16) $(-7x^5 + 14 - 2x) + (10x^4 + 7x + 5x^5)$

$-2x^5 + 10x^4 + 5x + 14$

visually helpful... but very time consuming.

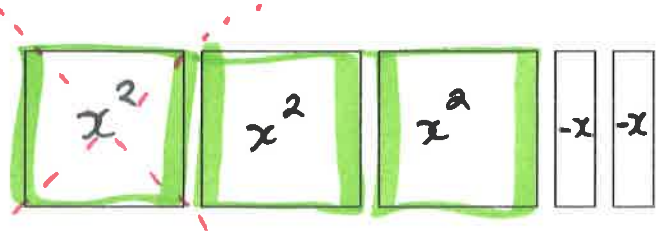
Subtracting Polynomials

Method #1: Subtracting Polynomials Using Algebra Tiles

Example #1: Subtract $(3x^2 - 2x) - (x^2 + 4x)$

$-x^2 - 4x$ read as

Start with the first polynomial:



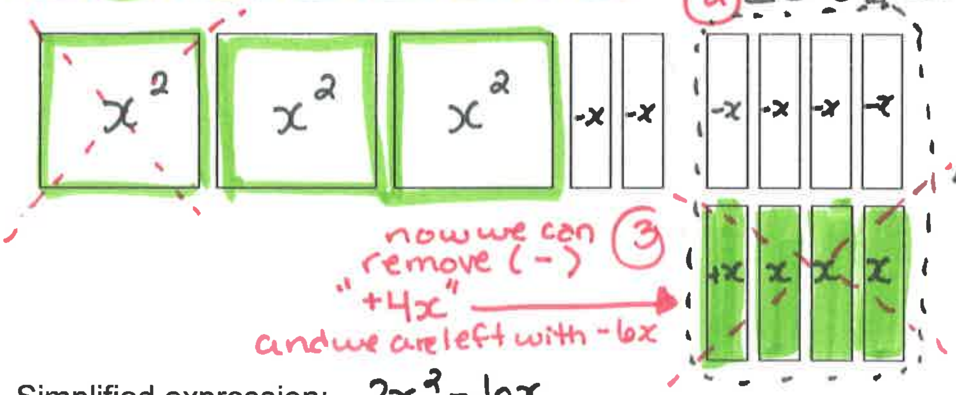
We need to

① *take away one x^2 tile from three x^2 tiles*

take away four x tiles from two negative x tiles

If you don't have enough positive tiles, you need to add more positive tiles and balance by also adding the same number of negative tiles.

② zero pair



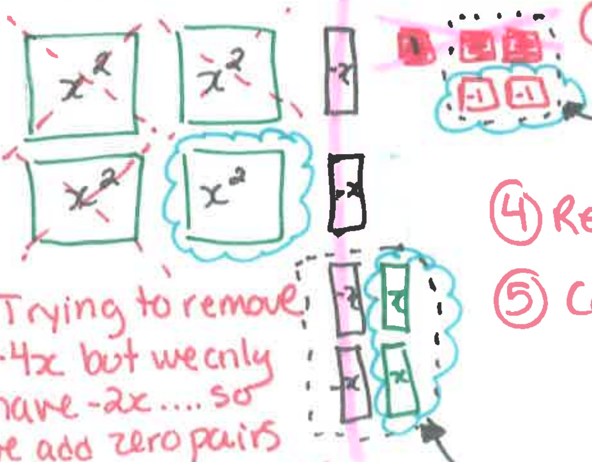
now we can remove (−) "$+4x$" ③ and we are left with $-6x$

because there aren't an $+x$ tiles, we add a group of zero pairs of the number we are trying to remove ie: $4x$

Simplified expression: $2x^2 - 6x$

Example #2: Use algebra tiles to subtract $(4x^2 - 2x + 1) - (3x^2 - 4x + 3)$

① $-3x^2$



③ Trying to remove $+3$, but there is only $+1$ ... add 2 more as a zero pair so that there is 3 to remove.

zero pair

④ Remove $-4x$ and $+3$ (shown in pink) crossing out.

⑤ Count what Remains (circled in blue)

$$\boxed{x^2 + 2x - 2}$$

② Trying to remove $-4x$ but we only have $-2x$.... so we add zero pairs until we have enough.

added $-2x$ zero pairs so that there is now $-4x$ to remove.

→ Faster method. (good if you tend to get mixed up with your +/- signs)

Method #2: Add the Opposite

Example #3: (5x + 4) - ~~(2x +1)~~ The opposite of (2x + 1) is $\boxed{(-2x - 1)}$

$(\boxed{5x}\boxed{+4}) + (\boxed{-2x}\boxed{-1})$

Remove brackets and add the opposite

$\underbrace{5x - 2x}\ \underbrace{+4 - 1}$

Collect like terms

$\boxed{3x + 3}$

Combine like terms

Example #4: $(3x^2 + 4x - 2)$ - ~~$(2x^2 + 6x + 2)$~~ The opposite of $(2x^2 + 6x + 2)$ is $+ (-2x^2 - 6x - 2)$

$\boxed{3x^2}+\boxed{4x}-2+(\boxed{-2x^2}\boxed{-6x}-2)$

Remove brackets and add the opposite

$\underbrace{3x^2 - 2x^2}\ \underbrace{+4x - 6x}_{(4-6=-2)}\ \underbrace{-2-2}_{(-2-2=-4)}$

Collect like terms

$x^2 \quad -2x \quad -4$

Combine like terms

→ Fastest.

Method #3: Subtracting Using Integer Properties (*"paper an pencil method"*))

Warm Up: Subtract. (give students 2mins... then go over answers)

a) 8 - 7 $= +1$

b) -3 – 5 $= -8$

c) 6 – (-4) $6+4 = 10$

d)-2 – (-5) $-2 + 5 = +3$

Example Subtract using properties of integers.

a) $(3x^2 - 2x) - (x^2 + 4x)$ The negative applies to everything inside the brackets.

$3x^2 - 2x - x^2 - 4x$

$\underbrace{3x^2 - x^2}\ \underbrace{-2x - 4x}$

3-2=1

$\boxed{2x^2 - 6x}$

b) $(-8a^2 + 3a - 7) - (-2a^2 - a +5)$

$-8a^2 + 3a - 7 - (-2a^2) - (-a) - (+5)$

$\boxed{-8a^2}\boxed{+3a}-7+\boxed{2a^2}\boxed{+a}-5$

$\underbrace{-8a^2 + 2a^2}_{-8+2=6}\ \underbrace{+3a + a}\ \underbrace{-7-5}$ -7-5=-12

$\boxed{-6a^2 + 4a - 12}$

ASSIGNMENT #2
Section 4.2 pg 123 - 126

Required questions	Extra practice	Extension
1, 2a, 3, 4, 6, 7abcd, 9, 10, 11, 12, 13, 16, 19, 21, 22, 23a	2b, 7ef, 8, 14, 15, 17, 18, 20, 23bcd	25