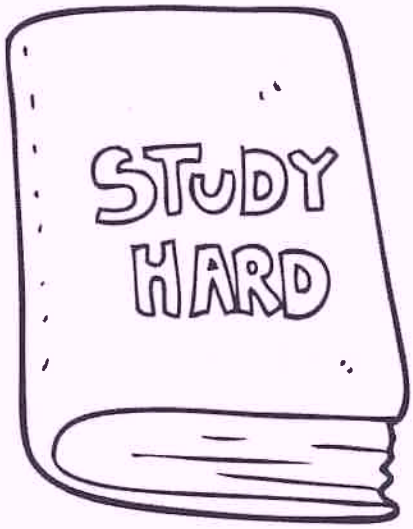
FOM & PRE-CALC 10

Final Exam



BOOK 3:

- RELATIONS & FUNCTIONS
- LINEAR FUNCTIONS
- SYSTEMS OF LINEAR EQUATIONS
- ARITHMETIC SEQUENCES & SERIES

NAME: KEY BLOCK: _____

Study Checklist

This review booklet is by no means a "practice final". It is a collection of practice questions on each unit, meant to guide your final exam studying and prepare you for the types of questions you will see. DO NOT treat this booklet as a practice test. DO NOT go straight to the answer key when you come across a question you cannot remember how to do. Difficult questions SHOULD guide your study! Always look up a concept in your class notes if you are stuck, then attempt the question again.

BEFORE beginning this booklet you should:

- read through your class notes booklet on *each topic*
- make your own "quick summary page" of important formulas & key concepts for the unit
- review quizzes & tests from the unit to recall strengths & weaknesses *(a great study method would be to re-do old quizzes & tests on a separate piece of paper)*

WHILE working through this booklet you should:

- look up concepts & example problems in your class notes when you come across a problem you are stuck on
- make a list of "questions to ask my teacher" so you can come to class and use your time efficiently.

Questions I'm having difficulty with:

Page	Question Number #	Topic

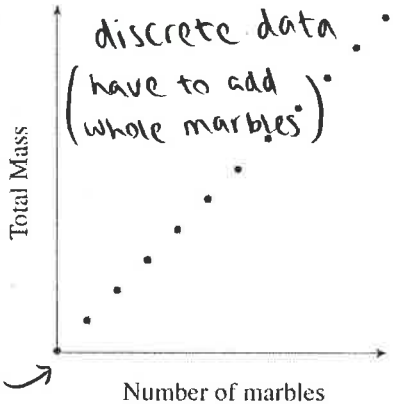
UNIT 5: RELATIONS & FUNCTIONS

MY NOTES AND THINGS TO REMEMBER...

UNIT 5: Relations and Functions Multiple Choice

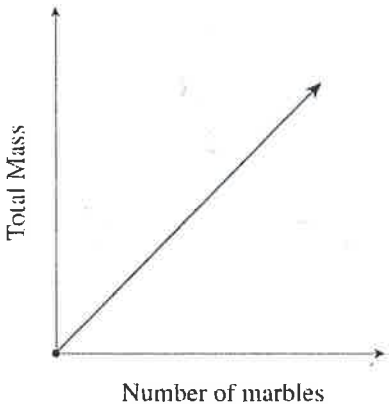
1. Marbles are placed in a jar one at a time. Which graph below best represents the total mass of the jar and marbles as the marbles are added?

A.



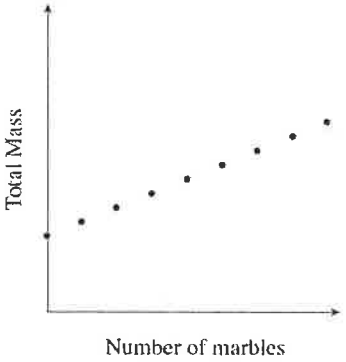
discrete data. (have to add whole marbles)

B.

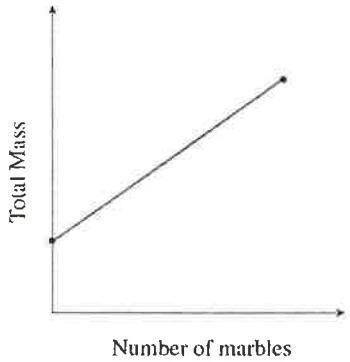


Should have 0 mass if no marbles

C.

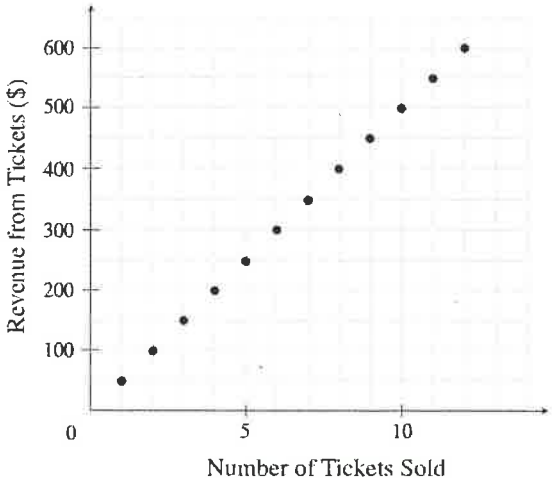


D.



2. What does the slope represent in the graph below?

rate of change



A. price per ticket
B. profit from tickets
C. revenue from tickets
D. number of tickets sold

3. The cost C, in dollars, to rent a car is determined by the formula $C(k) = 0.15k + 22$, where k is the number of kilometres driven. Calculate the value of k if $C(k) = 166$. Answer to the nearest kilometre.

$C(k) = 0.15k + 22$

$166 = 0.15k + 22$

$-22 \qquad -22$

$\frac{144}{0.15} = \frac{0.15k}{0.15}$ $\Rightarrow$ **k = 960 km**

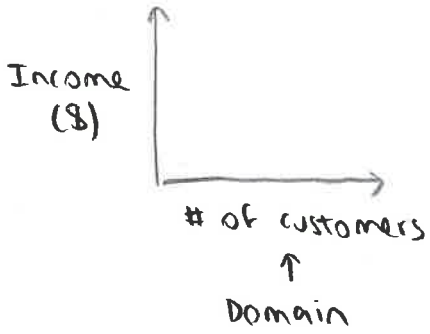
4. Damien has a list of 37 potential customers for his house-painting business. In order to get a business grant, he must graph his income versus the number of customers. Determine the domain of the graph.

A. $\{0, 1, 2, 3, \ldots\}$

B. $\{0, 1, 2, 3, \ldots 37\}$

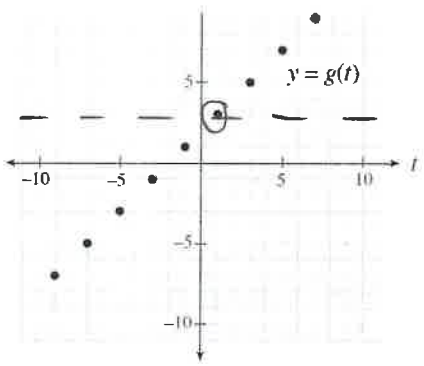
C. all real numbers

D. all real numbers between 0 and 37



5. Given the graph of $y = g(t)$ below, determine the value of t for which $g(t) = -3$. Answer as an integer.

y

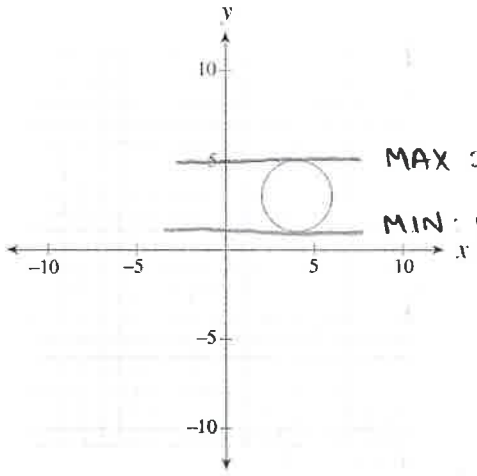


→ where does $y = 3$?

WHEN $t = 1$

t = 1

6. What is the range of the graph below?



MAX : y = 5

MIN : y = 1

[1, 5]

$1 \leq y \leq 5$

X	All x values between 2 and 6 inclusive.
X	(2, 6)
II.	[1, 5]
IV.	$1 \leq y \leq 5$

} Domain.

A. III only
B. IV only
C. I and II only
D. III and IV only

7. Which ordered pair represents $f(3) = -5$?

x y

(3, -5)

A. (–5, 3)
B. (–3, 5)
C. (3, –5)
D. (5, –3)

8. The cost C, in dollars, of renting a hall for the prom is given by the formula $C(n) = 500 + 4n$, where n is the number of students attending the prom. Calculate the cost of renting the hall if 70 students attend.

A. $108
B. $500
C. $780
D. $970

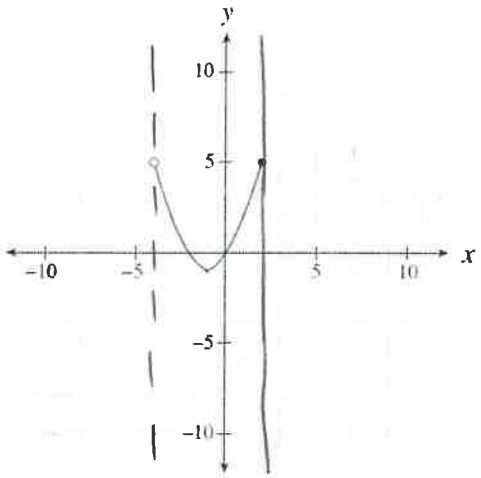
$C(n) = 500 + 4n$

$C(70) = 500 + 4(70)$

$= 500 + 280$

$= 780$

9. Determine the domain of the relation graphed below.



MIN = -4 MAX = 2
(not equal) (equal)

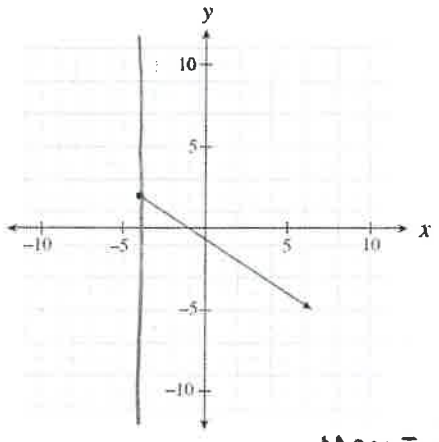
(A) $(-4, 2]$
B. $[-4, 2)$
C. $[-1, 5)$
D. $[-1, 5]$

$(-4, 2]$

10. Which of the following scenarios is **not** linear?

(A) the height of a football thrown over time
B. the total weight of a jar of pennies as more pennies are added
C. the distance travelled by a car moving at a constant speed over time
D. the pay of a truck driver who earns $2500 a month, plus $0.50 for every kilometre he drives

11. Determine the range of the linear relation graphed below.

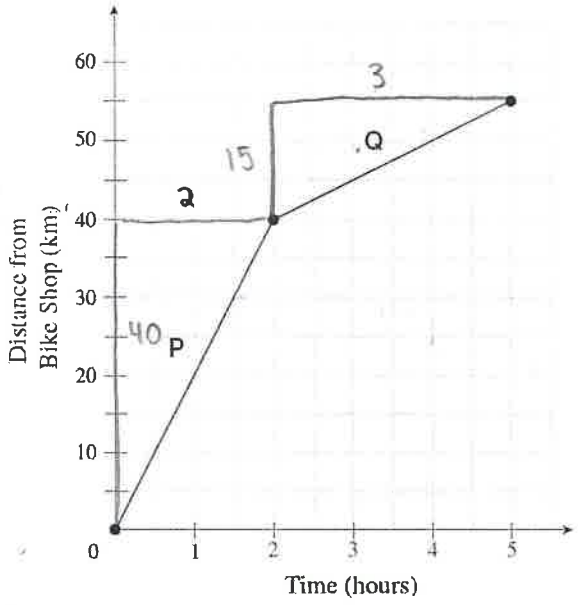


MIN = -4 MAX = ∞

A. $y \leq -4$
B. $y \leq 2$
(C) $y \geq -4$
D. $y \geq 2$

$y \geq -4$

12. The graph below models a bicycle's distance from a bike shop over time.



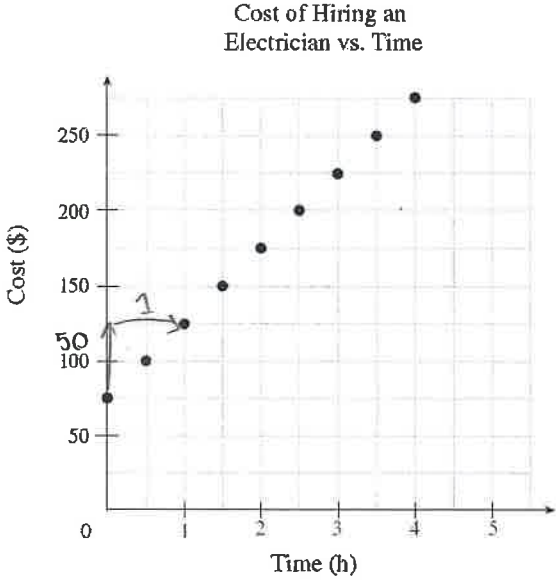
Calculate the change in the speed of the bike from segment P to segment Q.

(A) decreased by 15 km/h
B. decreased by 5 km/h
C. increased by 15 km/h
D. increased by 11 km/h

P: Speed $= \frac{40\text{ km}}{2\text{ h}} = 20\ \text{km/h}$

Q: Speed $= \frac{15\text{ km}}{3\text{ h}} = 5\ \text{km/h}$

13. **Use the graph below to answer question 13.**



13. What is the cost of hiring an electrician for 8 hours?

A. $550
(B.) $475
C. $400
D. $275

slope $= \frac{50}{1} = 50$

y-int = 75

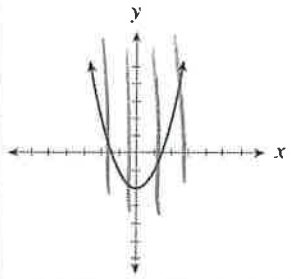
$C(t) = 50t + 75$

$C(8) = 50(8) + 75$

$= 400 + 75$

$= 475$

14. Which of the following relations are also functions?

X	$\{(0, 2), (1, 4), (3, 6), (4, 5), (4, 3), (7, -8)\}$
II.	$y = 2x + 5$ ✓ (line)
III.	The output is 6 more than half the input. ✓ (line)
IV.	 passes VLT ✓

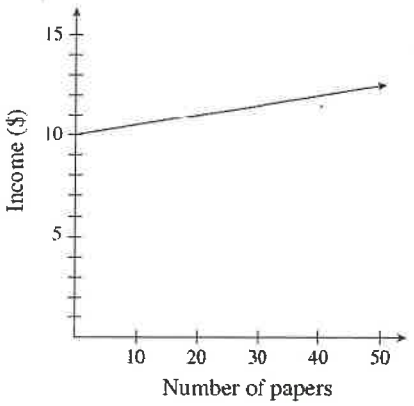
$y = \frac{1}{2}x + 6$

A. I only
B. I and IV only
C. II and III only
D. II, III and IV only

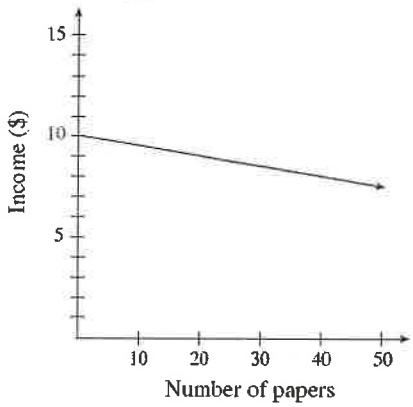
15. Jim delivers newspapers. He gets paid 10 dollars for every day of work, plus 5 cents for every paper he delivers. Which of the following graphs best represents Jim's possible income for one day?

($)
y-int = 10
slope = 0.05

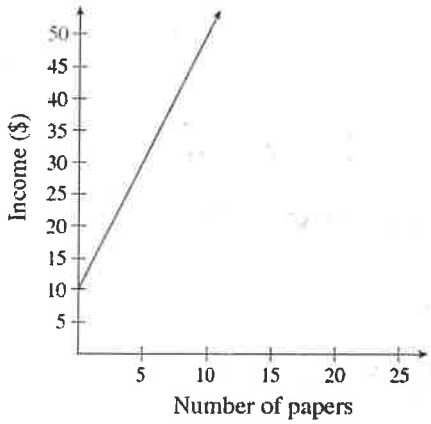
A.



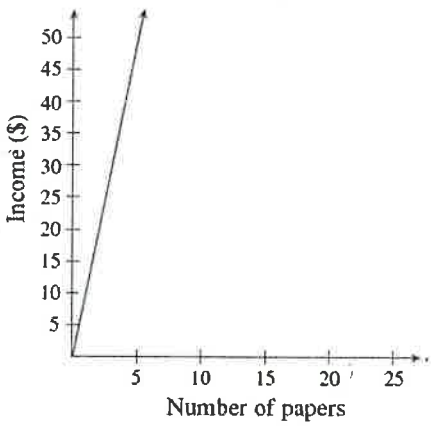
B.



C.

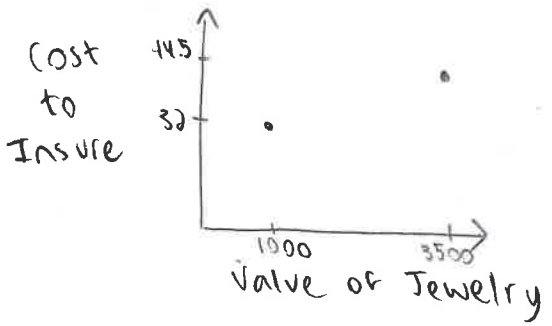


D.



16. The cost to insure jewellery is a fixed amount plus a percentage of the value of the jewellery. It costs $32 to insure $1000 worth of jewellery or $44.50 to insure $3500 worth of jewellery. What is the fixed amount to insure jewellery?

A. $27.00
B. $31.25
C. $44.65
D. $58.82



① rate of change $= \frac{\Delta \text{ dep variable}}{\Delta \text{ indep variable}}$

$= \frac{44.5 - 32}{3500 - 1000}$

$= \frac{12.5}{2500}$

$= 0.005$

② fixed cost: $y = 0.05x + b$ ← fixed cost

$32 = 0.05(1000) + b$ [plugged in $(\overset{x}{1000}, \overset{y}{32})$]

$32 = 5 + b$

$-5 \quad -5$

$b = 27$

OR USE SYSTEMS: Let x be the fixed cost
Let y be the variable cost

① $1000y + x = 32 \Rightarrow x = 32 - 1000y$

② $3500y + x = 44.50$

$3500y + (32 - 1000y) = 44.50$

$2500y + 32 - 32 = 44.50 - 32$

$\frac{2500y}{2500} = \frac{12.5}{2500}$

$y = 0.005$

$x = 32 - 1000y$

$= 32 - 1000(0.005)$

$= 27$

UNIT 5: Relations and Functions Written Response

1. Determine the domain and range of the relation.

   a) $(-1, 4), (-2, 4), (-3, 4)$

   Domain: $\{-1, -2, -3\}$

   Range: $\{4\}$

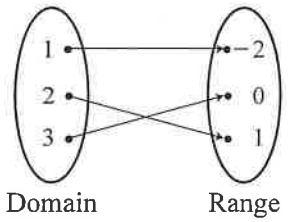
   b) $(4, -1), (4, -2), (4, -3)$

   Domain: $\{4\}$

   Range: $\{-1, -2, -3\}$

2. Write the mapping diagram in ordered pair notation.

   a)

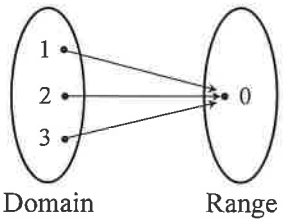
   

   $(1, -2)$

   $(2, 1)$

   $(3, 0)$

   b)

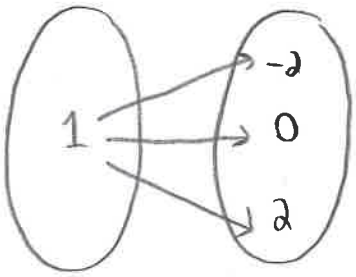
   $(1, 0)$

   $(2, 0)$

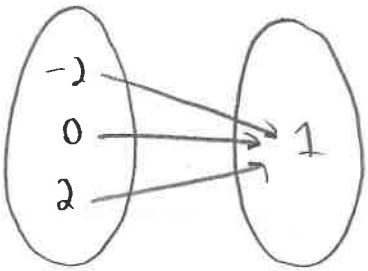
   $(3, 0)$

3. Draw a mapping diagram for the ordered pair.

   a) $(1, 2), (1, -2), (1, 0)$

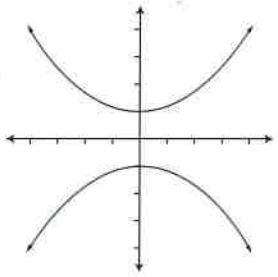
   

   b) $(2, 1), (-2, 1), (0, 1)$

4. Determine the domain and range. (in set notation)

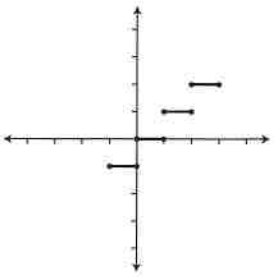
   a)

   Domain: $\{x \mid x \in \mathbb{R}\}$

   Range: $\{y \mid y \geq 1, y \leq -1, y \in \mathbb{R}\}$

   b)

   Domain: $\{x \mid -1 \leq x \leq 3, x \in \mathbb{R}\}$

   Range: $\{y \mid -1 \leq y \leq 2, y \in I\}$

   OR

   $\{y \mid y = -1, 0, 1, 2\}$

5. Are the sets of ordered pairs functions?

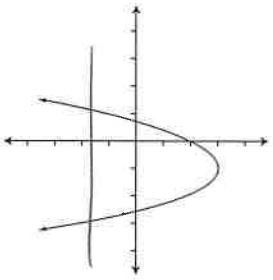
a) (2, 4), (2, 3), (2, 2) y / n

b) (4, 2), (3, 2), (2, 2) y / n

c) (2, 4), (4, 2), (1, 3), (3, 1) y / n

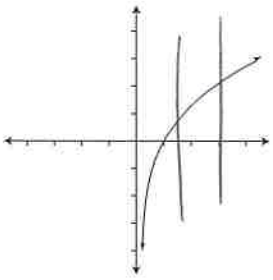
6. Apply the vertical line test to determine if the relation is a function.

a)



y / n

b)

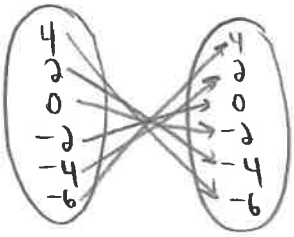


y / n

7. The table below defines a function.

x	4	2	0	−2	−4	−6
y	−6	−4	−2	0	2	4

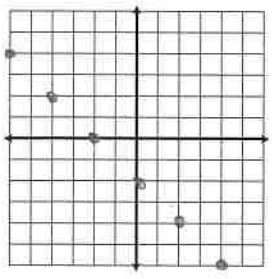
a) Express this function using mapping notation.



b) Express this function using ordered pair notation.

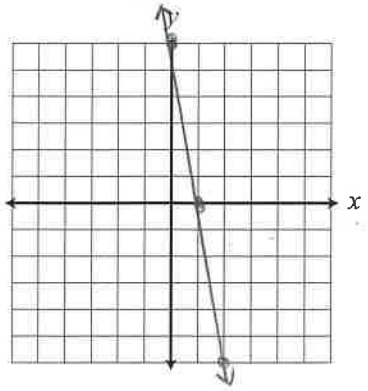
(4, -6), (2, -4), (0, -2), (-2, 0), (-4, 2), (-6, 4)

c) Graph this function.



8. Graph the linear equations, and determine which are functions.

a) $3x + \frac{1}{2}y = 3$



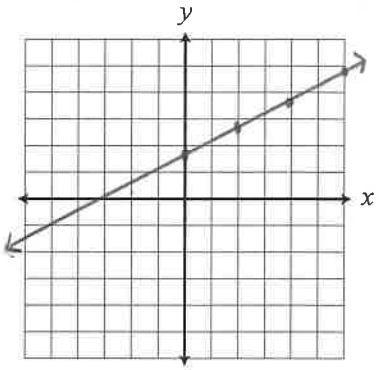
$\left(3x + \frac{1}{2}y = 3\right) \times 2$

$6x + y = 6$

$y = -6x + 6$ ← $b = +6$

↑ $m = -6/1$

b) $2x - 4y = -7$



$2x - 4y = -7$

$2x = -7 + 4y$

$2x + 7 = 4y$

$y = \frac{2x+7}{4} = \frac{1}{2}x + \frac{7}{4}$

↑ m ↑ b

$\left(\frac{7}{4} = 1\frac{3}{4}\right)$

9. A tool rental store charges \$20.00, plus \$5.00 per hour, to rent a rototiller.

a) Write a linear equation describing the relationship between the rental cost and the number of hours the rototiller is used.

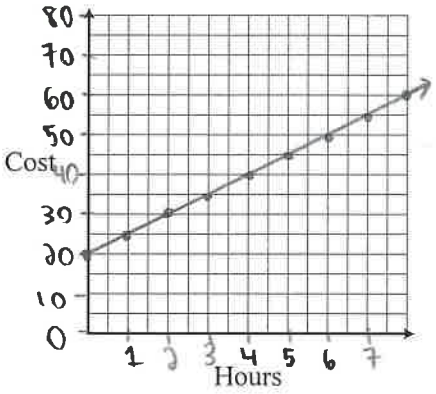
$Cost = 5 \times hours + 20$

↑ rate of change.

↑ fixed cost

$C(t) = 5t + 20$

b) Graph this relationship.



c) Determine the cost of renting the rototiller for 2.5 hours.

$C(t) = 5t + 20$

$C(2.5) = 5(2.5) + 20$

$= 12.5 + 20$

$= 32.5$

\$32.5

d) If the cost was \$37.50, how many hours was the rototiller used for?

$C(t) = 5t + 20$

$37.50 = 5t + 20$

$-20 \quad -20$

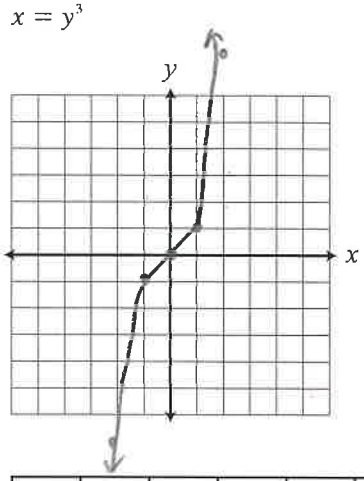
$\frac{17.50}{5} = \frac{5t}{5}$

$t = 3.5$

3.5h

10. Graph the non-linear equation, and determine if the relation is also a function by the vertical line test.

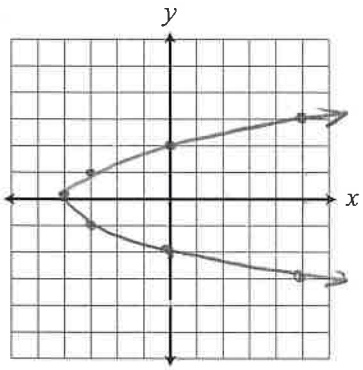
a) $x = y^3$



x	-27	-8	-1	0	1	8	27
y	-3	-2	-1	0	1	2	3

FUNCTION ✓

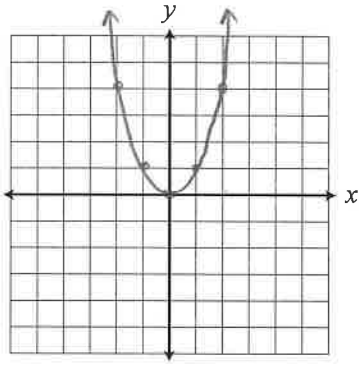
b) $x = y^2 - 4$



x	5	0	-3	-4	-3	0	5
y	-3	-2	-1	0	1	2	3

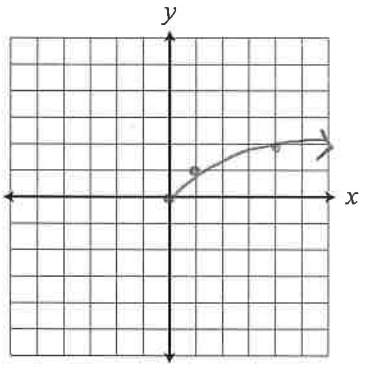
NOT A FUNCTION X

c) $y = x^2$



x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

d) $y = x^{1/2}$ $\quad y = \sqrt{x}$



x	0	1	4	9			
y	0	1	2	3			

11. For $f(x) = -3x - 2$, find:

a) $f(3)$

$f(3) = -3(3) - 2$

$= -9 - 2$

$= -11$

b) $f(-8)$

$f(-8) = -3(-8) - 2$

$= 24 - 2$

$= 22$

c) $f(x) = 3$

$3 = -3x - 2$

$+2 \quad +2$

$\frac{5}{-3} = \frac{-3x}{-3}$

$x = 5/-3$

d) $f(x) = -4$

$-4 = -3x - 2$

$+2 \quad +2$

$\frac{-2}{-3} = \frac{-3x}{-3}$

$x = 2/3$

e) $f(a)$

$f(a) = -3a - 2$

f) $f(x) = a$

$a = -3x - 2$

$+2$

$\frac{a+2}{-3} = \frac{-3x}{-3}$

$x = \frac{a+2}{-3}$

g) $f(x+h)$

$f(x+h) = -3(x+h) - 2$

$= -3x - 3h - 2$

h) $f(x) + f(h)$

$= -3x - 2 + (-3h - 2)$

$= -3x - 2 - 3h - 2$

$= -3x - 3h - 4$

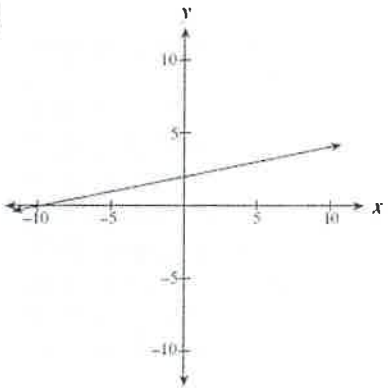
UNIT 6: SLOPE & LINEAR RELATIONS

MY NOTES AND THINGS TO REMEMBER...

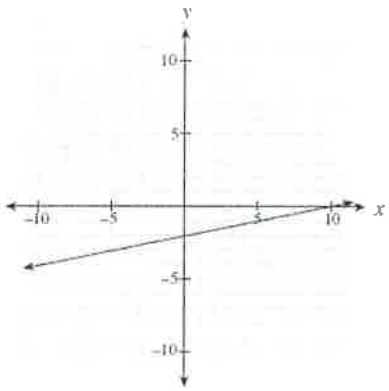
UNIT 6: Slope & Linear Relations Multiple Choice

1. Which graph represents the relation $x-5y+10=0$?

A.

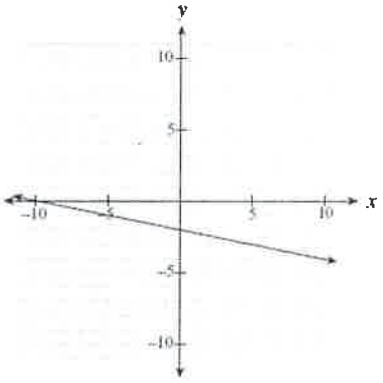


B.

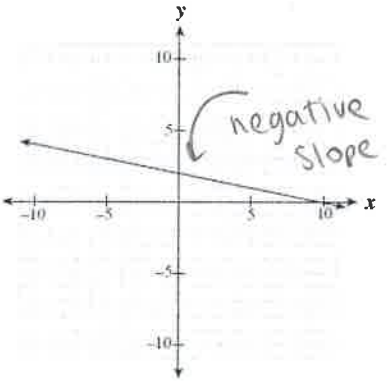


$x-5y+10=0$
$-5y \quad +5y$
$x+10=5y$
$\frac{x+10}{5}=\frac{5y}{5}$
$y=\frac{1}{5}x+2$

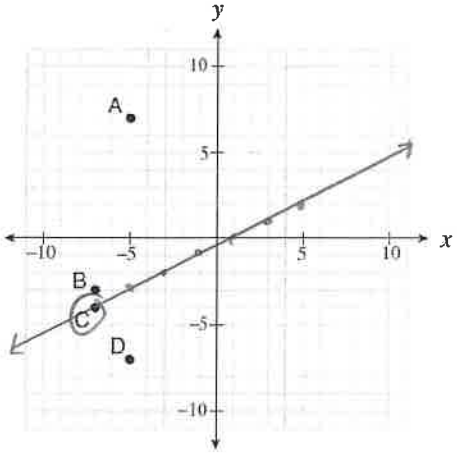
C.



D.



Use the following graph to answer question 2

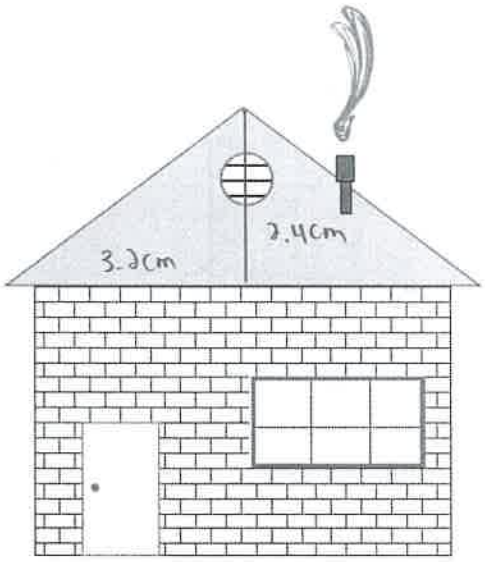


2. The line $y-2=\frac{1}{2}(x-5)$ passes through which point on the graph?

A. A
B. B
C. C
D. D

point (5, 2)
slope $\frac{1}{2}$

3. Use a ruler to determine the slope of the roof shown below.



Note: This diagram is drawn to scale.

rise = 2.4cm
run = 3.2cm

$\frac{\text{rise}}{\text{run}} = \frac{2.4}{3.2} = 0.75$

A. $\frac{3}{8} = 0.375$

B. $\frac{3}{4} = 0.75$

C. $\frac{4}{5} = 0.8$

D. $\frac{4}{3} = 1.\overline{3}$

4. Calculate the slope between the points $(7, -3)$ and $(4, 3)$.

(x_1, y_1) (x_2, y_2)

A. -2

B. $-\frac{1}{2}$

C. 2

D. 10

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-3)}{4 - 7} = \frac{6}{-3} = -2$

5. A line with an undefined slope passes through the points $(-2, 1)$ and (p, q). Which of the following points could be (p, q)?

A. $(1, 0)$

B. $(0, 1)$

C. $(0, -2)$

D. $(-2, 0)$

$x = -2$

m = undefined

6. Determine the slope of the linear relation $3x+5y+15=0$.

A. $\frac{5}{3}$

B. $\frac{3}{5}$

C. $-\frac{3}{5}$

D. $-\frac{5}{3}$

$-3x \quad +15 \qquad -3x \quad -15$

$\frac{5}{5}y = \frac{-3}{5}x - \frac{15}{5}$

$y = \left(\frac{-3}{5}\right)x - 3$

m

7. Determine the slope-intercept equation of the line that is parallel to $y=\frac{2}{5}x-3$ and passes through the point $(0, 5)$.

A. $y=-\frac{5}{2}x-3$

B. $y=-\frac{5}{2}x+5$

C. $y=\frac{2}{5}x+3$

D. $y=\frac{2}{5}x+5$

$x_1 \; y_1$

① $m = \frac{2}{5}$ & $(0,5)$

② $y-y_1 = m(x-x_1)$

$y-5 = \frac{2}{5}(x-0)$

$y-5 = \frac{2}{5}x$

$+5 \qquad +5$

$y = \frac{2}{5}x+5$

8. Lines A and B are perpendicular and have the same x-intercept. The equation of line A is $x+2y-4=0$. Determine the y-intercept of line B.

A. -8

B. -2

C. 4

D. 8

① SLOPE OF B:

A: $x+2y-4=0$

$-x \qquad +4 \quad -x+4$

$\frac{2}{2}y = \frac{-1}{2}x + \frac{4}{2}$

$y = \frac{-1}{2}x + 2$

m of B = +2

② X-INT OF B ($y=0$):

A: $x+2y-4=0$

$x+2(0)-4=0$

$x-4=0$

$+4 \quad +4$

$x=4$

x-int of B = +4 (4,0)

③ EQUATION OF B:

$y-y_1 = m(x-x_1)$

$y-0 = 2(x-4)$

$y = 2x-8$

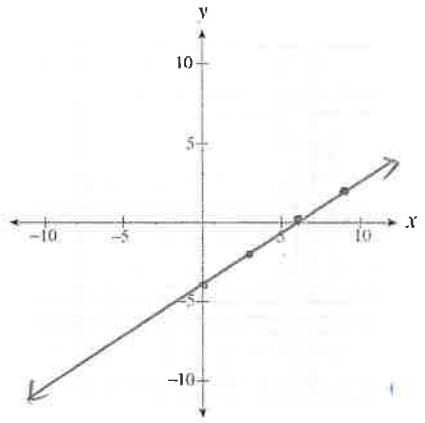
④ Y-INT OF B ($x=0$):

$y = 2x-8$

$y = 2(0)-8$

$y = -8$

The grid below may be used for rough work to answer question 9



9. A line has a slope of $\frac{2}{3}$ and passes through the point $(6,\ 0)$. Which of the following points must also be on the line?

A. $(-3,\ -6)$
B. $(3,\ 8)$
C. $(4,\ -3)$
D. $(9,\ 3)$

10. Rewrite $y = \frac{x}{5} - 6$ in general form.

A. $\frac{x}{5} - y - 6 = 0$
B. $x + 5y - 6 = 0$
C. $x - 5y - 30 = 0$
D. $5x - 5y - 30 = 0$

$y = \frac{x}{5} - 6 \quad \times 5$

$5y = x - 30$

$-5y \quad -5y$

$0 = x - 5y - 30$

11. Given the equation $Ax + By + C = 0$, which of the following conditions must be true for the graph of the line to have a positive slope and a positive y-intercept?

A. $A > 0,\ B > 0,\ C > 0$
B. $A > 0,\ B < 0,\ C > 0$
C. $A > 0,\ B > 0,\ C < 0$
D. $A > 0,\ B < 0,\ C < 0$

$Ax + By + C = 0$

$-Ax \quad -C \quad -Ax - C$

$By = -Ax - C$

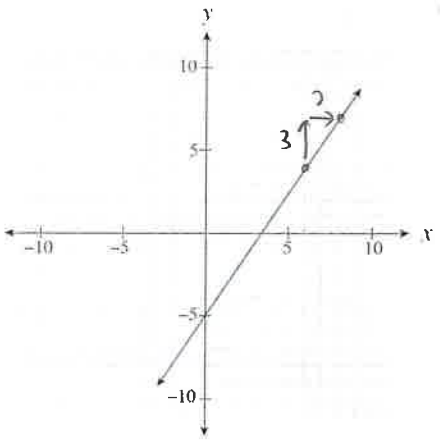
$\frac{By}{B} = \frac{-Ax}{B} - \frac{C}{B}$

$y = \frac{-A}{B}x - \frac{C}{B}$

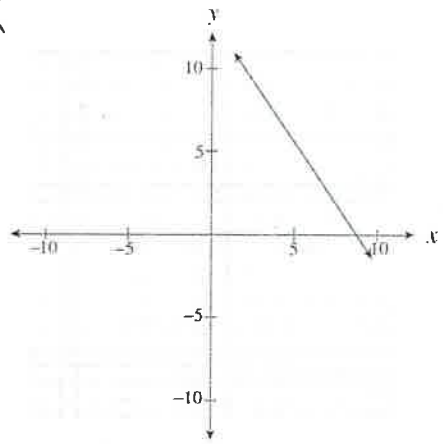
12. Which of the following graphs represents a line that passes through $(6, 4)$ and is perpendicular to $y = -\frac{2}{3}x$?

$m = +\frac{3}{2}$

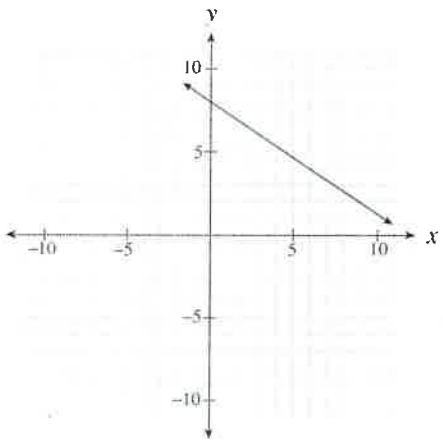
A.



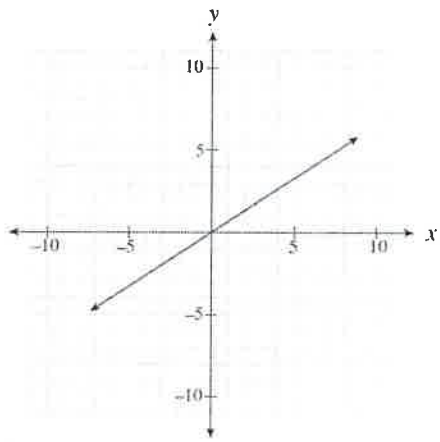
B



C



D.



13. Determine the slope-intercept form of the line that passes through the point $(-4, 3)$ and is parallel to the line segment that joins $A(-1, -5)$ and $B(-3, 1)$.

A. $y = -3x - 9$
B. $y = -3x + 5$
C. $y = -3x + 15$
D. $y = 3x + 15$

① SLOPE: $\frac{y_2 - y_1}{x_2 - x_1}$

A $(-1, -5)$ x_1 y_1

B $(-3, 1)$ x_2 y_2

$= \frac{1 + (5)}{-3 + (1)}$

$= \frac{6}{-2}$

$= -3$

m of line $= -3$

x_1 y_1

$(-4, 3)$ -3 -4

② $y - y_1 = m(x - x_1)$

$y - 3 = -3(x + 4)$

$y - 3 = -3x - 12$

$+3 \quad +3$

$y = -3x - 9$

14. Which of the following statements are true for $2x + 3y = 6$?

X	The y-intercept is -2.
X	The line is parallel to $y = 2x$.
X	The slope-intercept form of the line is $y = \frac{2}{3}x + 2$.
IV.	The range is all real numbers. ✓ (line)

A. IV only
B. I and II only
C. I and IV only
D. III and IV only

$x=0$

I: $2x + 3y = 6$

$2(0) + 3y = 6$

$\frac{3y}{3} = \frac{6}{3}$

$y = 2$

II: $2x + 3y = 6$

III: $-2x \quad -2x$

$\frac{3y}{3} = \frac{-2x + 6}{3}$

$y = -\frac{2}{3}x + 2$

15. A hot-dog stand owner makes a profit of \$100 when he sells 90 hot dogs a day. He has a loss of \$30 when he sells 25 hot dogs a day. Which linear relation represents his profit?

all positive y-value for ②

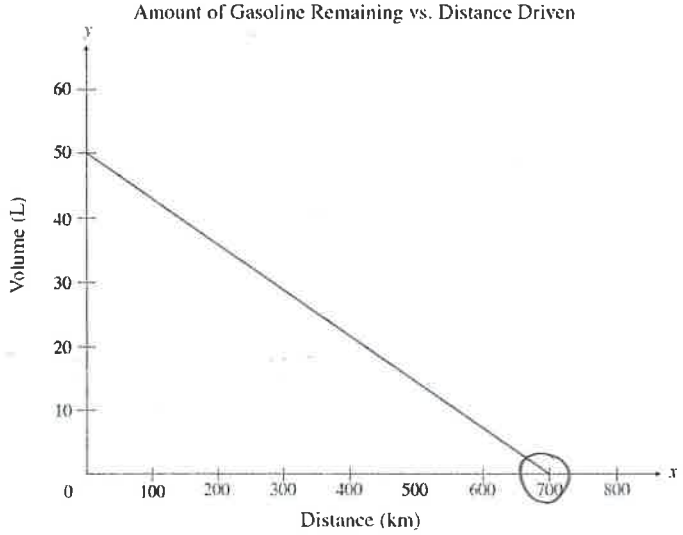
X. $y = 0.5x + 55$
B. $y = 1.08x + 3.08$
X. $y = 1.11x$
D. $y = 2x - 80$

① (90, 100)
② (25, -30)

See which equation is true for BOTH ① & ②

Use the following graph to answer question 16

Amount of Gasoline Remaining vs. Distance Driven



16. The graph above shows the relationship between the amount of gasoline remaining in a 50 L tank and the distance driven for a certain car.

What does the x-intercept represent in this situation?

A. fuel capacity of the gasoline tank
B. total distance travelled during a long trip
C. total distance driven until the car is out of gas
D. number of kilometres driven per litre of gasoline

(AB is parallel to CD)

17. The slope of AB is $-\frac{2}{3}$. The slope of CD is $\frac{w}{24}$. Given AB $\parallel$ CD, determine the value of w.
Answer as an integer.

Parallel $\Rightarrow$ EQUAL SLOPES :

$-\frac{2}{3} = \frac{w}{24}$

$3w = -48 \Rightarrow \boxed{w = -16}$

18. Determine the equation of a line, in slope-intercept form, that passes through the points $(6, 1)$ and $(-10, 9)$.

$x_1\ y_1 \quad x_2\ y_2$

A. $y = -\frac{1}{2}x + 4$

B. $y = -\frac{1}{2}x - 2$

C. $y = -2x + 8$

D. $y = -2x + 13$

① $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-1}{-10-6} = \frac{8}{-16} = -\frac{1}{2}$

② $y - y_1 = m(x - x_1)$

$\left(y - 1 = -\frac{1}{2}(x-6)\right) \times 2$

$2y - 2 = -1(x-6)$

$2y - 2 = -x + 6$

$+2 \quad +2$

$\frac{2}{2}y = \frac{-x+8}{2}$

$y = -\frac{1}{2}x + 4$

19. Which of the following lines have a negative slope?

I.	$y + 3 = 0$
II.	$2x + y = 6$
III.	$(y+2) = -4(x-5)$

A. II only
B. III only
C. I and III only
D. II and III only

I: $y + 3 = 0$

$-3 \quad -3$

$y = -3$

HORIZONTAL $\Rightarrow m = 0$ X

II: $2x + y = 6$

$-2x \quad -2x$

$y = -2x + 6$

$\uparrow$

$m = -2$ ✓

III: $(y+2) = -4(x-5)$

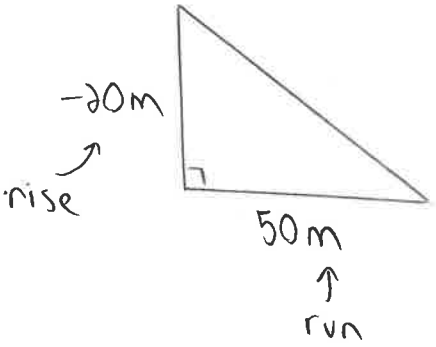
$y + 2 = -4x + 20$

-2

$y = -4x + 18$

$\uparrow m = -4$ ✓

20. A waterslide descends 20 m over a horizontal distance of 50 m. What is the slope of the waterslide? Answer, with a positive value, to the nearest tenth.



$$m = \frac{\text{rise}}{\text{run}} = \frac{-20}{50} = -0.4$$

$\boxed{m = 0.4}$

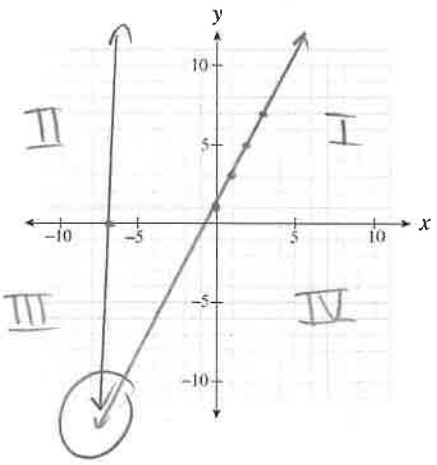
21. In which quadrant do the graphs of $x = -7$ and $y = 2x + 1$ intersect?

A. Quadrant I
B. Quadrant II
C. Quadrant III
D. Quadrant IV

① $x = -7$

VERTICAL LINE @ $x = -7$

② $y = 2x + 1$, $m = \frac{2}{1}$, $b = +1$



22. Which of the following coordinates are intercepts of the linear relation $2x - 3y + 30 = 0$?

I.	$(0, 10)$
~~II.~~	$\left(0, \frac{2}{3}\right)$
~~III.~~	$(-10, 0)$
IV.	$(-15, 0)$

A. I only
B. I and IV only
C. II and III only
D. II and IV only

① x-int ($y=0$):

$2x - 3(0) + 30 = 0$

$2x + 30 = 0$

$2x = -30$

$x = -15$

② y-int ($x=0$):

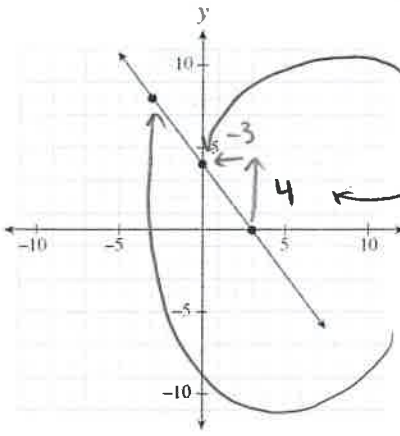
$2(0) - 3y + 30 = 0$

$-3y + 30 = 0$

$-3y = -30$

$y = 10$

Use the following graph to answer question 23



$= 4$

$m = -\frac{4}{3}$

point $= (-3, 8)$

23. Which of the following equations describes the linear relation graphed above?

~~I.~~	$y = -\frac{4}{3}x + 4$
II.	$y - 8 = -\frac{4}{3}(x+3)$ ✓
III.	$4x + 3y - 12 = 0$

A. II only
B. I and II only
C. I and III only
D. II and III only

$\left(y = -\frac{4}{3}x + 4\right) \times 3$

$3y = -4x + 12$

$+4x \quad -12 \quad +4x \quad -12$

$4x + 3y - 12 = 0$

24. Kelly explained her method for graphing the linear relation $y = -\frac{2}{3}x + 7$ as follows:

Steps	
I.	Place a dot on the y-axis at positive 7. ✓
II.	Move up two on the y-axis to positive 9. ✓ ↑
III.	From the positive 9, move to the left three spots and place a dot there. ✓ ←
IV.	Draw a line through the two dots. ✓

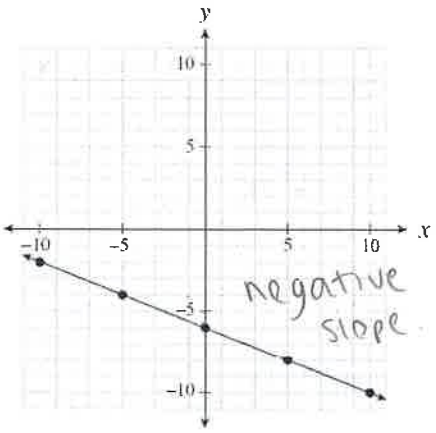
negative ✓ slope

Where did Kelly make the first mistake in her explanation?

A. Step I
B. Step II
C. Step III
D. There is no mistake.

25. Which of the following relations could be produced by $y = \frac{2}{5}x - 6$?

check all points

I.	$2x - 5y - 30 = 0$
II.	$\{(15, 0), (10, -2), (-5, -8), (-10, -10)\}$
~~III.~~	

$\left(y = \frac{2}{5}x - 6\right) \times 5$

$5y = 2x - 30$

$-5y \quad -5y$

$0 = 2x - 5y - 30$

A. I only
B. II only
C. I and II only
D. I, II and III

UNIT 6: Slope & Linear Relations Written Response

1. Find the slope of the line containing each pair of points.

a) $(-2, 5)$ and $(4, -3)$ $\quad -4/3$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-5}{4-(-2)} = \frac{-8}{6} = \frac{-4}{3}$

b) $(6, -2)$ and $(-4, -3)$ $\quad 1/10$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-(-2)}{-4-6} = \frac{-1}{-10} = \frac{1}{10}$

c) $(3, 1)$ and $(-4, 6)$ $\quad -5/7$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-1}{-4-3} = \frac{5}{-7}$

d) $(a, -b)$ and $(-b, a)$ $\quad -1$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{a-(-b)}{-b-a} = \frac{a+b}{-b-a}$

$= \frac{a+b}{-(b+a)} = -1$

TRICKY!

e) $(-3, 0)$ and $(-3, 4)$ $\quad$ undefined

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{-3-(-3)} = \frac{4}{0}$

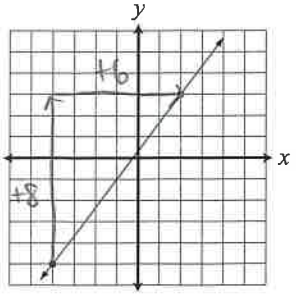
= undefined

f) $(4, -1)$ and $(-2, -1)$ $\quad 0$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-(-1)}{-2-4} = \frac{0}{-6} = 0$

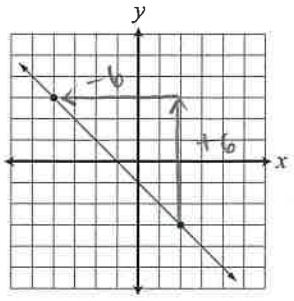
2. Determine the slope.

a)



$m = \frac{\text{rise}}{\text{run}} = \frac{8}{6} = \frac{4}{3}$

b)

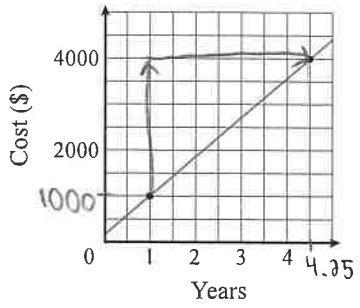


$m = \frac{\text{rise}}{\text{run}} = \frac{6}{-6} = -1$

slope!

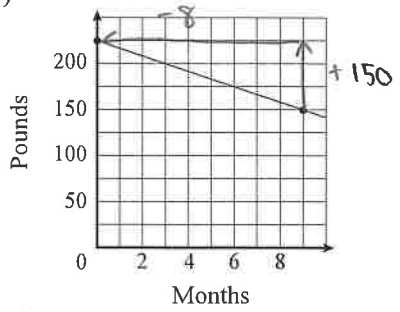
3. Determine the rate of change.

a) Car Repairs



$m = \frac{\text{rise}}{\text{run}} = \frac{3000}{3.25} = 923.08\ \$/\text{year}$.

b) Weight Loss



$m = \frac{\text{rise}}{\text{run}} = \frac{+150}{-8} = -18.75\ \frac{\text{pounds}}{\text{month}}$

4. George rents a motor scooter for three hours to travel around Crescent Beach. It cost him $36.00 for travelling 42 km.

a) Determine George's average speed in km/h.

$\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{42\text{ km}}{3\text{ hr}} = \boxed{14\ \text{km/hr}}$

b) Determine the rental rate in dollars per hour.

$\frac{\$}{\text{hr}} = \frac{36.00}{3} = \boxed{12\ \$/\text{hr}}$

c) Determine the rental rate in cents per km.

$\frac{¢}{\text{hr}} = \frac{36.00 \times 100}{3} = \boxed{1200\ ¢/\text{hr}}$

5. Marelee rented a stall at a craft market for four hours at a cost of $120. She sold $600 worth of pottery.

a) Determine her rental cost per hour.

$\frac{\$}{\text{hr}} = \frac{120}{4} = \boxed{30\ \$/\text{hr}}$

b) Determine her average sales per hour.

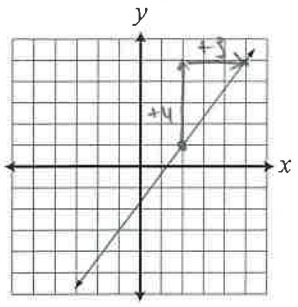
$\frac{\text{sales}}{\text{hr}} = \frac{600}{4} = \boxed{150\ \$/\text{hr sold}}$

c) Determine her average profit per hour.

$\frac{\text{profit}}{\text{hr}} = \frac{600-120}{4} = \frac{480}{4} = \boxed{120\ \$/\text{hr}}$

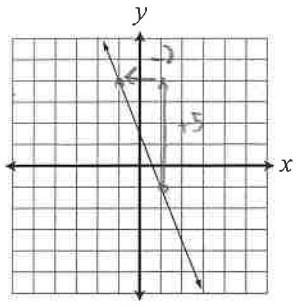
6. Determine the slope.

a)



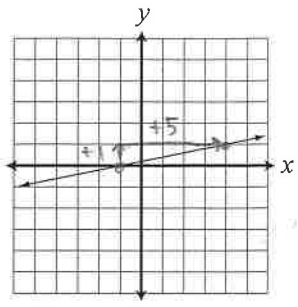
$m = \frac{4}{3}$

b)



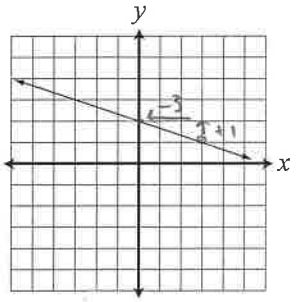
$m = \frac{5}{-2}$

c)



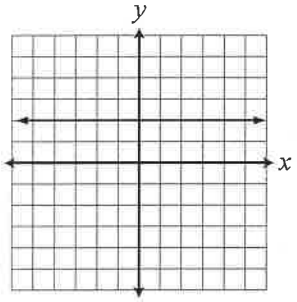
$m = \frac{1}{5}$

d)



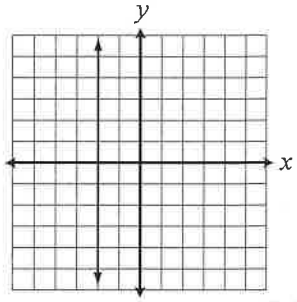
$m = \frac{1}{-3}$

e)



$m = 0$

f)



$m = \text{undefined}$

7. Find the number n, so that the line passing through the point $(-3, 5)$ and $(-4, n)$ has slope 3.

$x_1, y_1 \quad x_2, y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$3 = \frac{n-5}{-4-(-3)}$$

$$3 = \frac{n-5}{-4+3}$$

$$\frac{3}{1} \times \frac{n-5}{-1}$$

$$n - 5 = -3$$
$$+5 \quad +5$$

$$\boxed{n = 2}$$

8. The line through the point $(8, y)$ and $(2, -3)$ has a slope parallel to a line with x-intercept 3 and y-intercept -1. Determine y.

$x_1, y_1 \quad x_2, y_2$

$$① \quad m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-1-0}{0-3}$$

$$m = \frac{-1}{-3} = \frac{1}{3} \text{ (parallel slope)}$$

x_1, y_1

$x\text{-int} = 3 \Rightarrow (3, 0)$

$y\text{-int} = -1 \Rightarrow (0, -1)$

x_2, y_2

$$② \quad m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{1}{3} = \frac{-3-y}{2-8}$$

$$\frac{1}{3} \times \frac{-3-y}{-6}$$

$$-6 = 3(-3-y)$$

$$-6 = -9 - 3y$$
$$+9 \quad +9$$

$$\frac{3}{-3} = \frac{-3y}{-3}$$

$$\boxed{y = -1}$$

9. Find the value of c so that the line through the points $(-2, -4)$ and $(-1, -1)$ is parallel to the line through the points $(6, -2)$ and $(3, c)$.

① $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-1 - (-4)}{-1 - (-2)}$

$= \frac{3}{1}$

$= 3$

parallel slope $= 3$

② $m = \frac{y_2 - y_1}{x_2 - x_1}$

$3 = \frac{c - (-2)}{3 - 6}$

$\frac{3}{1} = \frac{c+2}{-3}$

$-9 = c + 2$

$-2 \quad -2$

$c = -11$

10. Find the value of c so that the line through the points $(-2, -4)$ and $(-1, -1)$ is perpendicular to the line through the points $(6, -2)$ and $(3, c)$.

① $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-1 - (-4)}{-1 - (-2)}$

$= \frac{3}{1}$

perp slope $= -\frac{1}{3}$

② $m = \frac{y_2 - y_1}{x_2 - x_1}$

$-\frac{1}{3} = \frac{c - (-2)}{3 - 6}$

$-\frac{1}{3} = \frac{c+2}{-3}$

$3 = 3(c+2)$

$3 = 3c + 6$

$-6 \quad -6$

$-3 = 3c$

$\Rightarrow c = -1$

11. Find the value of c so that the line through the points $(0, 3)$ and $(-1, 0)$ is parallel to the line through the points $(c, 1)$ and $(-2, 3)$.

① $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{0 - 3}{-1 - 0}$

$= \frac{-3}{-1}$

$= 3$

Parallel slope $= 3$

② $m = \frac{y_2 - y_1}{x_2 - x_1}$

$3 = \frac{3 - 1}{-2 - c}$

$\frac{3}{1} = \frac{2}{-2 - c}$

$-6 - 3c = 1$

$+6 \quad +6$

$-3c = 7$

$c = -\frac{7}{3}$

12. Find the value of c so that the line through the points $(0, 3)$ and $(-1, 0)$ is perpendicular to the line through the points $(c, 1)$ and $(-2, 3)$.

① $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{0 - 3}{-1 - 0}$

$= \frac{-3}{-1}$

$= 3$

Perp. Slope

$= -\frac{1}{3}$

② $m = \frac{y_2 - y_1}{x_2 - x_1}$

$-\frac{1}{3} = \frac{3 - 1}{-2 - c}$

$-\frac{1}{3} = \frac{2}{-2 - c}$

$6 = 2 + c$

$-2 \quad -2$

$c = 4$

12. Find the slope and y-intercept.

a) $2x - 5y = 7$

$-2x \quad -2x$

$\frac{-5y}{-5} = \frac{-2x}{-5} + \frac{7}{-5}$

$y = \frac{2}{5}x - \frac{7}{5}$

slope $\frac{2}{5}$

y-intercept $-\frac{7}{5}$

b) $5x + y = -2$

$-5x \quad -5x$

$y = -5x - 2$

slope -5

y-intercept -2

13. Write the equation in slope-intercept form.

a) $6x - y = 3$

$+y \quad +y$

$6x = 3 + y$

$-3 \quad -3$

$y = 6x - 3$

b) $2x + 5y = 7$

$-2x \quad -2x$

$\frac{5y}{5} = \frac{-2x}{5} + \frac{7}{5}$

$y = -\frac{2}{5}x + \frac{7}{5}$

14. Write the slope-intercept equation in general form.

a) $\left(y = -\frac{2}{3}x + 4\right) \times 3$

$3y = -2x + 12$

$+2x \quad -12 \quad +2x \quad -12$

$2x + 3y - 12 = 0$

b) $\left(y = -3x + \frac{2}{5}\right) \times 5$

$5y = -15x + 2$

$+15x \quad -2 \quad +15x \quad -2$

$15x + 5y - 2 = 0$

15. Write the point-slope equation in slope-intercept form.

a) $\left(y + 1 = -\frac{2}{3}(x - 4)\right) \times 3$

$3y + 3 = -2(x - 4)$

$3y + 3 = -2x + 8$

$-3 \quad -3$

$\frac{3}{3}y = -\frac{2}{3}x + \frac{5}{3}$

$y = -2/3\,x + 5/3$

b) $\left(y - \frac{2}{3} = -4\left(x + \frac{1}{2}\right)\right) \times 3$

$3y - 2 = -12\left(x + \frac{1}{2}\right)$

$3y - 2 = -12x - 6$

$+2 \quad +2$

$\frac{3}{3}y = -\frac{12}{3}x - \frac{4}{3}$

$y = -4x - 4/3$

16. Write the point-slope equation in general form.

a) $\left(y + 1 = -\frac{2}{3}(x - 4)\right) \times 3$

$3y + 3 = -2(x - 4)$

$3y + 3 = -2x + 8$

$+2x \quad -8 \quad +2x \quad -8$

$2x + 3y - 5 = 0$

b) $\left(y - \frac{2}{3} = -4\left(x + \frac{1}{2}\right)\right) \times 3$

$3y - 2 = -12\left(x + \frac{1}{2}\right)$

$3y - 2 = -12x - 6$

$+12x \quad +6 \quad +12x \quad +6$

$12x + 3y + 4 = 0$

17. Write the equation of each line in general form.

a) $(0, -3)$, $m = -4$ (x_1 y_1)

$y - y_1 = m(x - x_1)$

$y + 3 = -4(x - 0)$

$y + 3 = -4x$

$+4x \quad +4x$

$4x + y + 3 = 0$

b) $(2, 0)$, $m = -\frac{1}{3}$ (x_1 y_1)

$y - y_1 = m(x - x_1)$

$y - 0 = -\frac{1}{3}(x - 2)$

$\left(y = -\frac{1}{3}x + \frac{2}{3}\right) \times 3$

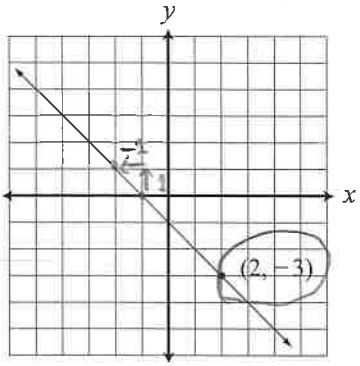
$3y = -x + 2$

$+x \quad -2 \quad +x \quad -2$

$x + 3y - 2 = 0$

18. Determine the equation in: general form, slope-intercept form and point-slope form.

a)



$m = \frac{1}{-1} = -1$

① general form $x + y + 1 = 0$

② slope-intercept form $y = -x - 1$

③ point-slope form $y + 3 = -1(x - 2)$

$y - y_1 = m(x - x_1)$

$y + 3 = -1(x - 2)$ ③

$y + 3 = -1x + 2$

$-3 \quad -3$

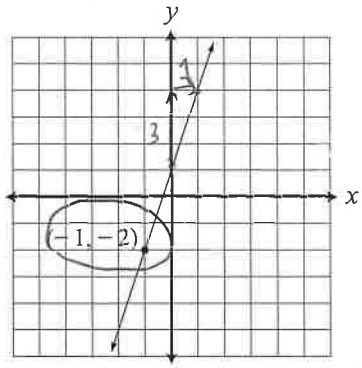
$y = -1x - 1$ ②

$y = -x - 1$

$+x \quad +1 \quad +x \quad +1$

$x + y + 1 = 0$ ①

b)



$m = \frac{3}{1} = 3$

① general form $0 = 3x - y + 1$

② slope-intercept form $y = 3x + 1$

③ point-slope form $y + 2 = 3(x + 1)$

$y - y_1 = m(x - x_1)$

$y + 2 = 3(x + 1)$ ③

$y + 2 = 3x + 3$

$-2 \quad -2$

$y = 3x + 1$ ②

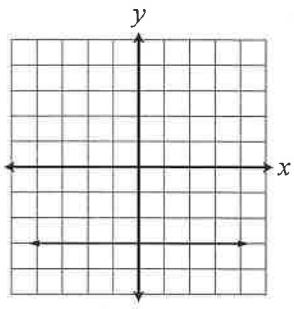
$y = 3x + 1$

$-y \quad -y$

$0 = 3x - y + 1$ ①

19. Determine the equation of the graph.

a)

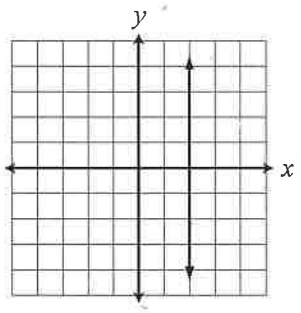


y constant

@ −3

$y = -3$

b)



x constant

@ 2

$x = 2$

20. Write the equation of the line with the given characteristics.

a) vertical, passes through $(-2, 5)$

$x = -2$

b) horizontal, passes through $(-2, 5)$

$y = 5$

c) vertical, passes through (a, b)

$x = a$

d) horizontal, passes through (a, b)

$y = b$

21. For each pair of equations, determine whether the lines are parallel, perpendicular or neither parallel nor perpendicular.

a) ① $3x + 2y = 7$
② $4x + 6y = 2$

NEITHER

① $3x + 2y = 7$
$-3x \quad -3x$
$\frac{2}{2}y = \frac{-3}{2}x + \frac{7}{2}$
$y = \frac{-3}{2}x + \frac{7}{2}$

② $4x + 6y = 2$
$-4x \quad -4x$
$\frac{6}{6}y = \frac{-4}{6}x + \frac{2}{6}$
$y = \frac{-2}{3}x + \frac{1}{3}$

b) ① $5x - 2y = 4$
② $4x + 10y = 3$

PERPENDICULAR

① $5x - 2y = 4$
$-5x \quad -5x$
$\frac{-2}{-2}y = \frac{-5}{-2}x + \frac{4}{-2}$
$y = \frac{5}{2}x - 2$

② $4x + 10y = 3$
$-4x \quad -4x$
$\frac{10}{10}y = \frac{-4}{10}x + \frac{3}{10}$
$y = \frac{-2}{5}x + \frac{3}{10}$

c) ① $y = 2x - 3$
② $2x + y = -3$

NEITHER

① $y = 2x - 3$

② $2x + y = -3$
$-2x \quad -2x$
$y = -2x - 3$

d) ① $3x - y = 2$
② $6x - 2y = 2$

PARALLEL

① $3x - y = 2$
$-3x \quad -3x$
$\frac{-1}{-1}y = \frac{-3}{-1}x + \frac{2}{-1}$
$y = 3x - 2$

② $6x - 2y = 2$
$-6x \quad -6x$
$\frac{-2}{-2}y = \frac{-6}{-2}x + \frac{2}{-2}$
$y = 3x - 1$

22. Write the equation of the line passing through the given set of points in general form.

a) $(-3, 1)$ and $(-4, -6)$ (x_1, y_1 and x_2, y_2)

① $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-1}{-4-(-3)} = \frac{-7}{-1} = 7$

② $y - y_1 = m(x - x_1)$
$y - 1 = 7(x + 3)$
$y - 1 = 7x + 21$
$-y \quad +1 \quad -y \quad +1$
$0 = 7x - y + 22$

b) $(-2, -3)$ and $(-5, -1)$ (x_1, y_1 and x_2, y_2)

① $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1+3}{-5+2} = \frac{2}{-3}$

② $y - y_1 = m(x - x_1)$
$\left(y + 3 = -\frac{2}{3}(x + 2)\right) \times 3$
$3y + 9 = -2(x + 2)$
$3y + 9 = -2x - 4$
$+2x \quad +4 \quad +2x \quad +4$
$2x + 3y + 13 = 0$

23. Find the slopes of lines parallel and perpendicular to the following equations.

a) $3x - 4y = -6$

$m_{\parallel}$ 3/4 $m_{\perp}$ -4/3

$-3x \quad -3x$

$\frac{-4y}{-4} = \frac{-3x}{-4} - \frac{6}{-4}$

$y = \left(\frac{3}{4}\right)x + \frac{3}{2}$

$m = \frac{3}{4}$

b) $x = 3y + 2$

$m_{\parallel}$ 1/3 $m_{\perp}$ -3

$-2 \quad -2$

$\frac{x}{3} - \frac{2}{3} = \frac{3y}{3}$

$y = \left(\frac{1}{3}\right)x - \frac{2}{3}$

$m = \frac{1}{3}$

24. Find the equation of the line that passes through the given point and is parallel to the given line. in general form

a) $P(-2, 4)$; $2x - 3y = 5$ (x_1, y_1)

① Find m: $2x - 3y = 5$

$-2x \quad -2x$

$-3y = -2x + 5$

$y = \left(\frac{2}{3}\right)x - \frac{5}{3}$

$m_{\parallel} = \frac{2}{3}$

② $y - y_1 = m(x - x_1)$

$\left(y - 4 = \frac{2}{3}(x+2)\right) \times 3$

$3y - 12 = 2(x+2)$

$3y - 12 = 2x + 4$

$-3y + 12 \quad -3y + 12$

$0 = 2x - 3y + 16$

b) $P(4, -1)$; $4x + 7y = -2$ (x_1, y_1)

① Find m: $4x + 7y = -2$

$-4x \quad -4x$

$\frac{7y}{7} = \frac{-4x}{7} - \frac{2}{7}$

$y = \left(\frac{-4}{7}\right)x - \frac{2}{7}$

$m_{\parallel} = \frac{-4}{7}$

② $y - y_1 = m(x - x_1)$

$\left(y + 1 = \frac{-4}{7}(x-4)\right) \times 7$

$7y + 7 = -4(x-4)$

$7y - 7 = -4x + 16$

$+4x - 16 \quad +4x - 16$

$4x + 7y - 23 = 0$

25. Find the equation of the line that passes through the given point and is perpendicular to the given line in slope-intercept form.

a) $P(-2, 4)$; $2x - 3y = 5$ (x_1, y_1)

① $y = \frac{2}{3}x - \frac{5}{3}$ (see 24a)

$m_{\perp} = -\frac{3}{2}$

② $y - y_1 = m(x - x_1)$

$\left(y - 4 = -\frac{3}{2}(x+2)\right) \times 2$

$2y - 8 = -3(x+2)$

$2y - 8 = -3x - 6$

$+8 \quad +8$

$\frac{2y}{2} = \frac{-3}{2}x + \frac{2}{2}$ ⇒ $y = \frac{-3}{2}x + 1$

b) $P(4, -1)$; $4x + 7y = -2$ (x_1, y_1)

① $y = \frac{-4}{7}x - \frac{2}{7}$ (see 24b)

$m_{\perp} = \frac{7}{4}$

② $y - y_1 = m(x - x_1)$

$\left(y + 1 = \frac{7}{4}(x-4)\right) \times 4$

$4y + 4 = 7(x-4)$

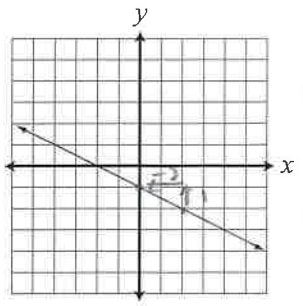
$4y + 4 = 7x - 28$

$-4 \quad -4$

$\frac{4y}{4} = \frac{7x}{4} - \frac{32}{4}$ ⇒ $y = \frac{7}{4}x - 8$

26. Determine the equation of a line, in general form which is parallel to the line and which goes through the given point.

a) $(5, 2)$ $x_1\ y_1$



$m = \frac{1}{-2}$

$m_{\parallel} = \frac{-1}{2}$

$y - y_1 = m(x - x_1)$

$\left(y - 2 = \frac{-1}{2}(x - 5)\right) \times 2$

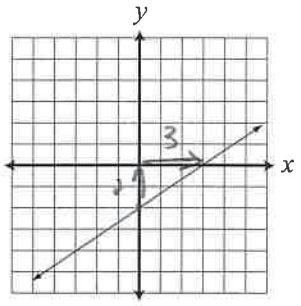
$2y - 4 = -1(x - 5)$

$2y - 4 = -x + 5$

$+x \quad -5 \qquad +x \quad +5$

$x + 2y - 9 = 0$

b) $(-3, 4)$ $x_1\ y_1$



$m = \frac{2}{3}$

$m_{\parallel} = \frac{2}{3}$

$y - y_1 = m(x - x_1)$

$\left(y - 4 = \frac{2}{3}(x + 3)\right) \times 3$

$3y - 12 = 2(x + 3)$

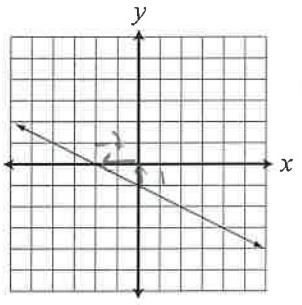
$3y - 12 = 2x + 6$

$-3y \quad +12 \qquad -3y + 12$

$0 = 2x - 3y + 18$

27. Determine the equation of a line, in general form which is perpendicular to the line and which goes through the given point.

a) $(5, 2)$ $x_1\ y_1$



$m = \frac{1}{-2}$

$m_{\perp} = 2$

$y - y_1 = m(x - x_1)$

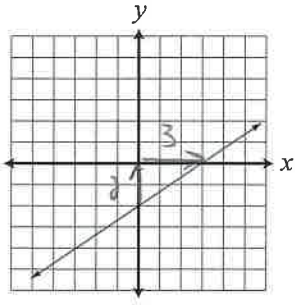
$y - 2 = 2(x - 5)$

$y - 2 = 2x - 10$

$-y \quad +2 \qquad -y \quad +2$

$0 = 2x - y - 8$

b) $(-3, 4)$ $x_1\ y_1$



$m = \frac{2}{3}$

$m_{\perp} = \frac{-3}{2}$

$\left(y - 4 = \frac{-3}{2}(x + 3)\right) \times 2$

$2y - 8 = -3(x + 3)$

$2y - 8 = -3x - 9$

$+3x \quad +9 \qquad +3x \quad +9$

$3x + 2y + 1 = 0$

UNIT 7: SYSTEMS OF EQUATIONS

MY NOTES AND THINGS TO REMEMBER...

UNIT 7: Systems of Linear Equations Multiple Choice

1. Solve for y in the following system of equations:

$$\begin{matrix} x - y = -1 \\ 3x + 5y = 21 \end{matrix} \quad \times -3$$

$\Rightarrow \quad -3x + 3y = 3$
$\quad\quad 3x + 5y = 21$

$\frac{8y}{8} = \frac{24}{8}$

$y = 3$

A. 2
(B) 3
C. 9
D. 12

2. Which of the following systems of linear equations has a solution of $(-3, 4)$?

~~A~~ $\begin{cases} 2x - 3y = 6 \\ y = 3x - 13 \end{cases}$ $2(-3) - 3(4) = 6 \,?\, \Rightarrow -6 - 12 = 6 \,?\, \Rightarrow -18 \neq 6$ X

~~B~~ $\begin{cases} 2x - 3y = 6 \\ y = 3x + 13 \end{cases}$ X

~~C~~ $\begin{cases} 2x + 3y = 6 \\ y = 3x - 13 \end{cases}$ $2(-3) + 3(4) = 6 \,?\, \Rightarrow -6 + 12 = 6 \,?\, \quad 6 = 6$ ✓

$4 = 3(-3) - 13 \,?\, \Rightarrow 4 = -9 - 13 \,?\, \Rightarrow 4 \neq -22$ X

(D.) $\begin{cases} 2x + 3y = 6 \\ y = 3x + 13 \end{cases}$ ✓ $4 = 3(-3) + 13 \,?\, \Rightarrow 4 = -9 + 13 \,?\, \Rightarrow 4 = 4$ ✓

3. Two planes have a cruising speed of 570 km/h without wind. The first plane flies for 12 hours against a constant headwind. The second plane flies for 10 hours in the opposite direction with the same wind (a tailwind). The second plane flies 370 km less than the first plane.

Determine two equations that could be used to solve for the wind speed, w, and the distance travelled by the first plane, d.

A. $(570-w)(12)=d$
$(570+w)(10)=d-370$

B. $(570-w)(12)=d$
$(570+w)(10)=d+370$

C. $(570+w)(12)=d$
$(570-w)(10)=d-370$

D. $(570+w)(12)=d$
$(570-w)(10)=d+370$

① $570+w=\frac{d-370}{10} \Rightarrow 10(570+w)=d-370$

② $570-w=\frac{d}{12} \Rightarrow 12(570-w)=d$

4. How many solutions does this system of equations have?

$y=3x+7$
$y=3x-4$

PARALLEL $\Rightarrow$ NO SOL'N
(same m & different b)

A. no solution
B. one solution
C. an infinite number of solutions
D. cannot be determined without solving

5. Solve for x:

$3x+4y=-16$
$x=4y$
$-4y \quad -4y$

$\Rightarrow$ $3x+4y=-16$
$x-4y=0$

$\frac{4x}{4}=\frac{-16}{4}$

$x=-4$

6. A package of 12 hex bolts and 10 anchor bolts weighs 7 pounds. A second package of 5 hex bolts and 15 anchor bolts weighs 4 pounds. How much does a single hex bolt weigh? Answer in pounds to one decimal place.

Let h = # of hex bolts

Let a = # of anchor bolts

① $12h + 10a = 7$ | $\times -3$

② $5h + 15a = 4$ | $\times 2$ $\Rightarrow$

$-36h - 30a = -21$

$10h + 30a = 8$

$-26h = -13$

$\frac{-26h}{-26} = \frac{-13}{-26}$

$h = 0.5$

① $12h + 10a = 7$

$12(0.5) + 10a = 7$

$6 + 10a = 7$

$10a = 1$

$a = 0.1$

A single hex bolt weighs 0.5 pounds

7. Joey bought 8 books. Some books cost \$12 each the rest cost \$18 each. He spent a total of \$108. Which of the following systems of linear equations could represent the given situation?

A. $x + y = 8$
$12x + 18y = 108$

B. $x + y = 108$
$12x + 18y = 8$

C. $x + 12y = 8$
$x + 18y = 108$

D. $12x + y = 8$
$x + 18y = 108$

$x \rightarrow$ # of \$12 books

$y \rightarrow$ # of \$18 books

① $x + y = 8$

② $12x + 18y = 108$

8. Kim invested a total of $1500 between two bonds. One bond earned 8% per annum and the other bond earned 10% per annum. In one year, Kim earned $132 on her investments. How much did she invest in the bond that earned 10%?

(A) $600
B. $750
C. $900
D. $1000

Let x be the amount invested at 8%.

Let y be the amount invested at 10%.

① $x + y = 1500 \Rightarrow x = 1500 - y$

② $0.08x + 0.10y = 132$

② $0.08x + 0.10y = 132$

$0.08(1500 - y) + 0.10y = 132$

$120 - 0.08y + 0.10y = 132$

$120 + 0.02y = 132$

$-120 \quad -120$

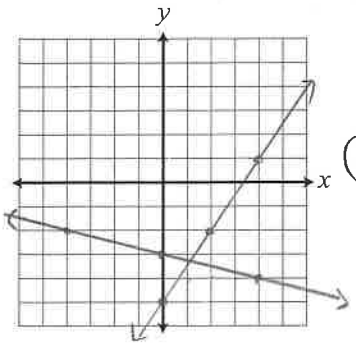
$\frac{0.02y}{0.02} = \frac{12}{0.02}$

$y = 600$

UNIT 7: Systems of Linear Equations Written Response

1. Solve by graphing.

a) ① $3x - 2y = 10$
② $-x + 4y = -12$

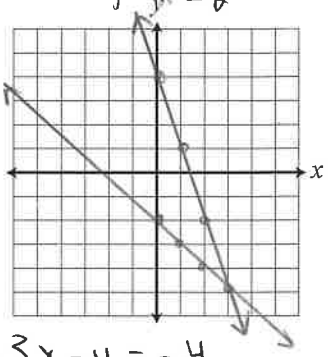


APPROX
(1.1, −3.2)

① $3x - 2y = 10$
$-2y = -3x + 10$
$y = \frac{3}{2}x - 5$

② $-x + 4y = -12$
$4y = x - 12$
$y = \frac{1}{4}x - 3$

b) ① $y + x = -2$
② $3x - y = -8$

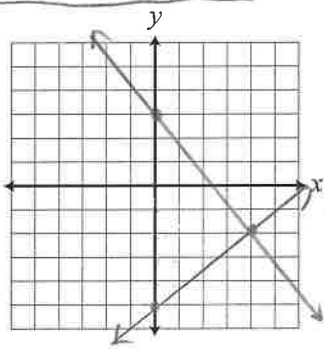


(3, −5)

① $y + x = -2$
$y = -x - 2$

② $3x - y = -4$
$-y = -3x - 4$
$y = -3x + 4$

c) ① $5x + 4y = 12$
② $3x - 4y = 20$

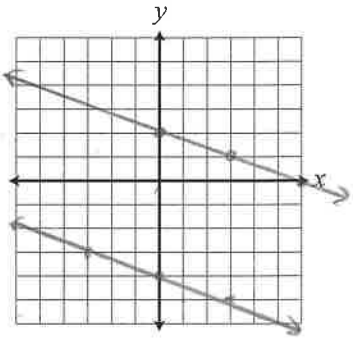


(4, −2)

① $5x + 4y = 12$
$4y = -5x + 12$
$y = \frac{-5}{4}x + 3$

② $3x - 4y = 20$
$-4y = -3x + 20$
$y = \frac{3}{4}x - 5$

d) ① $3x + 9y = 18$
$2x + 6y = -24$



PARALLEL

NO SOLUTION

① $3x + 9y = 18$
$9y = -3x + 18$
$y = -\frac{1}{3}x + 2$

② $2x + 6y = -24$
$6y = -2x - 24$
$y = -\frac{1}{3} - 4$

2. The solution of the system $\left\{\begin{matrix} Ax - 5y = 2 \\ -Ax + By = -8 \end{matrix}\right\}$ is $(-4, 2)$. Find A and B.

(x, y)

① $Ax - 5y = 2$
$A(-4) - 5(2) = 2$
$-4A - 10 = 2$
$+10 \quad +10$
$\frac{-4A}{-4} = \frac{12}{-4}$
$A = -3$

② $-Ax + By = -8$
$-A(-4) + B(2) = 8$
$4A + 2B = 8$
$4(-3) + 2B = 8$
$-12 + 2B = 8$
$2B = 20$
$B = 10$

3. Solve by the elimination method.

a) ① $3x + y = 6$) $\times -1$
② $3x + 2y = 3$

$\Rightarrow -3x - y = -6$
$3x + 2y = 3$
$y = 9$

① $3x + y = 6$
$3x + 9 = 6$
$3x = -3$
$x = -1$

$(-1, 9)$

b) $(3x - y = -7) \times 3$
$2x + 3y = 10$

$\Rightarrow 9x - 3y = -21$
$2x + 3y = 10$
$11x = -11$
$x = -1$

① $3x - y = -7$
$3(-1) - y = -7$
$-3 - y = -7$
$+3 \quad +3$
$-y = -4$
$-1 \quad -1$
$y = 4$

$(-1, 4)$

c) $(2x - 3y = -4) \times 2$
$-4x + 6y = -4$

$\Rightarrow 4x - 6y = -8$
$-4x + 6y = -4$
$0 = -12$ X

NO SOLUTION

(parallel lines)

d) ① $0.2x + 0.3y = 0$
② $3x - 2y = \frac{13}{2}$

① $(0.2x + 0.3y = 0) \times 10$
$\Rightarrow 2x + 3y = 0$

② $(3x - 2y = \frac{13}{2}) \times 2$
$\Rightarrow 6x - 4y = 13$

$(2x + 3y = 0) \times -3$
$6x - 4y = 13$

$-6x - 9y = 0$
$6x - 4y = 13$
$-13y = 13$
$y = -1$

① $2x + 3y = 0$
$2x + 3(-1) = 0$
$2x - 3 = 0$
$2x = 3$
$x = \frac{3}{2}$

$(\frac{3}{2}, -1)$

e) ① $\left(\frac{4}{5}x+\frac{3}{5}y=1\right)\times 5 = 4x+3y=5$

② $\left(\frac{3}{8}x-\frac{1}{4}y=1\right)\times 8 = 3x-2y=8$

$(4x+3y=5)\times 2 \Rightarrow 8x+6y=10$

$(3x-2y=8)\times 3 \Rightarrow 9x+6y=24$

$17x=34$

$x=2$

① $4x+3y=5$

$4(2)+3y=5$

$8+3y=5$

$3y=-3$

$y=-1$

$(2,-1)$

f) ① $3x-4y=6$

② $2(2y+3)=3x \Rightarrow 4y+6=3x$

① $3x-4y=6$

② $4y+6=3x \Rightarrow -3x+4y=-6$

$-3x \quad -6 \quad -3x \quad -6$

$3x-4y=6$

$-3x+4y=-6$

$0=0$ ✓

∞ SOLUTIONS

(same line)

g) ① $x-2y=16$

② $y+3=3x$

① $x-2y=16$

② $y+3=3x \Rightarrow -3x+y=-3$

$-3x \quad -3 \quad -3x \quad -3$

$(x-2y=16)\times 3 \Rightarrow 3x-6y=48$

$-3x+y=-3 \Rightarrow -3x+y=-3$

$-5y=45$

$y=-9$

② $y+3=3x$

$-9+3=3x$

$-6=3x$

$x=-2$

$(-2,-9)$

h) ① $1.5x-0.2y=14$

② $0.4x+17y=89$

① $(1.5x-0.2y=14)\times 10$

$15x-2y=140$

② $(0.4x+17y=89)\times 10$

$4x+170y=890$

$(15x-2y=140)\times -4$

$(4x+170y=890)\times 15$

$\cup$

$-60x+8y=-560$

$60x+2550y=13350$

$2558y=12790$

$y=5$

① $15x-2y=140$

$15x-2(5)=140$

$15x-10=140$

$15x=150$

$x=10$

$(10,5)$

4. Solve by the substitution method.

a) ① $y = 2x + 7$
② $x + 2y = -6$

② $x + 2(2x+7) = -6$

$x + 4x + 14 = -6$

$5x + 14 = -6$

$5x = -20$

$x = -4$

① $y = 2x + 7$

$y = 2(-4) + 7$

$y = -8 + 7$

$y = -1$

$(-4, -1)$

b) ① $2x = 3y - 13$
② $-y = -2x - 7$

② $\frac{-y}{-1} = \frac{-2x-7}{-1}$

$y = 2x + 7$

① $2x = 3y - 13$

$2x = 3(2x+7) - 13$

$2x = 6x + 21 - 13$

$2x = 6x + 8$

$-2x \quad -2x$

$0 = 4x + 8$

$-8 = 4x$

$x = \frac{-8}{4} = -2$

$y = 2x + 7$

$y = 2(-2) + 7$

$y = -4 + 7$

$y = 3$

$(-2, 3)$

c) ① $6x - 3y = 4$
② $y + 2x = 0$

② $y + 2x = 0$

$y = -2x$

① $6x - 3y = 4$

$6x - 3(-2x) = 4$

$6x + 6x = 4$

$12x = 4$

$x = \frac{4}{12} = \frac{1}{3}$

② $y = -2x$

$y = -2\left(\frac{1}{3}\right)$

$y = -\frac{2}{3}$

$\left(\frac{1}{3}, -\frac{2}{3}\right)$

d) ① $6x = 3y + 12$
② $4 = 2x - y$

② $4 = 2x - y$

$4 - 2x = -y$

$y = 2x - 4$

① $6x = 3y + 12$

$6x = 3(2x - 4) + 12$

$6x = 6x - 12 + 12$

$6x = 6x$

$0 = 0$

∞ SOLUTIONS

(same line)

e) ① $2x - 3y = x - y$
② $-2x + 4y = 6$

① $2x - 3y = x - y$
$-x + y \quad -x + y$
$x - 2y = 0$
$x = 2y$

② $-2x + 4y = 6$
$-2(2y) + 4y = 6$
$-4y + 4y = 6$
$0 = 6$ ✗

NO SOLUTIONS
(parallel lines)

f) ① $4x = \frac{1}{2}y - 2$
② $\frac{1}{6}y = 8x - 1$

① $\left(4x = \frac{1}{2}y - 2\right) \times 2 \Rightarrow 8x = y - 4$

② $\left(\frac{1}{6}y = 8x - 1\right) \times 6 \Rightarrow y = 48x - 6$

① $8x = y - 4$
$8x = (48x - 6) - 4$
$8x = 48x - 10$
$-48x \quad -48x$
$-40x = -10$
$x = \frac{-10}{-40} = \frac{1}{4}$

$y = 48x - 6$
$y = 48\left(\frac{1}{4}\right) - 6$
$y = 12 - 6$
$y = 6$

$\left(\frac{1}{4}, 6\right)$

g) ① $y = \frac{x}{3} + \frac{5}{6}$
② $\frac{3x}{2} - \frac{y}{4} = -\frac{11}{12}$

① $\left(y = \frac{x}{3} + \frac{5}{6}\right) \times 6 \Rightarrow 6y = 2x + 5$

② $\left(\frac{3x}{2} - \frac{y}{4} = \frac{-11}{12}\right) \times 12 \Rightarrow 18x - 3y = -11$

① $y = \frac{x}{3} + \frac{5}{6}$

② $18x - 3y = -11$
$18x - 3\left(\frac{x}{3} + \frac{5}{6}\right) = -11$
$\left(18x - x - \frac{5}{2} = -11\right) \times 2$
$36x - 2x - 5 = -11$
$+5 \quad +5$
$34x = -6$
$x = \frac{-6}{34} = \frac{-3}{17}$

① $y = \frac{x}{3} + \frac{5}{6}$
$y = \frac{-3}{17} + \frac{5}{6}$
$y = \frac{-3}{17} \times \frac{1}{3} + \frac{5}{6}$
$y = \frac{-1}{17} \times 6 + \frac{5}{6} \times 17$
$y = \frac{-6}{102} + \frac{85}{102}$
$y = 79/102$

$\left(\frac{-3}{17}, \frac{79}{102}\right)$

h) ① $2x + y = a$
② $x - 2y = b$

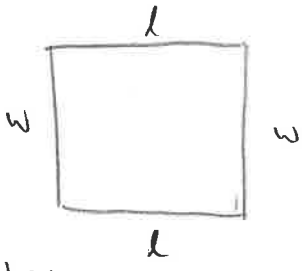
① $2x + y = a$
$y = -2x + a$

② $x - 2y = b$
$x - 2(-2x + a) = b$
$x + 4x - 2a = b$
$5x - 2a = b$
$5x = 2a + b$
$x = \frac{2a + b}{5}$

① $y = -2x + a$
$y = -2\left(\frac{2a + b}{5}\right) + a$
$y = -\frac{4a - 2b}{5} + \frac{5a}{5}$
$y = -\frac{4a - 2b + 5a}{5}$
$y = \frac{a - 2b}{5}$

$\left(\frac{2a + b}{5}, \frac{a - 2b}{5}\right)$

5. Jerry has 150 m of fencing to enclose a rectangular chicken pen. If the length of the pen is 15 m less than twice the width, find the area of the chicken pen.



Let l = length
Let w = width

Area = $l \times w$
= 45×30
= $1350 m^2$

① $2l + 2w = 150$

② $l = 2w - 15$

① $2l + 2w = 150$

$2(2w - 15) + 2w = 150$

$4w - 30 + 2w = 150$

$6w - 30 = 150$

$6w = 180$

$w = 30$

② $l = 2w - 15$

$l = 2(30) - 15$

$l = 60 - 15$

$l = 45$

6. Trudene invests part of her savings of $150 000 in mutual funds that average 8% interest annually. The remainder is invested in bonds that average 7% interest annually. The combined interest earned the first year is $11 500. How much money was invested in each account?

Let x be $ invested at 8%.
Let y be $ invested at 7%.

① $x + y = 150\,000 \Rightarrow y = 150\,000 - x$

② $0.08x + 0.07y = 11\,500$

② $0.08x + 0.07y = 11500$

$0.08x + 0.07(150\,000 - x) = 11500$

$0.08x + 10500 - 0.07x = 11500$

$0.01x = 1000$

$x = 100\,000$

① $y = 150\,000 - x$

$y = 150\,000 - 100\,000$

$y = 50\,000$

$100 000 invested at 8% and $50 000 invested at 7%

7. A boat travelled 60 km downstream in four hours and made the return trip in five hours. Find the speed of the boat in still water.

Let b = speed of boat
Let c = speed of current

① $b + c = \frac{60}{4} \Rightarrow b + c = 15$

② $b - c = \frac{60}{5} \Rightarrow b - c = 12$

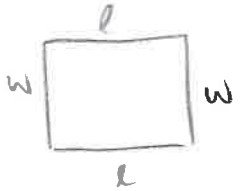
$b + c = 15$
$b - c = 12$

$2b = 27$

$b = 13.5$

The speed of the boat is 13.5 km/hr

8. The perimeter of a basketball court is 288 ft, and the length of the court is 44 ft longer than its width. What are the dimensions of a basketball court?



① $2l + 2w = 288$

② $l = 44 + w$

Let l = length

Let w = width

① $2l + 2w = 288$

$2(44 + w) + 2w = 288$

$88 + 2w + 2w = 288$

$4w = 200$

$w = 50$

② $l = 44 + w$

$l = 44 + 50$

$l = 90$

The length is 90ft and the width is 50ft

*

9. The sum of the digits of a two-digit number is 12. The number formed by reversing the digits is 36 more than the original number. Find the original number.

Let the two digit number be $10x + y$

($10x$: tens place; y: ones place)

① $x + y = 12 \Rightarrow y = 12 - x$

② $10y + x = 36 + (10x + y)$

$10y + x = 36 + 10x + y$

$-y \quad -x \qquad -x \quad -y$

$9y = 36 + 9x$

$9(12 - x) = 36 + 9x$

$108 - 9x = 36 + 9x$

$+9x \qquad +9x$

$108 = 36 + 18x$

$-36 \qquad -36$

$72 = 18x$

$x = 4$

① $y = 12 - x$

$y = 12 - 4$

$y = 8$

The original number is $10(4) + 8 = 48$

UNIT 8: ARITHMETIC SEQUENCES & SERIES

MY NOTES AND THINGS TO REMEMBER...

UNIT 8: Arithmetic Sequences & Series Written Response

1. Determine the 25th term of the arithmetic sequence $-2, -8, -14, -20, \ldots$

$t_1 = -2$
$d = -6$
$n = 25$
$t_{25} = ?$

$t_n = t_1 + (n-1)d$
$t_{25} = -2 + (25-1)(-6)$
$= -2 + 24(-6)$
$= -2 - 144$

$\boxed{t_{25} = -146}$

2. Determine the number of terms in the arithmetic sequence $5, 1, -3, \ldots, -111$.

$t_1 = 5$
$d = -4$
$n = ?$
$t_n = -111$

$t_n = t_1 + (n-1)d$
$-111 = 5 + (n-1)(-4)$
$-111 = 5 - 4n + 4$
$-111 = 9 - 4n$
$-9 \quad -9$
$-120 = -4n$
$\frac{-120}{-4} = \frac{-4n}{-4}$

$\boxed{n = 30}$

3. An auditorium has eight seats in the first row. Each subsequent row has four more seats than the previous row. How many seats are in the 28th row?

8, 12, 16, ...

$t_1 = 8$
$d = 4$
$n = 28$
$t_{28} = ?$

$t_n = t_1 + (n-1)d$
$t_{28} = 8 + (28-1)(4)$
$= 8 + 27(4)$
$= 8 + 108$

$\boxed{t_{28} = 116}$

4. Determine the sum of the arithmetic series $5 + 1 + (-3) + \cdots + (-111)$.

$t_1 = 5$
$d = -4$
$t_n = -111$
$n = ?$

① Find n:
$t_n = t_1 + (n-1)d$
$-111 = 5 + (n-1)(-4)$
$-111 = 5 - 4n + 4$
$-111 = 9 - 4n$
$-120 = -4n$
$n = 30$

② $S_n = \frac{n}{2}(t_1 + t_n)$
$S_{30} = \frac{30}{2}(5 - 111)$
$S_{30} = 15(-106) = \boxed{-1590}$

5. Find the sum of the first 30 terms of the series $10 + 14 + 18 + \cdots$.

$t_1 = 10$
$d = 4$
$n = 30$
$S_{30} = ?$

$S_n = \frac{n}{2}[2t_1 + (n-1)d]$
$S_{30} = \frac{30}{2}[2(10) + (30-1)(4)]$
$= 15(20 + (29)(4)]$
$= 15(20 + 116)$
$= 15(136)$

$\boxed{S_{30} = 2040}$

6. Find the sum of all multiples of 6 between 100 and 1000.

102 + 108 + 114 + 120 + ... + 996

$t_1 = 102$
$d = 6$
$t_n = 996$
$n = ?$

① Find n:
$t_n = t_1 + (n-1)d$
$996 = 102 + (n-1)(6)$
$996 = 102 + 6n - 6$
$996 = 96 + 6n$
$900 = 6n$
$n = 150$

② $S_n = \frac{n}{2}(t_1 + t_n)$
$= \frac{150}{2}(102 + 996)$
$= 75(1098)$
$= \boxed{82350}$